



Course of
"Automatic Control Systems"
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Analytic results for P and I controllers

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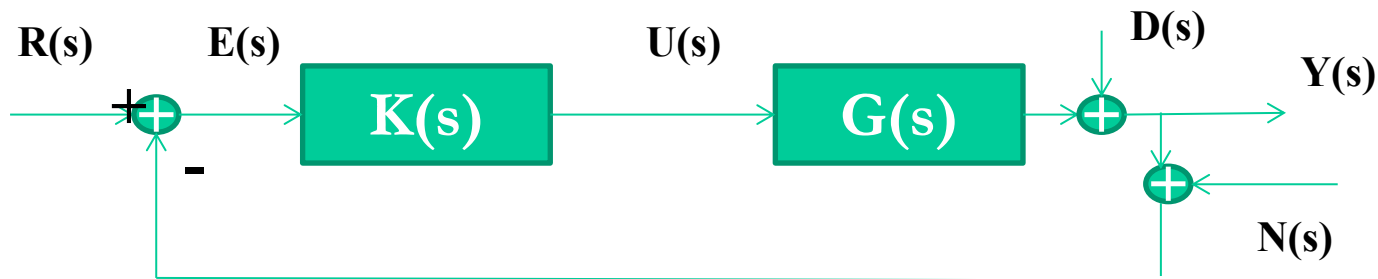
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P controller with a plant of first order

Let us consider a closed loop system in the form



where

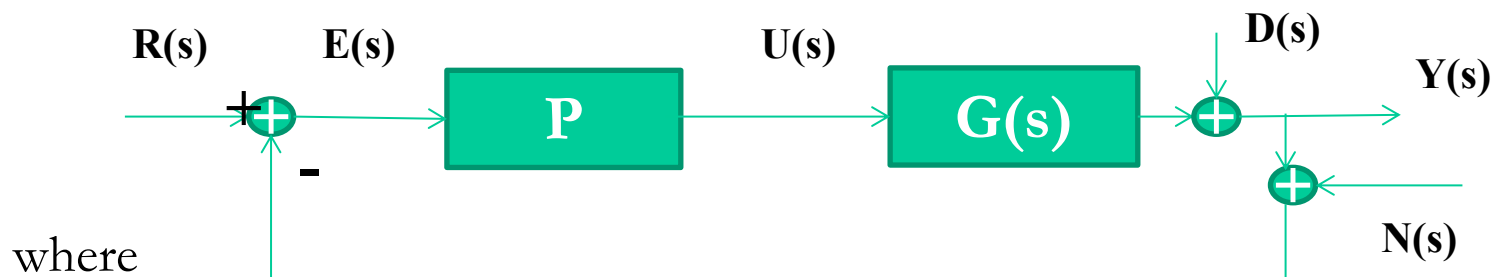
$$G(s) = \frac{G_0}{(1 + s\tau)}$$



$$W(s) = \frac{Y(s)}{R(s)} = \frac{\frac{G_0 K_p}{1 + G_0 K_p}}{\left(1 + s \frac{\tau}{1 + G_0 K_p}\right)}$$

P controller with a plant of second order

Let us consider a closed loop system in the form



where

$$G(s) = \frac{G_0}{\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1}$$

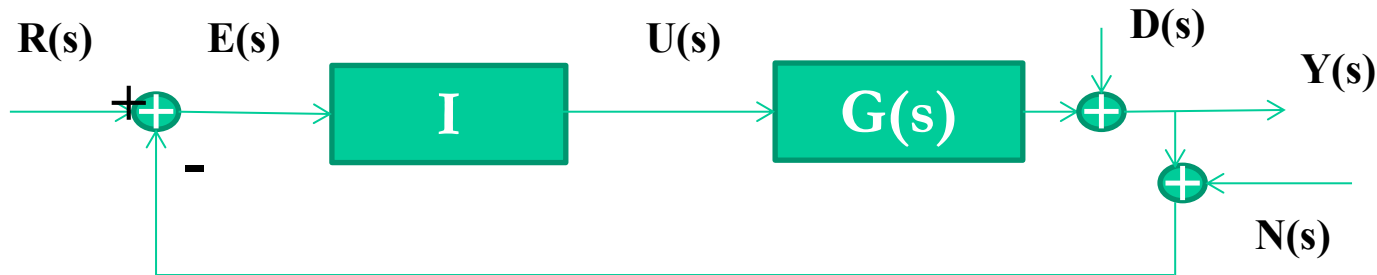
→

$$W(s) = \frac{\frac{G_0 K_p}{1 + G_0 K_p}}{\left(\frac{s^2}{(1 + G_0 K_p)\omega_n^2} + \frac{2\zeta}{(1 + G_0 K_p)\omega_n}s + 1 \right)}$$

- $\omega_{nc} = \omega_n \sqrt{1 + G_0 K_p}$
- $\zeta_c = \zeta / \sqrt{1 + G_0 K_p}$

I controller with a plant of first order

Let us consider a closed loop system in the form



where

$$G(s) = \frac{G_0}{(1 + s\tau)}$$

→
$$W(s) = \frac{1}{\left(\frac{\tau}{G_0 K_I} s^2 + \frac{1}{G_0 K_I} s + 1\right)}$$

$$\bullet \omega_{nc} = \sqrt{G_0 K_I / \tau} \quad \bullet \zeta_c = \frac{1}{2\sqrt{G_0 K_I \tau}}$$