Course of “Automatic Control Systems”
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PID controller

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PID controller

- A PID controller is characterized by a Proportional-Integral-Derivative control actions with respect to the tracking error $e(t) = r(t) - y(t)$.

- A PID can be written in the time domain as

$$u(t) = K_p e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$$

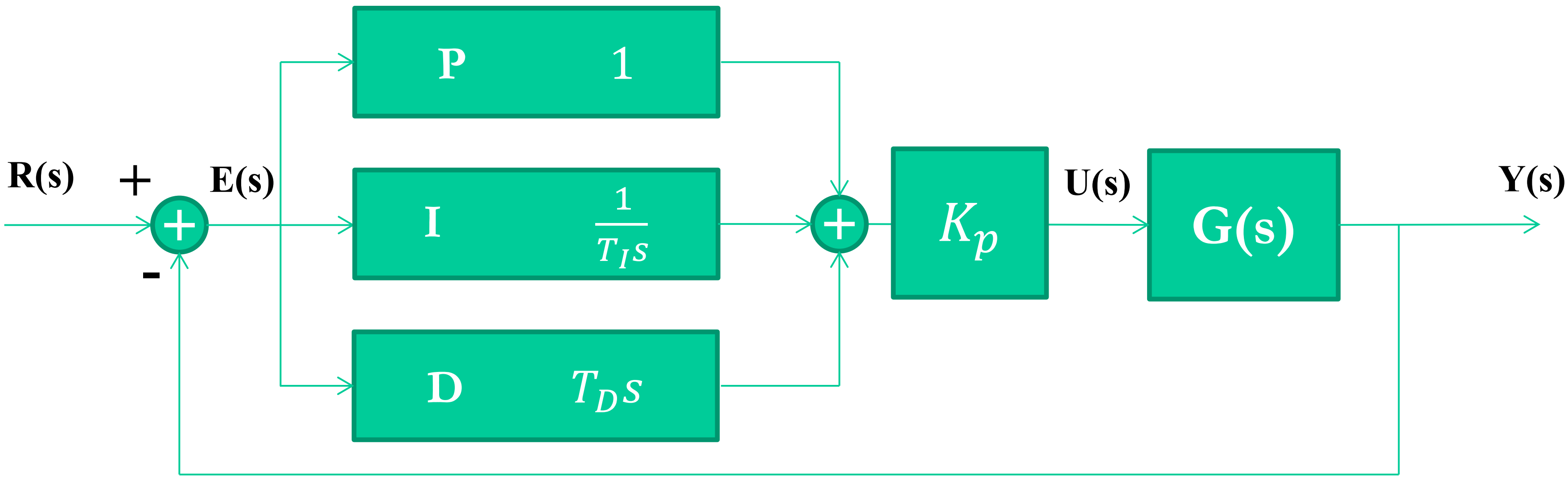
even if it is usually defined in the form

$$u(t) = K_p \left(e(t) + \frac{1}{T_I} \int e(t)dt + T_D \frac{de(t)}{dt} \right)$$

where $T_I = \frac{K_P}{K_I}$ (Integral time) and $T_D = \frac{K_D}{K_P}$ (Derivative time)

- A PID controller in the Laplace domain can be defined as

$$U(s) = K_p\left(E(s) + \frac{1}{T_I s}E(s) + T_D s E(s)\right)$$



PID controller

- Usually, only a subset of the possible PID control actions are implemented.
- In particular we have
  - ***Proportional controller (P)***
  - ***Integral controller (I)***
  - ***Proportional-Integral controller (PI)***
  - ***Proportional-Derivative controller (PD)***
  - ***Proportional-Integral-Derivative controller (PID)***

P controller

- The proportional controller has already considered in the previous lesson.
- **P controllers are used to reduce the steady-state error** when
  - the integral action is not required for the steady-state performance
  - the bandwidth can be increased without violating the requirements
  - the phase margin can be reduced without violating the requirements

I controller

- The integral controller has already considered in the previous lesson.
- **I controllers are used to reduce or eliminate the steady-state error** when
  - the phase margin can be reduced of 90° without violating the requirements

PI controller

- The proportional-integral controller in the Laplace domain can be written as

$$U(s) = K_p E(s) + \frac{K_I}{s} E(s) = K_p\, E(s) + \frac{K_P}{T_I s} E(s) \rightarrow U(s) = \left(K_p + \frac{K_P}{T_I s}\right) E(s)$$

$$K(s) = \frac{K_P}{T_I} \frac{(1 + T_I s)}{s}$$

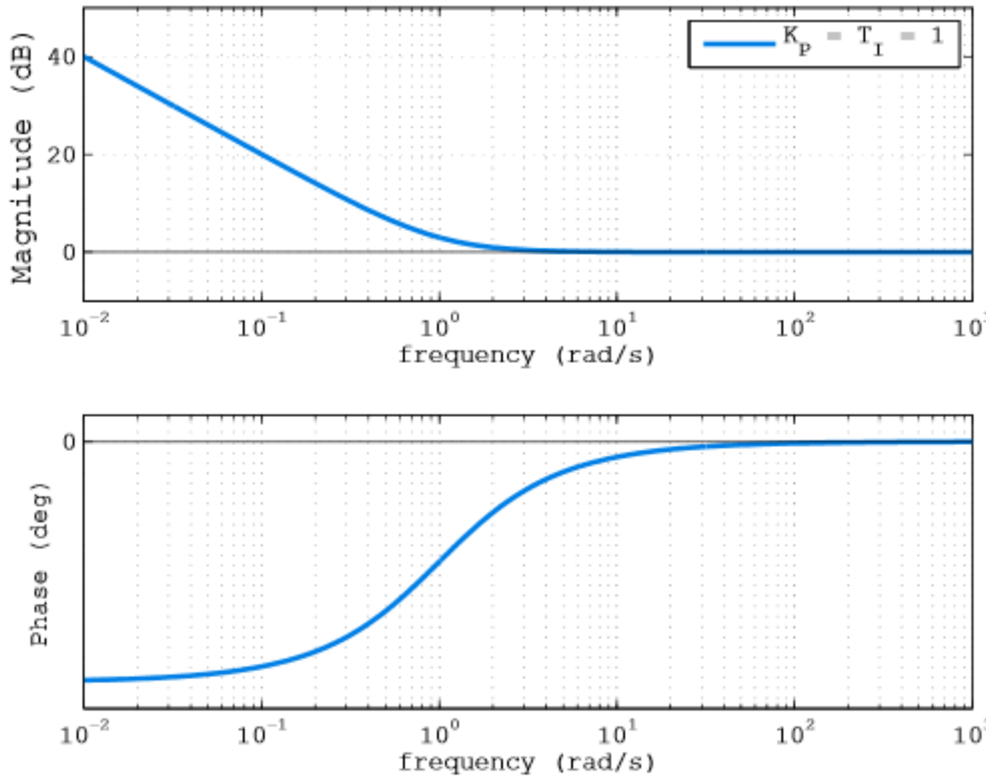
- The PI controllers are composed by
  - a gain $\frac{K_P}{T_I}$ $\left(T_I = \frac{K_P}{K_I}\right)$
  - a pole in the origin
  - a zero in $z = -\frac{1}{T_I}$

PI controller

- PI controllers are used to improve the steady-state performance of the system

- Due to the presence of the zero, a PI controller preforms better than a pure integral controller in terms of transient requirements

- Indeed, the zero can reduce or eliminate the phase lag at the crossing frequency ω_c



PD controller

- The proportional-derivative controller in the Laplace domain can be written as

$$U(s) = K_p E(s) + K_P T_D s E(s) \quad \rightarrow \quad U(s) = \left(K_p + K_P T_D s\right) E(s)$$

$$\Downarrow$$

$$K(s) = K_P(1 + T_D s)$$

- In the present form, the controller has an improper transfer function due to the presence of the ideal derivative action.

PD controller

- In the common practice, a real derivative action is implemented

$$U(s) = K_p\left(E(s) + \frac{T_D s}{1 + \frac{T_D}{N}s}E(s)\right)$$

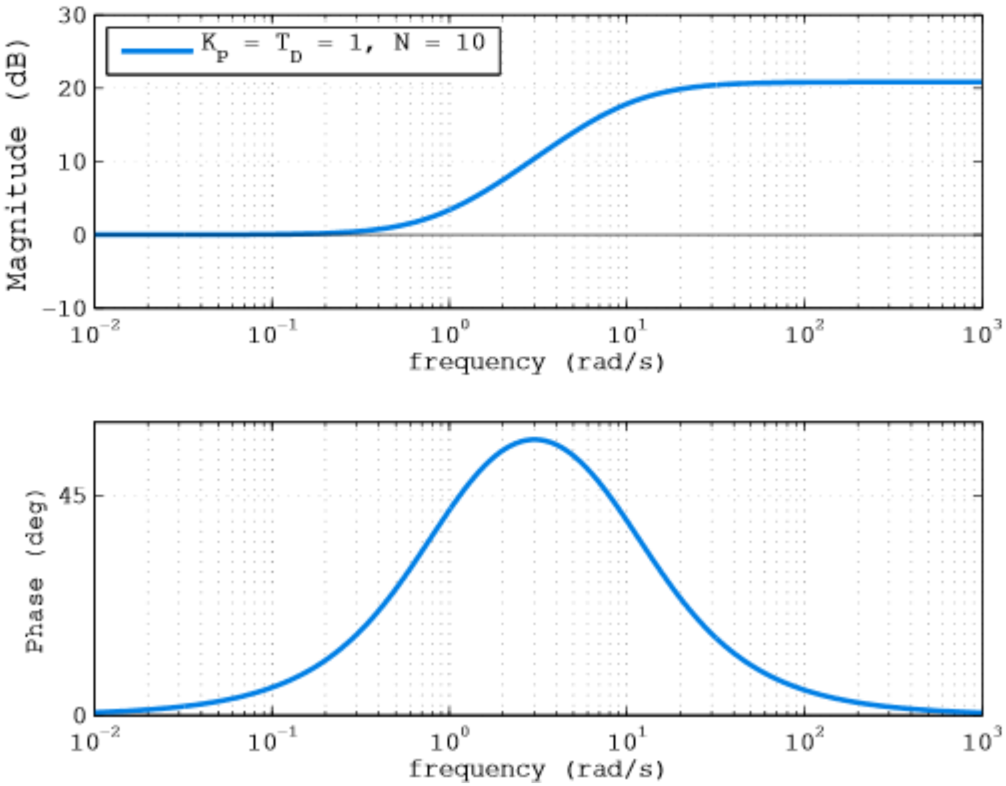
with $N \gg 1$.

- Taking into account that $T_D + \frac{T_D}{N} \cong T_D$, a real PD controller is in the form

$$K(s) = K_P\frac{(1 + T_D s)}{\left(1 + \frac{T_D}{N}s\right)}$$

- PD controllers has the same steady-state performance of the proportional controller

- However, due to the presence of a no-null zero and pole, it can also guarantee an increment of the phase margin



PID controller

- The proportional-integral-derivative controller in the Laplace domain can be written as

$$U(s) = K_p E(s) + \frac{K_P}{T_I s} E(s) + K_P T_D s E(s) \;\; \rightarrow$$

$$U(s) = \left(K_p + \frac{K_P}{T_I s} + K_P T_D s \right) E(s)$$

$$\Downarrow$$

$$K(s) = \frac{K_P}{T_I} \frac{(1 + T_I s + T_I T_D s^2)}{s}$$

- Also in this case a pole at high frequencies have to be added for the physical implementation.

PID controller

- PID controllers are used to improve both the steady-state and transient performance of the system
- Due to the pole in the origin, they are able
  - to reduce or eliminate the steady-stare error
- Due to the PD action, they are able to
  - increase or reduce the phase margin of the system