



MASTER IN ENTREPRENEURSHIP
INNOVATION MANAGEMENT
IN COLLABORATION WITH **MIT SLOAN**

IN COLLABORATION WITH

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UNIVERSITÀ DEGLI STUDI DI NAPOLI
PARTHENOPE

MASTER MEIM 2022-2023

Python Programming Course

Lesson 6

Functions and Data Structures

Lesson given by prof. Mariacarla Staffa

Prof. Computer Science at the University of Naples Parthenope

OVERVIEW

- structuring programs and hiding details
- functions
- specifications
- keywords: return **vs** print
- scope

HOW DO WE WRITE CODE?

- so far...
 - covered language mechanisms
 - know how to write different files for each computation
 - each file is some piece of code
 - each code is a sequence of instructions

- problems with this approach
 - easy for small-scale problems
 - messy for larger problems
 - hard to keep track of details
 - how do you know the right info is supplied to the right part of code

GOOD PROGRAMMING

- more code not necessarily a good thing
- measure good programmers by the amount of functionality
- introduce **functions**
- mechanism to achieve **decomposition** and **abstraction**

EXAMPLE – PROJECTOR

- a projector is a black box
- don't know how it works
- know the interface: input/output
- connect any electronic to it that can communicate with that input
- black box somehow converts image from input source to a wall, magnifying it
- **ABSTRACTION IDEA:** do not need to know how projector works to use it

EXAMPLE – PROJECTOR

- projecting large image for Olympics decomposed into separate tasks for separate projectors
- each projector takes input and produces separate output
- all projectors work together to produce larger image
- **DECOMPOSITION IDEA:** different devices work together to achieve an end goal



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.....APPLY THESE CONCEPTS
TO PROGRAMMING!

Create Structures with **DECOMPOSITION**

- in projector example, separate devices
- in programming, divide code into **modules**
 - are **self-contained**
 - used to **break up** code
 - intended to be **reusable**
 - keep code **organized**
 - keep code **coherent**
- this lecture, achieve decomposition with **functions**
- in next lecture , achieve decomposition with **classes**

Suppress Details with **ABSTRACTION**

- in projector example, instructions for how to use it are sufficient, no need to know how to build one
- in programming, think of a piece of code as a **black box**
 - cannot see details
 - do not need to see details
 - do not want to see details
 - hide tedious coding details
- achieve abstraction with **function specifications** or **docstrings**

FUNCTIONS

- write reusable pieces/chunks of code, called **functions**
- functions are not run in a program until they are “**called**” or “**invoked**” in a program
- function characteristics:
 - has a **name**
 - has **parameters** (0 or more)
 - has a **docstring** (optional but recommended)
 - has a **body**
 - **returns** something

HOW TO WRITE and CALL/Invoke A FUNCTION

```
def is_even( i ):  
    """  
        Input: i, a positive int  
        Returns True if i is even, otherwise False  
    """
```

```
    print("inside is_even")  
    return i%2 == 0
```

```
is_even(3)
```

later in the code, you call the
function using its name and
values for parameters

IN THE FUNCTION BODY

```
def is_even( i ):  
    """
```

Input: *i*, a positive int

Returns True if *i* is even, otherwise False

```
"""
```

```
    print("inside is_even")
```

```
    return i%2 == 0
```

keyword

expression to
evaluate and return

run some
commands

VARIABLE SCOPE

- **formal parameter** gets bound to the value of **actual parameter** when function is called
- new **scope/frame/environment** created when enter a function
- **scope** is mapping of names to objects

```
def f( x ):    formal  
    x = x + 1 parameter  
    print('in f(x): x =', x)  
    return x
```

x = 3
z = f(x) actual parameter

Function definition

Main program code
* initializes a variable x
* makes a function call f(x)
* assigns return of function to variable z

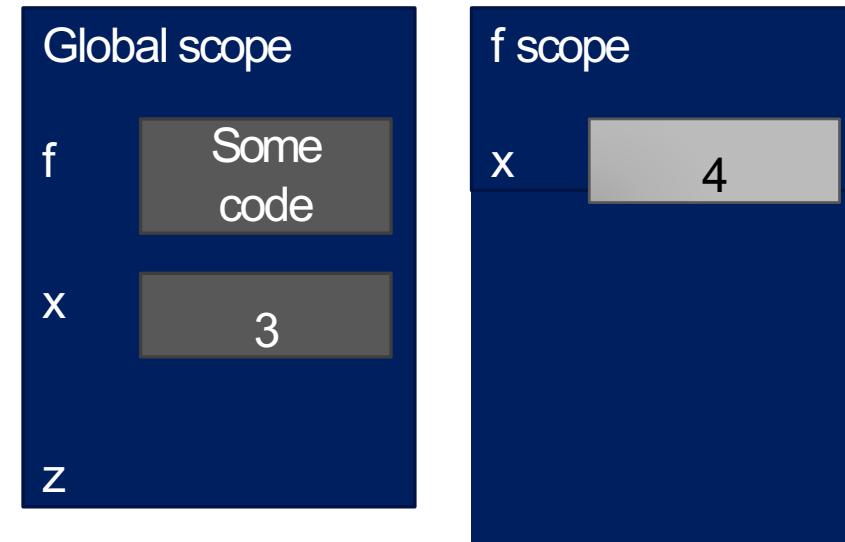
VARIABLE SCOPE

```
def f( x ):  
    x = x + 1  
    print('in f(x) : x = ', x)  
    return x  
  
x = 3  
z = f( x )
```



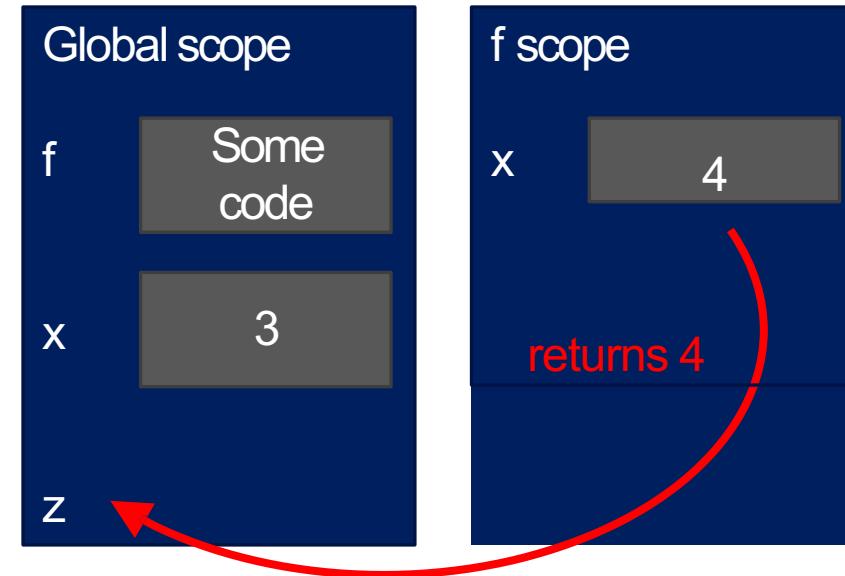
VARIABLE SCOPE

```
def f( x ):  
    x = x + 1  
    print('in f(x) : x =', x)  
    return x  
  
x = 3  
z = f( x )
```



VARIABLE SCOPE

```
def f( x ):  
    x = x + 1  
    print('in f(x) : x =', x)  
    return x  
  
x = 3  
z = f( x )
```



VARIABLE SCOPE

```
def f( x ):  
    x = x + 1  
    print('in f(x): x =', x)  
    return x  
  
x = 3  
z = f( x )
```



ONE WARNING IF NO return STATEMENT

```
def is_even( i ):  
    """  
        Input: i, a positive int  
        Does not return anything  
    """
```

```
i%2 == 0
```

*without a return
statement*

- Python returns the value **None**, if no **return** given
- represents the absence of a value

return

- return only has meaning **inside** a function
- only **one** return executed inside a function
- code inside function but after return statement not executed
- has a value associated with it, **given to function caller**

vs

print

- print can be used **outside** functions
- can execute **many** print statements inside a function
- code inside function can be executed after a print statement
- has a value associated with it, **outputted** to the console

FUNCTIONS AS ARGUMENTS

- arguments can take on any type, even functions

```
def func_a():  
    print 'inside func_a'  
  
def func_b(y):  
    print 'inside func_b'  
    return y  
  
def func_c(z):  
    print 'inside func_c'  
    return z()  
  
print func_a()  
print 5 + func_b(2)  
print func_c(func_a)
```

call func_a, takes no parameters
call func_b, takes one parameter
call func_c, takes one parameter, another function

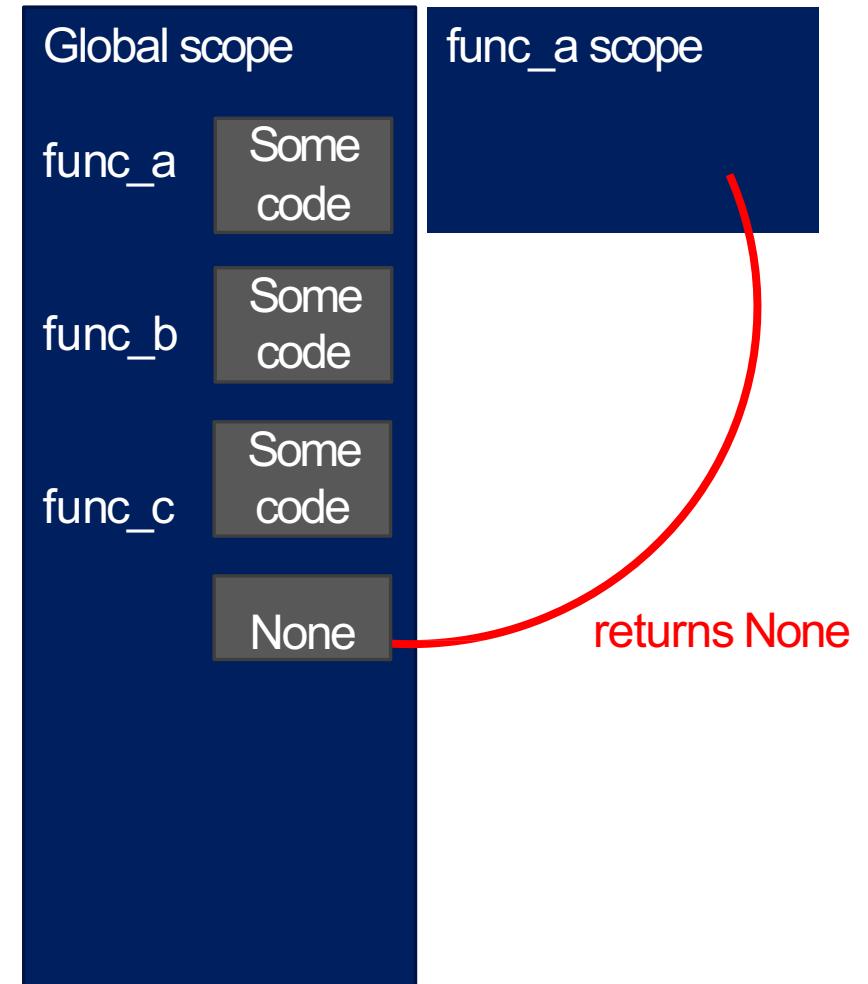
Functions As Arguments

```
def func_a():
    print 'inside func_a'

def func_b(y):
    print 'inside func_b'
    return y

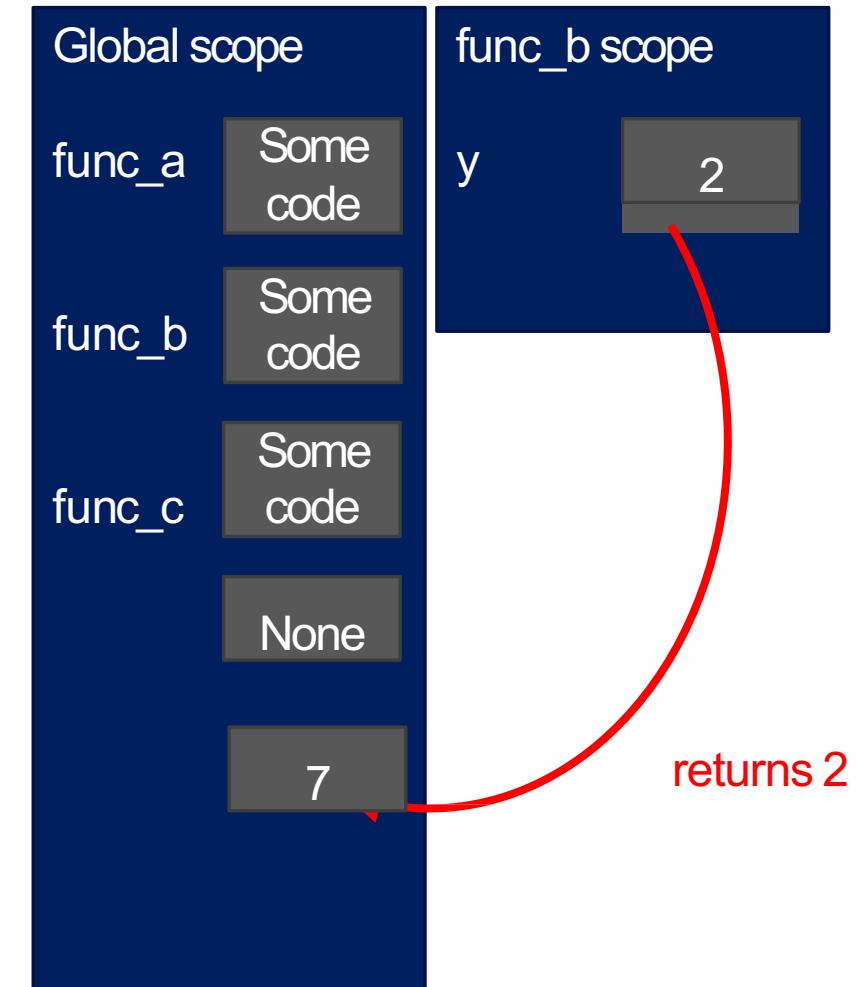
def func_c(z):
    print 'inside func_c'
    return z()

print func_a()
print 5 + func_b(2)
print func_c(func_a)
```



Functions As Arguments

- def func_a():
- print 'inside func_a' def func_b(y):
 - print 'inside func_b' return y
- def func_c(z):
 - print 'inside func_c' return z()
- print func_a() print 5 + func_b(2) print func_c(func_a)



Functions As Arguments

```

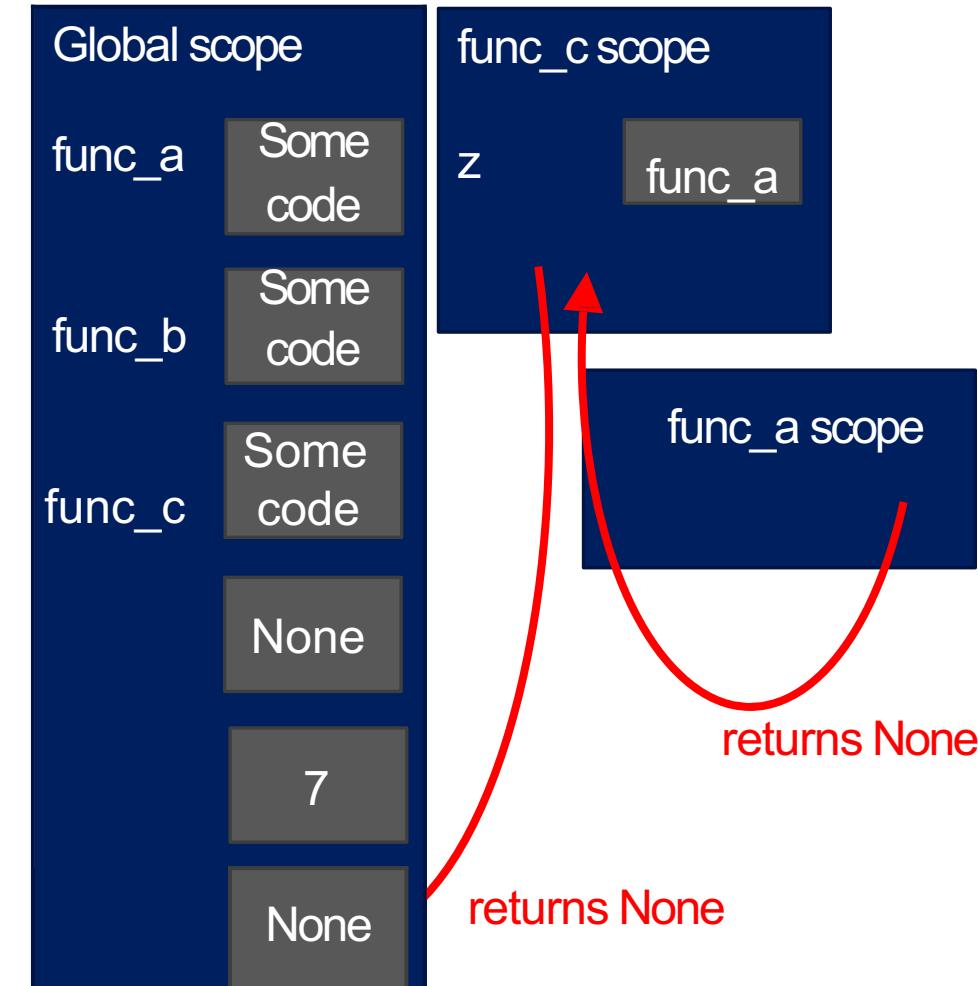
def func_a():
    print 'inside func_a'

def func_b(y):
    print 'inside func_b'
    return y

def func_c(z):
    print 'inside func_c'
    return z()

print func_a()
print 5 + func_b(2)
print func_c(func_a)

```



Scope Example

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside -- can use **global variables**, but frowned upon

```
def f(y):  
    x = 1  
    x += 1  
    print(x)  
  
    x = 5  
  
f(x)  
print(x)
```

*x is re-defined
in scope of f*

*different x
objects*

```
def g(y):  
    print(x)  
    print(x + 1)  
  
    x = 5  
  
g(x)  
print(x)
```

*x from
outside g*

*x inside g is picked up
from scope that called
function g*

```
def h(y):  
    x += 1  
  
    x = 5  
  
h(x)  
print(x)
```

*UnboundLocalError: local variable
'x' referenced before assignment*

SCOPE EXAMPLE

- inside a function, **can access** a variable defined outside
- inside a function, **cannot modify** a variable defined outside -- can use **global variables**, but frowned upon

```
def f(y):
    x = 1
    x += 1
    print(x)

x = 5
f(x)
print(x)
```

```
def g(y):
    print(x)

x = 5
g(x)
print(x)
```

```
def h(y):
    x += 1

x = 5
h(x)
print(x)
```

↳ from
global/main
program scope

Harder Scope Example



IMPORTANT
and
TRICKY!

*Python Tutor is your best friend to
help sort this out!*

<http://www.pythontutor.com/>

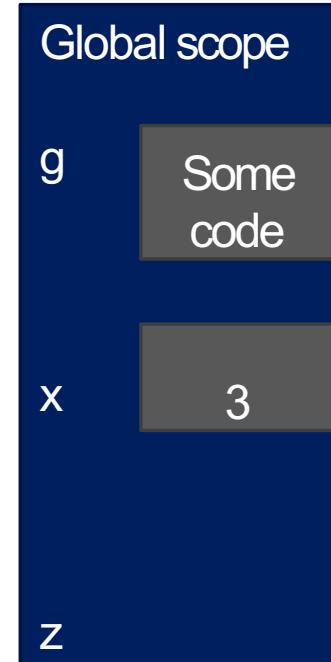
Scope Details

```
def g(x):
    def h():
        x = 'abc'
        x = x + 1
        print('g: x =', x)
        h()
    return x
```

x = 3

z = g(x)

Some code

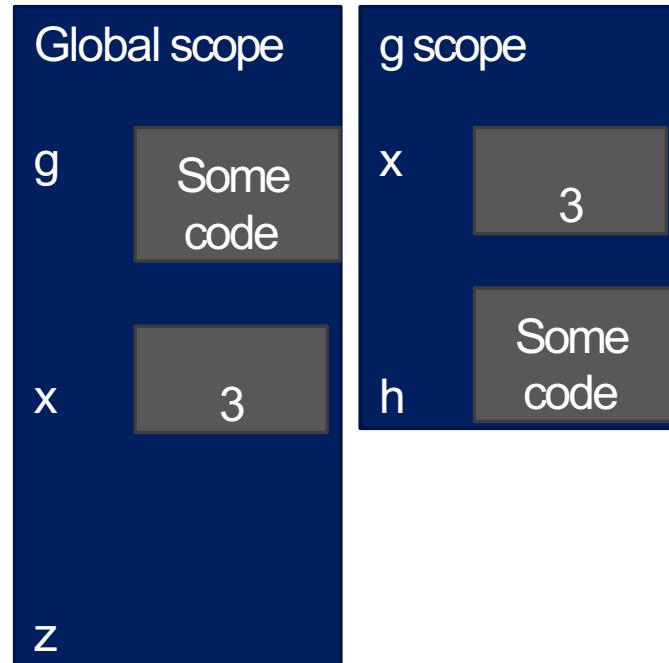


Scope Details

```
def g(x):
    def h():
        x = 'abc'
        x = x + 1
        print('g: x =', x)
        h()
    return x
```

x = 3

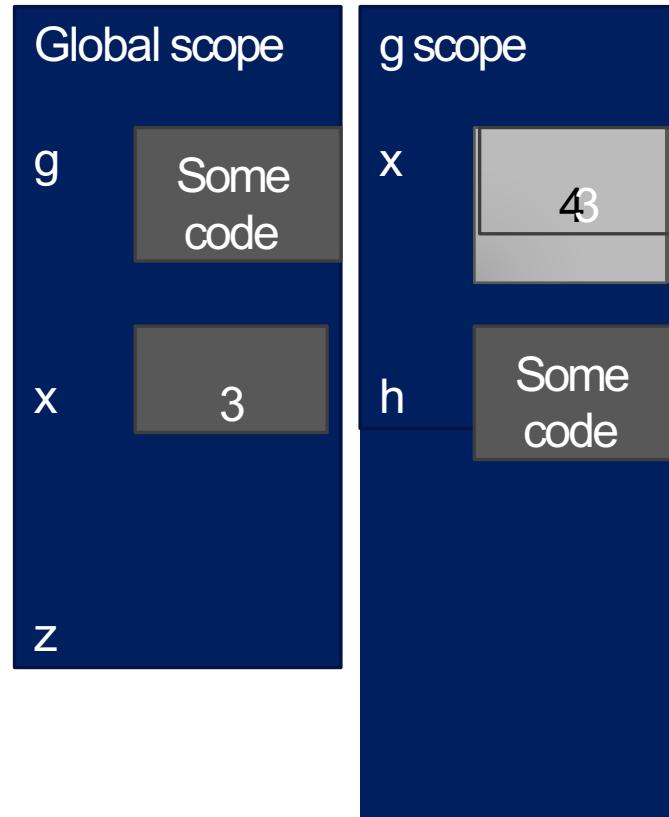
z = g(x)



Scope Details

```
def g(x):
    def h():
        x = 'abc'
        x = x + 1
        print('g: x =', x)
        h()
    return x

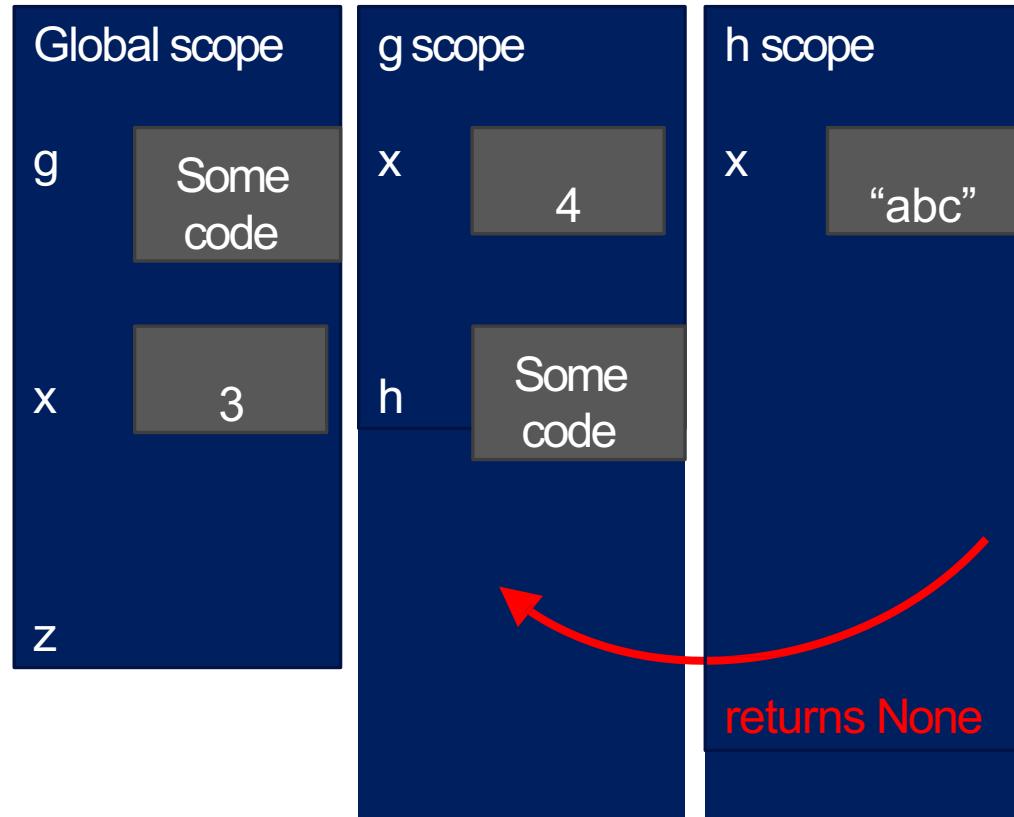
x = 3
z = g(x)
```



Scope Details

```
def g(x):
    def h():
        x = 'abc'
        x = x + 1
        print('g: x =', x)
        h()
    return x

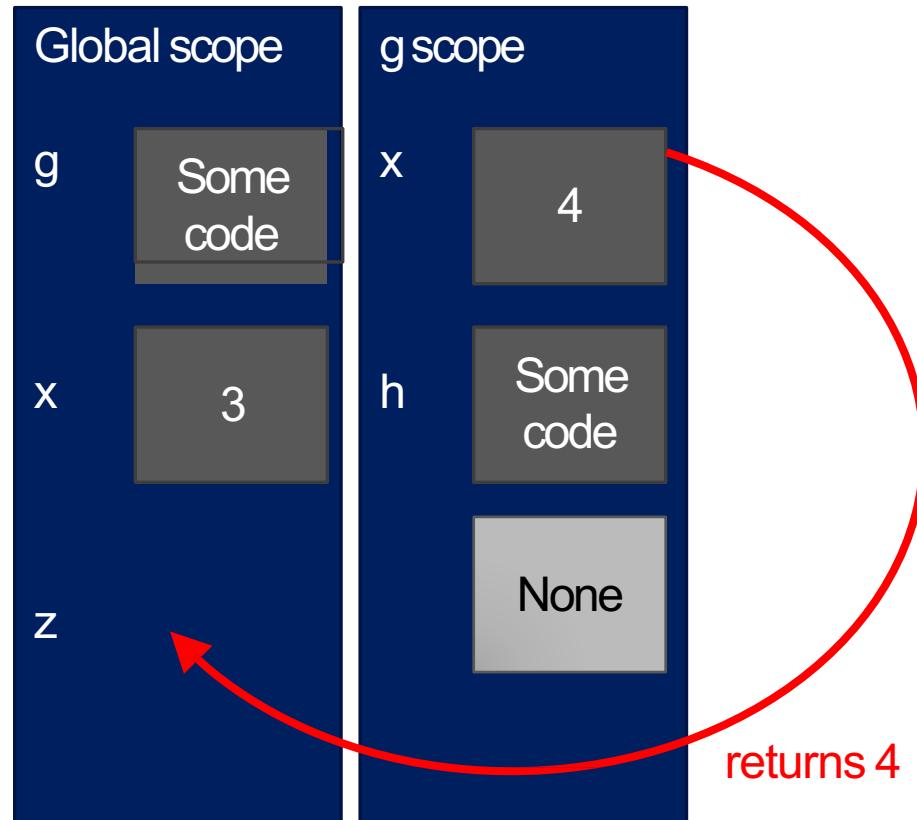
x = 3
z = g(x)
```



Scope Details

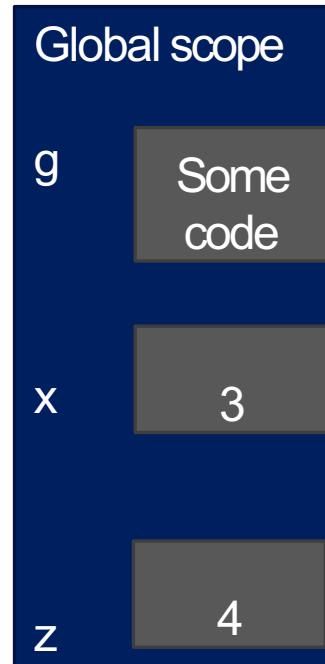
```
def g(x):
    def h():
        x = 'abc'
        x = x + 1
        print('g: x =', x)
        h()
    return x

x = 3
z = g(x)
```



Scope Details

```
def g(x):  
    def h():  
        x = 'abc'  
        x = x + 1  
        print('g: x =', x)  
        h()  
    return x  
  
x = 3  
z = g(x)
```





DECOMPOSITION & ABSTRACTION

- powerful together
- code can be used many times but only has to be debugged once!



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Data Structures

OVERVIEW

- have seen **variable types**: int, float, bool, string
- introduce new **compound data types**
 - tuples
 - lists
- idea of aliasing
- idea of mutability
- idea of cloning

TUPLES

- an ordered sequence of elements, can mix element types
- cannot change element values, **immutable**
- represented with parentheses

te = `()` empty tuple

t = `(2, "mit", 3)`

`t[0]` → evaluates to 2

`(2, "mit", 3) + (5, 6)` → evaluates to `(2, "mit", 3, 5, 6)`

`t[1:2]` → slice tuple, evaluates to `("mit",)`

`t[1:3]` → slice tuple, evaluates to `("mit", 3)`

`len(t)` → evaluates to 3

`t[1] = 4` → gives error, can't modify object

remember
strings?

extra comma
means a tuple
with one element

TUPLES

- conveniently used to **swap** variable values

x = y

y = x



temp = x

x = y

y = temp



(x, y) = (y, x)



- used to **return more than one value** from a function

```
def quotient_and_remainder(x, y):  
    q = x // y  
    r = x % y  
    return (q, r)
```

integer
division

```
(quot, rem) = quotient_and_remainder(4, 5)
```

MANIPULATING TUPLES

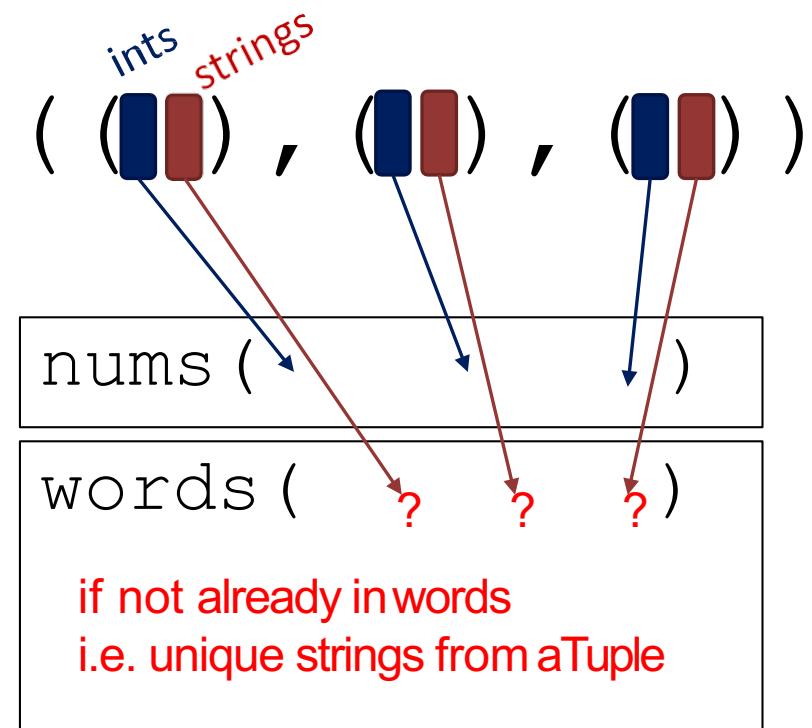
- can **iterate** over tuples

```
def get_data(aTuple):  
    nums = ()  
    words = ()  
    for t in aTuple:  
        nums = nums + (t[0],)  
        if t[1] not in words:  
            words = words + (t[1],)  
    min_n = min(nums)  
    max_n = max(nums)  
    unique_words = len(words)  
    return (min_n, max_n, unique_words)
```

empty tuple

singleton tuple

aTuple: ((blue box, red box), (blue box, red box), (blue box, red box))



LISTS

- **ordered sequence** of information, accessible by index
- a list is denoted by **square brackets**, []
- a list contains **elements**
 - usually homogeneous (ie, all integers)
 - can contain mixed types (not common)
- list elements can be changed so a list is **mutable**

INDICES AND ORDERING

```
a_list = []  
empty list  
L = [2, 'a', 4, [1,2]]
```

`len(L)` → evaluates to 4

`L[0]` → evaluates to 2

`L[2]+1` → evaluates to 5

`L[3]` → evaluates to `[1, 2]`, another list!

`L[4]` → gives an error

`i = 2`

`L[i-1]` → evaluates to 'a' since `L[1] = 'a'` above

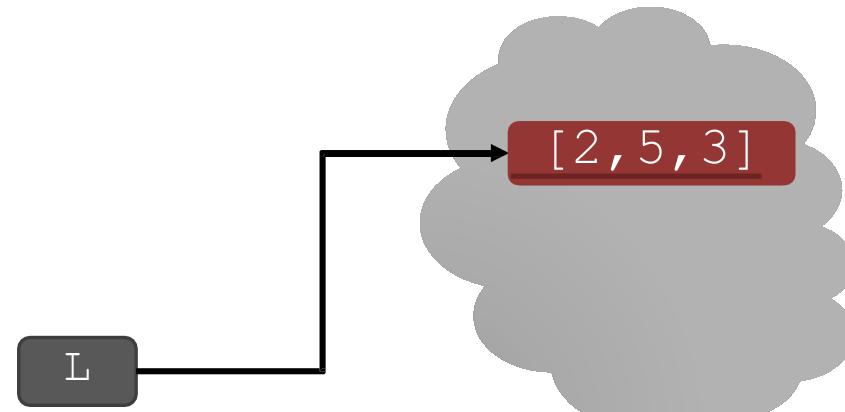
CHANGING ELEMENTS

- lists are **mutable**!
- assigning to an element at an index changes the value

L = [2, 1, 3]

L[1] = 5

- L is now [2, 5, 3], note this is the **same object** L



Iterating over a List

- compute the **sum of elements** of a list
- common pattern, iterate over list elements

```
total = 0
for i in range(len(L)):
    total += L[i]
print total
```

```
total = 0
for i in L:
    total += i
print total
```

like strings,
can iterate
over list
elements
directly

- notice
 - list elements are indexed 0 to `len(L) - 1`
 - `range(n)` goes from 0 to `n - 1`

OPERATIONS ON LISTS - ADD

- **add** elements to end of list with `L.append(element)`
- **mutates** the list!

`L = [2, 1, 3]`

`L.append(5)` → List now `[2, 1, 3, 5]`

- what is the dot?
 - lists are Python objects, everything in Python is an object
 - objects have data
 - objects have methods and functions
 - access this information by `object_name.do_something()`
 - will learn more about these later

OPERATIONS ON LISTS - ADD

- to combine lists together use **concatenation**, + operator, to give you a new list
- **mutate** list with `L.extend(some_list)`

`L1 = [2, 1, 3]`

`L2 = [4, 5, 6]`

`L3 = L1 + L2` → `L3` is `[2, 1, 3, 4, 5, 6]`
`L1, L2` unchanged

`L1.extend([0, 6])` → mutated `L1` to `[2, 1, 3, 0, 6]`

OPERATIONS ON LISTS- REMOVE

- delete element at a **specific index** with `del(L[index])`
- remove element at **end of list** with `L.pop()`, returns the removed element
- remove a **specific element** with `L.remove(element)`
 - looks for the element and removes it
 - if element occurs multiple times, removes first occurrence
 - if element not in list, gives an error

all these
operations
mutate
the list

```
L = [2,1,3,6,3,7,0] # do below in order
L.remove(2) → mutates L = [1,3,6,3,7,0]
L.remove(3) → mutates L = [1,6,3,7,0]
del(L[1])    → mutates L = [1,3,7,0]
L.pop()      → returns 0 and mutates L = [1,3,7]
```

Convert List to String and Back

- convert **string to list** with `list(s)`, returns a list with every character from `s` an element in `L`
- can use `s.split()`, to **split a string on a character** parameter, splits on spaces if called without a parameter
- use `' '.join(L)` to turn a **list of characters into a string**, can give a character in quotes to add char between every element

<code>s = "I<3 cs"</code>	→ <code>s</code> is a string
<code>list(s)</code>	→ returns <code>['I', '<', '3', ' ', 'c', 's']</code>
<code>s.split('<')</code>	→ returns <code>['I', '3 cs']</code>
<code>L = ['a', 'b', 'c']</code>	→ <code>L</code> is a list
<code>' '.join(L)</code>	→ returns <code>"abc"</code>
<code>'_'.join(L)</code>	→ returns <code>"a_b_c"</code>

Other List Operations

- `sort()` and `sorted()`
- `reverse()`
- and many more! <https://docs.python.org/3/tutorial/datastructures.html>

`L=[9 , 6 , 0 , 3]`

`sorted (L)`

→ returns sorted list, does **not mutate** L

`L.sort ()`

→ **mutates** L = [0 , 3 , 6 , 9]

`L.reverse ()`

→ **mutates** L = [9 , 6 , 3 , 0]

LISTS IN MEMORY

- lists are **mutable**
- behave differently than immutable types
- is an object in memory
- variable name points to object
- any variable pointing to that object is affected
- key phrase to keep in mind when working with lists is **side effects**

AN ANALOGY

- attributes of a person
 - singer, rich
- he is known by many names
- all nicknames point to the **same person**
 - add new attribute to **one nickname** ...

Justin Bieber

singer

rich

troublemaker

- ...**all his nicknames** refer to old attributes AND all new ones

The Bieb

singer

rich

troublemaker

JBeebs

singer

rich

troublemaker

ALIASES

- hot is an **alias** for warm – changing one changes the other!
- append() has a side effect

```

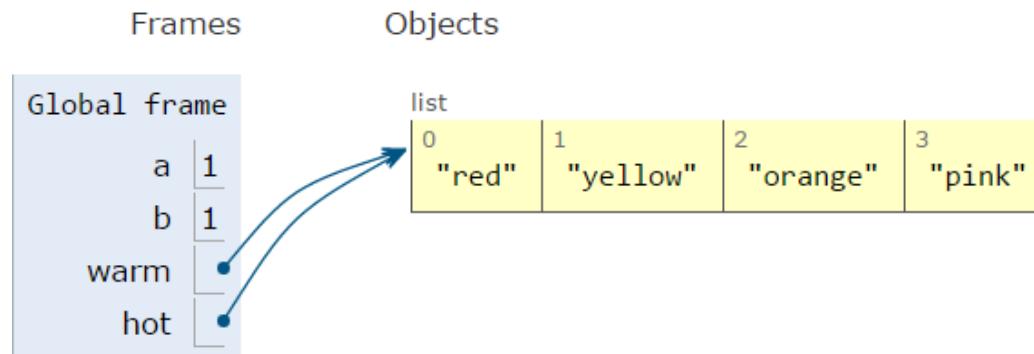
1 a = 1
2 b = a
3 print(a)
4 print(b)
5
6 warm = ['red', 'yellow', 'orange']
7 hot = warm
8 hot.append('pink')
9 print(hot)
10 print(warm)

```

```

1
1
['red', 'yellow', 'orange', 'pink']
['red', 'yellow', 'orange', 'pink']

```

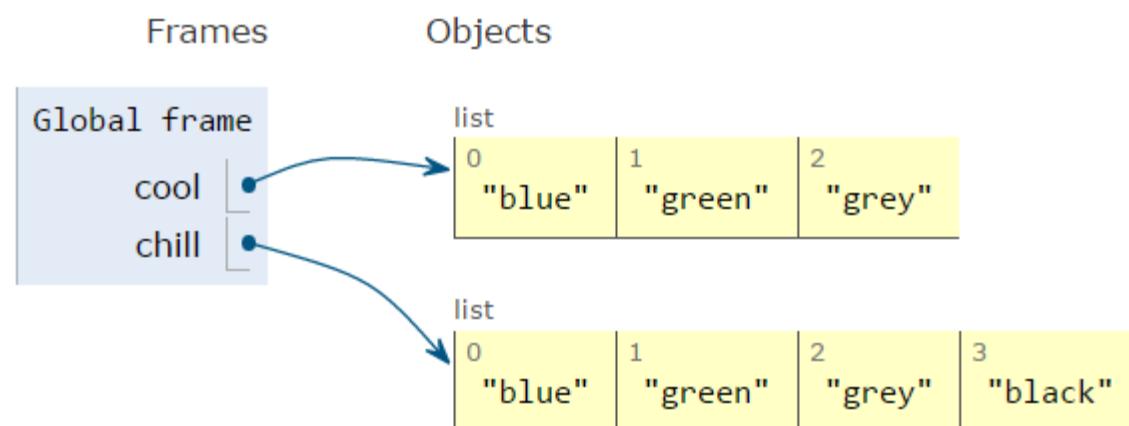


CLONING A LIST

- create a new list and **copy every element** using
chill = cool[:]

```
1 cool = ['blue', 'green', 'grey']
2 chill = cool[:]
3 chill.append('black')
4 print(chill)
5 print(cool)
```

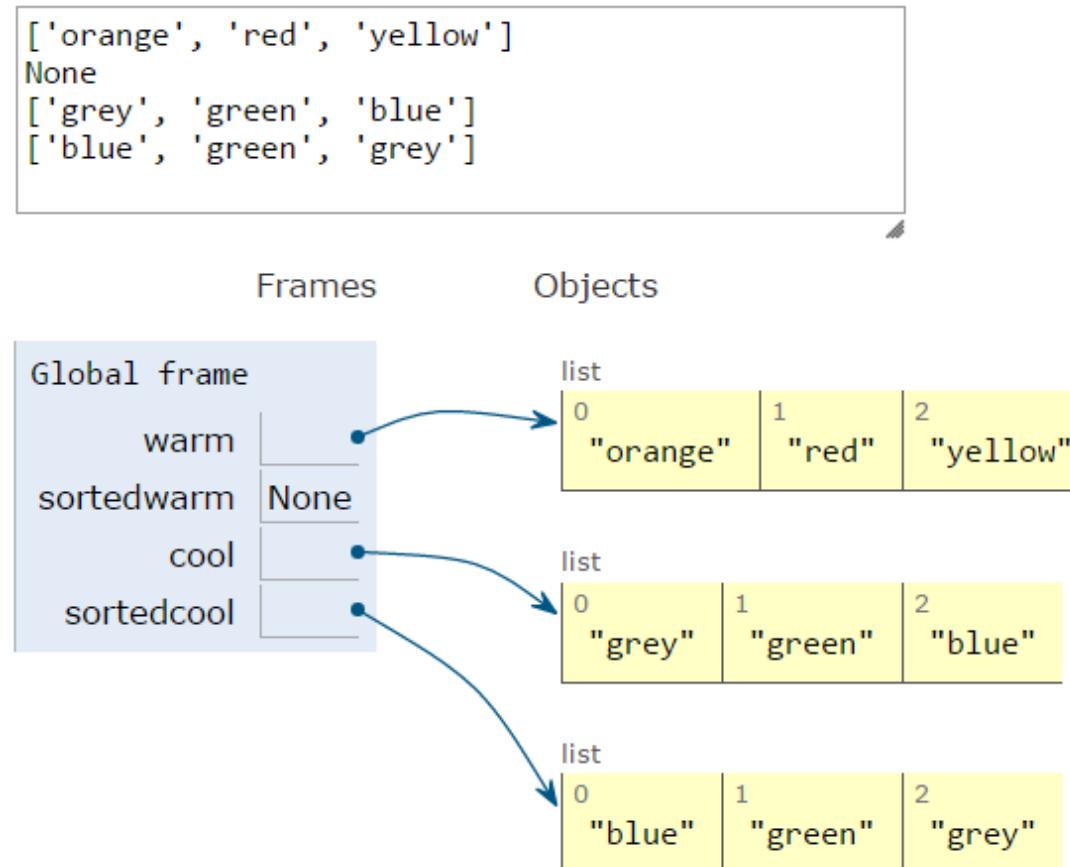
```
['blue', 'green', 'grey', 'black']  
['blue', 'green', 'grey']
```



SORTING LISTS

- calling `sort()` **mutates** the list, returns nothing
- calling `sorted()` **does not mutate** list, must assign result to variable

```
1 warm = ['red', 'yellow', 'orange']
2 sortedwarm = warm.sort()
3 print(warm)
4 print(sortedwarm)
5
6 cool = ['grey', 'green', 'blue']
7 sortedcool = sorted(cool)
8 print(cool)
9 print(sortedcool)
```

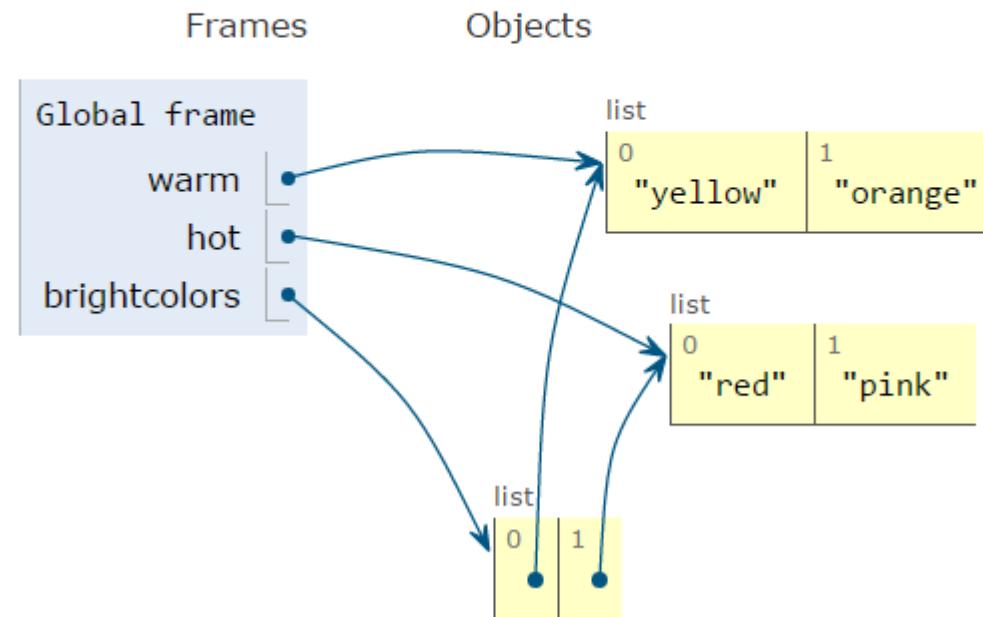


LIST OF LIST OF LIST OF...

- can have **nested** lists
 - side effects still possible after mutation

```
1 warm = ['yellow', 'orange']
2 hot = ['red']
3 brightcolors = [warm]
4 brightcolors.append(hot)
5 print(brightcolors)
6 hot.append('pink')
7 print(hot)
8 print(brightcolors)
```

```
[['yellow', 'orange'], ['red']]  
['red', 'pink']  
[['yellow', 'orange'], ['red', 'pink']]
```



MUTATION AND ITERATION

- **avoid** mutating a list as you are iterating over it

```
def remove_dups(L1, L2):
    for e in L1:
        if e in L2:
            L1.remove(e)
```



```
L1 = [1, 2, 3, 4]
L2 = [1, 2, 5, 6]
remove_dups(L1, L2)
```

- L1 is [2, 3, 4] not [3, 4] Why?

- Python uses an internal counter to keep track of index it is in the loop
- mutating changes the list length but Python doesn't update the counter
- loop never sees element 2



```
def remove_dups(L1, L2):
    L1_copy = L1[:]
    for e in L1_copy:
        if e in L2:
            L1.remove(e)
```

clone list first, note
that L1_copy = L1
does NOT clone



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Dictionaries

HOW TO STORE STUDENT INFO

- so far, can store using separate lists for every info

```
names = ['Ana', 'John', 'Denise', 'Katy']
grade = ['B', 'A+', 'A', 'A']
course = [2.00, 6.0001, 20.002, 9.01]
```

- a **separate list** for each item
- each list must have the **same length**
- info stored across lists at **same index**, each index refers to info for a different person

HOW TO UPDATE/RETRIEVE STUDENT INFO

```
def get_grade(student, name_list, grade_list, course_list): i =  
    name_list.index(student)  
    grade = grade_list[i]  
    course = course_list[i] return (course, grade)
```

messy if have a lot of different info to keep track of

must maintain **many lists** and pass them as arguments

must **always index** using integers

must remember to change multiple lists

A BETTER AND CLEANER WAY – A DICTIONARY

- nice to **index item of interest directly** (not always int)
- nice to use **one data structure**, no separate lists

0	Elem 1
1	Elem 2
2	Elem 3
3	Elem 4
...	...

index element

Key 1	Val 1
Key 2	Val 2
Key 3	Val 3
Key 4	Val 4
...	...

custom
index by
label element

A PYTHON DICTIONARY

- store pairs of data
 - key
 - value

'Ana'	'B'
'Denise'	'A'
'John'	'A+'
'Katy'	'A'

empty dictionary

```
my_dict = {}

grades = {'Ana':'B', 'John':'A+', 'Denise':'A', 'Katy':'A'}
```

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

key1 val1 key2 val2 key3 val3 key4 val4

custom index by label

element

DICTIONARY LOOKUP

- similar to indexing into a list
- **looks up** the **key**
- **returns** the **value** associated with the key
- if key isn't found, get an error

'Ana'	'B'
'Denise'	'A'
'John'	'A+'
'Katy'	'A'

```
grades = {'Ana':'B', 'John':'A+', 'Denise':'A', 'Katy':'A'}
grades['John']      → evaluates to 'A+'
grades['Sylvan']    → gives a KeyError
```

DICTIONARY OPERATIONS

```
grades = { 'Ana': 'B', 'John': 'A+', 'Denise': 'A', 'Katy': 'A' }
```

- **add** an entry

```
grades['Sylvan'] = 'A'
```

- **test** if key in dictionary

```
'John' in grades
'Daniel' in grades
```

- **delete** entry

```
del(grades['Ana'])
```

→ returns True
→ returns False

'Ana'	'B'
'Denise'	'A'
'John'	'A+'
'Katy'	'A'
'Sylvan'	'A'

DICTIONARY OPERATIONS

```
grades = {'Ana': 'B', 'John': 'A+', 'Denise': 'A', 'Katy': 'A'}
```

- get an **iterable that acts like a tuple of all keys**

grades.keys() → returns ['Denise', 'Katy', 'John', 'Ana']

*no guaranteed
order*

- get an **iterable that acts like a tuple of all values**

grades.values() → returns ['A', 'A', 'A+', 'B']

*no guaranteed
order*

'Ana'	'B'
'Denise'	'A'
'John'	'A+'
'Katy'	'A'

DICTIONARY KEYS and VALUES

- values
 - any type (**immutable and mutable**)
 - can be **duplicates**
 - dictionary values can be lists, even other dictionaries!
- keys
 - must be **unique**
 - **immutable** type (`int, float, string, tuple, bool`)
 - actually need an object that is **hashable**, but think of as immutable as all immutable types are hashable
 - careful with `float` type as a key
- **no order** to keys or values!

```
d = {4:{1:0}, (1,3) :"twelve",
'const':[3.14,2.7,8.44] }
```

Comparison between Lists and Dictionaries

list

vs

dict

- **ordered** sequence of elements
- look up elements by an integer index
- indices have an **order**
- index is an **integer**

- **matches** “keys” to “values”
- look up one item by another item
 - **no order** is guaranteed
 - key can be any **immutable** type

Functions to analyze Song Lyrics

- 1) create a **frequency dictionary** mapping str:int
- 2) find **word that occurs the most** and how many times
 - use a list, in case there is more than one word
 - return a tuple (list, int) for (words_list, highest_freq)
- 3) find the **words that occur at least X times**
 - let user choose “at least X times”, so allow as parameter
 - return a list of tuples, each tuple is a (list, int) containing the list of words ordered by their frequency
 - IDEA: From song dictionary, find most frequent word. Delete most common word. Repeat. It works because you are mutating the song dictionary.

Creating a Dictionary

```
def lyrics_to_frequencies(lyrics):
    myDict = {}
    for word in lyrics:
        if word in myDict:
            myDict[word] += 1
        else:
            myDict[word] = 1
    return myDict
```

can iterate over list
in dictionary

update value
associated with key

Using -the Dictionary

```
def most_common_words(freqs):  
    values = freqs.values()  
    best = max(values)      this is an iterable, so can  
apply built-in function  
    words = []  
    for k in freqs:         can iterate over keys  
in dictionary  
        if freqs[k] == best:  
            words.append(k)  
    return (words, best)
```

LEVERAGING DICTIONARY PROPERTIES

```
def words_often(freqs, minTimes):
    result = []
    done = False
    while not done:
        temp = most_common_words(freqs)
        if temp[1] >= minTimes:
            result.append(temp)
            for w in temp[0]:
                del(freqs[w])
        else:
            done = True
    return result

print(words_often(beatles, 5))
```

can directly mutate
dictionary; makes it
easier to iterate

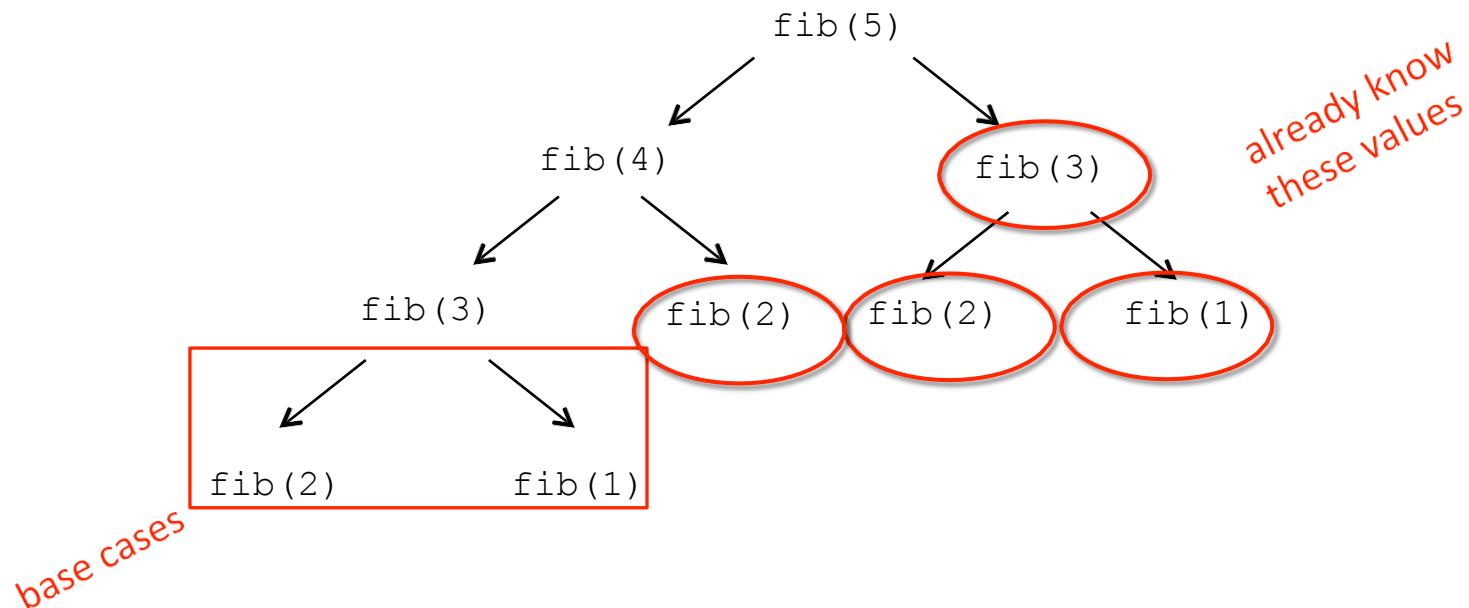
FIBONACCI: Recursive Code

```
def fib(n):  
    if n == 1:  
        return 1  
    elif n == 2:  
        return 2  
    else:  
        return fib(n-1) + fib(n-2)
```

- two base cases
- calls itself twice
- this code is inefficient

Inefficient FIBONACCI

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$



- **recalculating** the same values many times!
- could keep **track** of already calculated values

FIBONACCI with a Dictionary

- ```
def fib_efficient(n, d): if n in d:
```

    - `return d[n]`
    - `else:`
    - `ans = fib_efficient(n-1, d) + fib_efficient(n-2, d)`
    - `d[n] = ans`
    - `return ans`
  - `d = {1:1, 2:2}`
  - `print(fib_efficient(6, d))`
- 
- do a **lookup first** in case already calculated the value
  - **modify dictionary** as progress through function calls
- Method sometimes called "memoization"*
- Initialize dictionary with base cases*

# EFFICIENCY GAINS

- Calling `fib(34)` results in 11,405,773 recursive calls to the procedure
- Calling `fib_efficient(34)` results in 65 recursive calls to the procedure
- Using dictionaries to capture intermediate results can be very efficient
- But note that this only works for procedures without side effects (i.e., the procedure will always produce the same result for a specific argument independent of any other computations between calls)



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# Thank you for your attention



KEEP  
CALM  
AND  
LEARN  
PYTHON