Exercises

SC2_09 – Eigenvalues and eigenvectors.

1. Why, given a square matrix $\boldsymbol{A}$, are the following statements equivalent?
   - $\boldsymbol{A}$ is invertible;
   - $\det(\boldsymbol{A}) \neq 0$;
   - $\lambda = 0$ is not an eigenvalue of $\boldsymbol{A}$.

2. By means of the *Symbolic Math Toolbox*, prove that if $\boldsymbol{A}$ is a matrix of size 2×2 and $\boldsymbol{B}=2\boldsymbol{A} - I_2$, $\boldsymbol{A}$ and $\boldsymbol{B}$ have the same eigenvectors. What is the connection between the eigenvalues of $\boldsymbol{A}$ and those of $\boldsymbol{B}$?

3. Find eigenvalues and eigenspaces of the 3D orthogonal reflection across the line $r=\text{span}\{(2,1,1)^T\}$. Display such eigenspaces.

4. By means of the *Symbolic Math Toolbox*, find eigenvalues and eigenspaces of the 3D orthogonal reflection across a generic line $r=\text{span}\{\underline{\boldsymbol{a}}\}$, where $\underline{\boldsymbol{a}}=(a_1, a_2, a_3)^T$.

   [Hint: use the matrix form of the transformation.]

5. By means of the *Symbolic Math Toolbox*, find real eigenvalues and eigenspaces of 3D rotations around x, y, z axes.

6. Are diagonalizable the following transformations? Explain your answer. Use both the *Symbolic Math Toolbox* and numerical MATLAB to solve the exercise.
   - The 2D horizontal shear by a factor r=2;
   - The 2D rotation by 90°;
   - The mapping induced by the matrix: `A=[2 1;-9 8]`;
   - The mapping induced by the matrix: `A=[2 0 0 0;5 3 0 0;9 1 3 0;1 2 5 -1]`;
   - The 2D orthogonal projection onto the line $r=\text{span}\{(2,1)^T\}$.

7. Compute "efficiently" $\mathtt{A}^{100}$, where `A=[.8 .3;.2 .7]`.

8. Find those points that remain unchanged after applying the mapping t_A induced by the matrix: `A=[0.6 0.8;0.8 -0.6]`. Also find the lines of equation $y=mx$ that remain unchanged after the application of t_A. What 2D geometric transformation does t_A correspond to?

9. Factorize into elementary mappings the transformation induced by the matrix `A=[3 -1;-1 3]` starting from matrices of its diagonalization.

10. Write a MATLAB function to detect the number of connected components in a graph, given on input its adjacency matrix. Download the `graph2.mat` file containing an adjacency matrix, or use another adjacency matrix of your choice.