Exercises

SC2_12 – Discrete Fourier Transform.

1. Implement in a MATLAB function the recursive algorithm (sketched in the pdf file of slides) to compute a DFT(N), with $N=2^p$. Compare, as N increases, its execution time with that required by the MATLAB `fft()` function.

2. Since the matrix Ω_N of the DFT(N) is symmetric, explain why its eigenvalues $(+1, -1, +i, -i)$ are complex. What is the required assumption about a complex matrix for the eigenvalues to be real?

3. Implement in a MATLAB function the "efficient" algorithm to compute the cyclic convolution $\underline{\boldsymbol{w}}$ of two vectors $\underline{\boldsymbol{u}}$ and $\underline{\boldsymbol{v}}$, with N components. Compare, as N increases, its execution time with that required by the algorithm implementing the definition of a cyclic convolution, i.e.:

$$w_j = \sum_{k=0}^{N-1} u_k v_{(j-k) \bmod N}, \quad j = 0,1,2,\ldots,N$$