Exercises

SC2_13 – Fourier Series.

1. Using the *Symbolic Math Toolbox*, check the relationships between real and complex Fourier coefficients.

2. Given the Fourier coefficients of $f(x)$ for the interval $[-\pi, +\pi]$, obtain the formulas to get the Fourier coefficients in the interval $[0, 2\pi]$ by applying the *Shift Property* to the Fourier coefficients in the interval $[-\pi, +\pi]$. Similarly for the interval $[0,T]$ w.r.t. $[-T/2,+T/2]$. What changes between the formulas?

3. Draw a suitable partial sum $S_N(x)$ of the Fourier Series $S(x)$, w.r.t. the interval $[1-\pi, 1+\pi]$, of the following function $f(x)$:

$$f(x) = \begin{cases} -1 & x < 1 \\ +1 & x > 1 \end{cases}$$

   What can be said about the convergence of the sequence $\{S_N(x)\}_N$ to $f(x)$ in the given interval, and how could we estimate the error $\|f(x) - S_N(x)\|_2$ numerically.

4. Approximate numerically the function $f(x)=\cos(2x)-\sin(x)$ in the interval $[\pi/2-2\pi, \pi/2+2\pi]$ by means of Fourier partial sums of order 21, 41, 61 respectively. Approximate, again using the Fourier Series, its first and second derivatives, and compare the results with the derivatives of $f(x)$ computed symbolically.