Exercises

SC2_14 – Fourier Transform.

1. The following MATLAB code computes and displays a **rect pulse**, its double and triple convolutions, using the `conv()` function for the convolution between two vectors:

```
a=-2; b=2; T=b-a; t=a:.01:b; N=numel(t)-1;
R=rectpuls(t);
RR=conv(R,R,'same')*T/N;   % double convolution
RRR=conv(RR,R,'same')*T/N; % triple convolution
plot(t,R,'b'); axis equal; hold on
plot(t,RR,'r')
plot(t,RRR,'g')
```

   Compute the two convolutions by means of a **DFT** and by means of the `cconv()` function. Display the results.

2. The following MATLAB code computes and displays a **noisy rect pulse**, its double and triple convolutions, using the `conv()` function for the convolution between two vectors:

```
a=-2; b=2; T=b-a; t=a:.01:b; N=numel(t)-1;
rng('default')
R=rand(size(t)).*rectpuls(t); % with uniform random noise
RR=conv(R,R,'same')*T/N;      % double convolution
RRR=conv(RR,R,'same')*T/N;    % triple convolution
plot(t,R,'b'); axis equal; hold on
plot(t,RR/max(RR),'r')
plot(t,RRR/max(RRR),'g')
```

   Compute the two convolutions by means of a **DFT** and by means of the `cconv()` function. Display the results.

3. Explain why, from sinc(ω) being the Fourier Transform of the rect pulse function, it follows that the Fourier Transform of the triangular pulse function is $\text{sinc}^2(\omega)$.

4. The following symbolic code shows the effects of Windowing error:

```
syms t real;  f=exp(-abs(t)/2);          figure; fplot(f,[-10 10])
L=10;  w=rectangularPulse(-L/2,+L/2,t);  figure; fplot(w,[-10 10])
figure; fplot( f*w, [-10 10])
F=fourier(f);    figure; fplot(abs(F),[-10 10],'Color','k')
W=fourier(w);    figure; fplot(abs(W),[-10 10],'Color','r')
G=fourier(f*w); figure; fplot(abs(G),[-10 10],'Color','b')
```

   What happens for `L=5`?

5. Why, by sampling $2\cos(t)+\sin(5t)$ at a frequency $2/\pi$ < Nyquist frequency, are we able to reconstruct $2\cos(t)+\sin(t)$, and not $2\cos(t)+\sin(2t)$, or $2\cos(t)+\sin(3t)$, or $2\cos(t)+\sin(4t)$?

6. What is the Nyquist frequency of the signal $\cos(x)+\sin(10x)$? Check it by means of the Fourier Transform and Sampling Theorem. Display what happens with samples at a lower frequency.

7. How can we avoid an Aliasing Error in estimating numerically the Fourier Spectrum from a sample of the signal function $f(t)=\cos(1000\pi t)$ in the interval $[0,0.1]$? And how can we produce Aliasing Error? Write a MATLAB code to produce an example for both the questions.

8. The following MATLAB code displays informations about an audio file and then loads it into the MATLAB Workspace:

```
fileName='Star_Wars.mp3';
I=audioinfo(fileName)
[y,Fs]=audioread(fileName);
```

Download zip files containing some audio files from the Course page on the eLearning platform. Write a MATLAB function to detect the musical notes from an audio file by means of numerical Short Time Fourier Transform. Compare your results to those returned by the `spectrogram()` function (in Signal Processing Toolbox).