Course of
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Example of controller design

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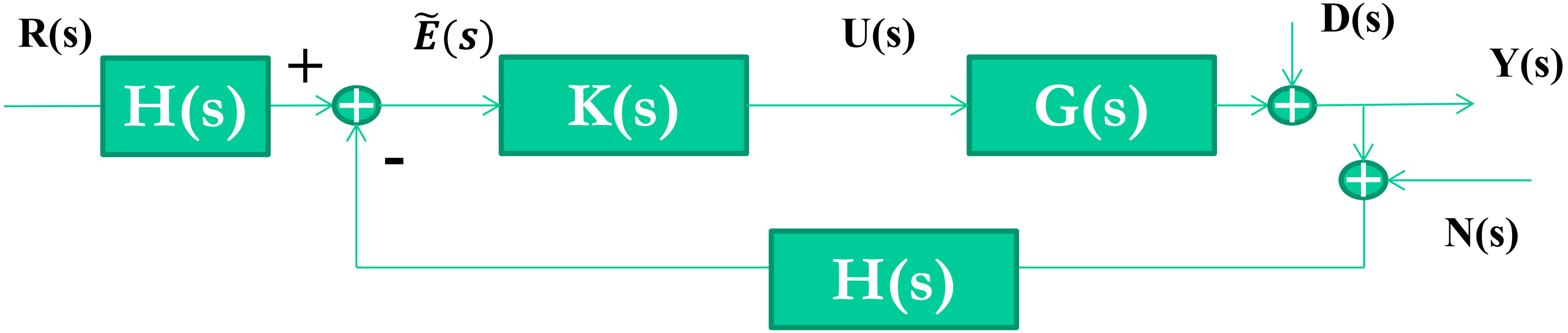
Example of controller design

- Let us consider a closed loop system in the form



where

$$H(s) = \frac{0.1}{1 + 0.2s}$$

$$G(s) = \frac{2(1 + s)}{(1 + 10s)(1 + s\tau)}$$

with $\tau \in [0 \quad 0.04]$

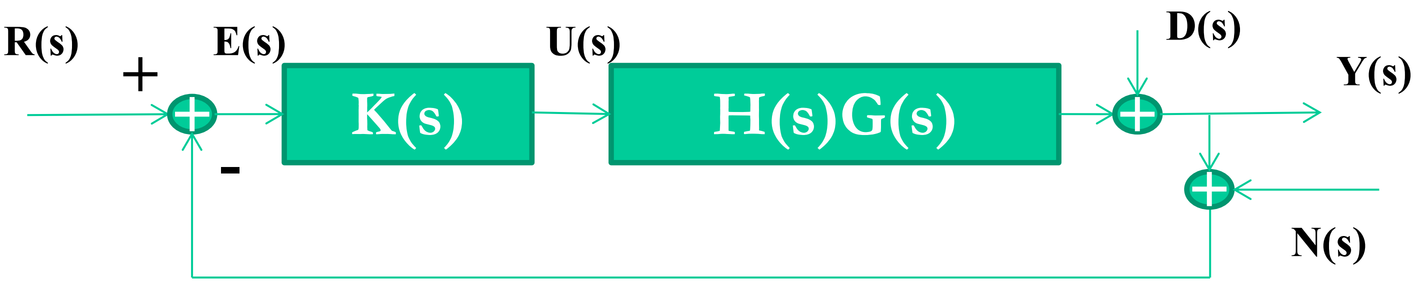
Requirements

Let us consider the following requirements:

1. $e_{\infty r} \leq 5\%$ for a reference signal $r(t) = r_0 t \cdot 1(t)$
2. $e_{\infty r} \leq 0.05$ for multi-frequency disturbs in the range $[0.01 \quad 0.5]$ rad/s
3. $e_{\infty r} \leq 0.1$ for multi-frequency reference signal in the range $[0 \quad 1]$ rad/s
4. Attenuation $\geq 20_{db}$ for multi-frequency noise in the range $[50 \quad 100]$ rad/s
5. Phase margin $\varphi_m > 30°$

Example of controller design

- Making use of the block diagrams algebra, the closed loop scheme can be equivalently written in the form



where

$$E(s) = H^{-1}(s)\tilde{E}(s)$$

Steady-state spec. for polynomial reference

1. $\boldsymbol{e_{\infty r} \leq 5\%}$ **for a reference signal** $\boldsymbol{r(t) = r_0 t \cdot 1(t)}$

- Steady-state requirement for polynomial reference signal of order 1.

- The subscript r in $\boldsymbol{e_{\infty r}}$ indicates that the requirement considers the relative steady-state error (independent on the amplitude of the reference signal r_0)

- To assure a finite steady state error for a polynomial signal of order 1 it is necessary that $F(s)$ is of type 1, that is $F(s)$ should have a pole in the origin.

- Taking into account that the plant transfer function $G(s)$ doesn't contain poles in the origin, the steady-state part of the controller is

$$K'(s) = \frac{k_0}{s}$$

where k_0 is chosen so that

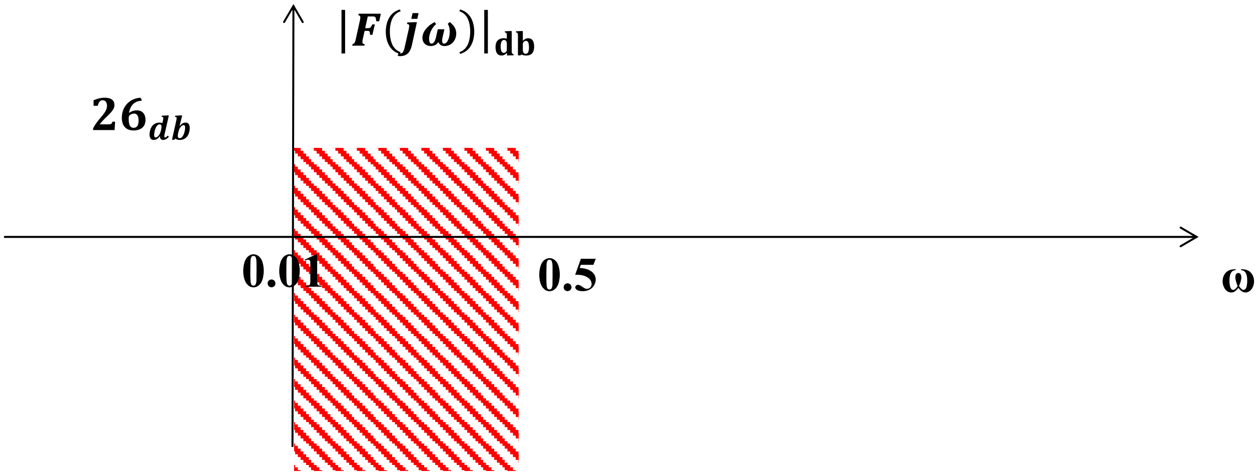
$$\lim_{s \to 0} s \frac{1}{1 + K'(s)G(s)H(s)} \frac{1}{s^2} < 0.05 \quad \rightarrow \quad \mathrm{k}_0 \geq 100$$

Steady-state spec. for multi-frequency disturbs

2. $\boldsymbol{e_{\infty r} \leq 0.05}$ **for multi-frequency disturbs in the range** $\boldsymbol{[0.01 \quad 0.5]}$ **rad/s**

- Steady-state requirement for multi-frequency disturbs
- It implies that

$$|S(s)| = \left|\frac{1}{1+K'(s)G(s)H(s)}\right| \leq 0.05 \quad \rightarrow \left|\frac{1}{F(s)}\right| \leq 0.05 \quad \rightarrow \quad |F(s)|_{db} \geq 26_{db}$$

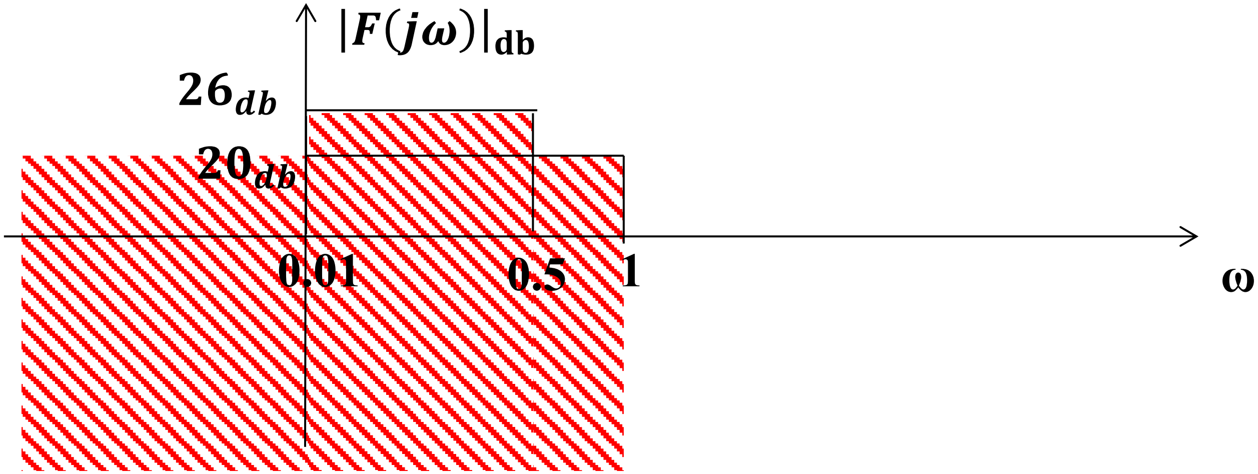


Steady-state spec. for multi-frequency reference

3. $e_{\infty r} \leq 0.1$ for multi-frequency reference signal in the range $[0 \quad 1]$ rad/s

- Steady-state requirement for multi-frequency reference
- It implies that

$$|S(s)| = \left|\frac{1}{1+F(s)}\right| \leq 0.1 \quad \rightarrow \left|\frac{1}{F(s)}\right| \leq 0.1 \quad \rightarrow \quad |F(s)|_{db} \geq 20_{db}$$

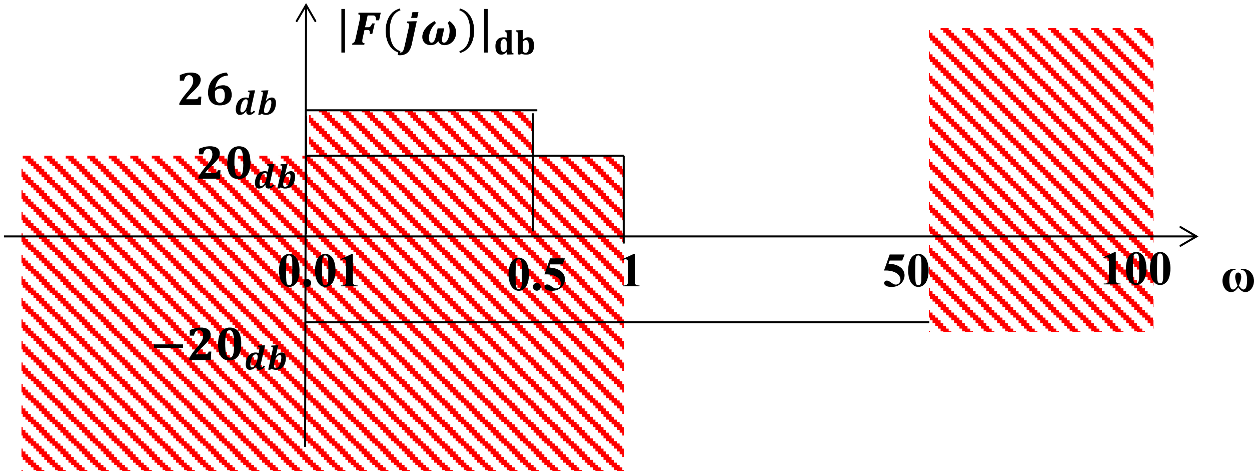


Steady-state spec. for multi-frequency noise

4. Attenuation $\geq 20_{db}$ for multi-frequency noise in the range $[50 \quad 100]$ rad/s

- Steady-state requirement for multi-frequency noise

- −It implies that $|T(s)| = \left|\frac{F(s)}{1+F(s)}\right|_{db} \leq -20_{db} \quad \rightarrow \quad |F(s)|_{db} \leq -20_{db}$

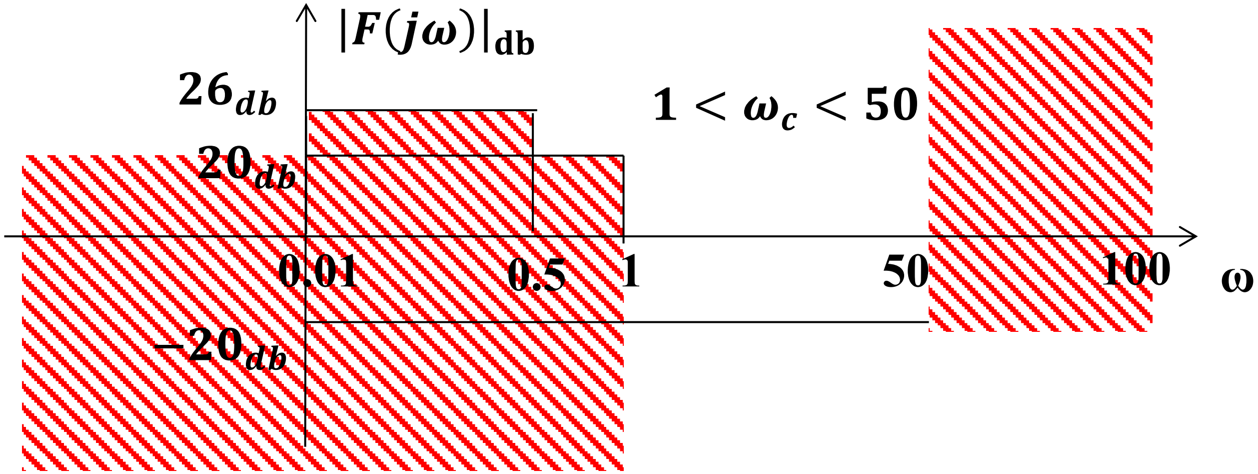


Transient requirements

5. $\varphi_m > 30°$

- The transient performance considers only the overshoot of the step response

- The crossing frequency of $F(s)$ is imposed by the multi-frequency requirements



- However, looking at the forbidden zones, it is reasonable to place the crossing frequency $\omega_c \in [10 \quad 20]$ rad/s

Controller design

- Design the controller $K(s)$ so that the open loop function $F(s) = K(s)G(s)$ satisfies the previous requirements

- The controller will be in the form

$$\boldsymbol{K(s) = K'(s) \cdot K''(s)}$$

where

- $K'(s)$ have been designed according to the steady-state requirements concerning polynomial reference and/or disturbs

$$\boldsymbol{K'(s) = \frac{100}{s}}$$

- $K''(s)$ have to be designed according to the steady-state multi-frequency requirements and the transient requirements

Step2: Controller design

- In order to design $K''(s)$, let us consider the uncertain transfer function

$$F'(s) = K'(s)H(s)G(s) = \frac{20(1+s)}{s(1+10s)(1+s\tau)(1+0.2s)}$$

with $\tau \in [0 \quad 0.04]$.

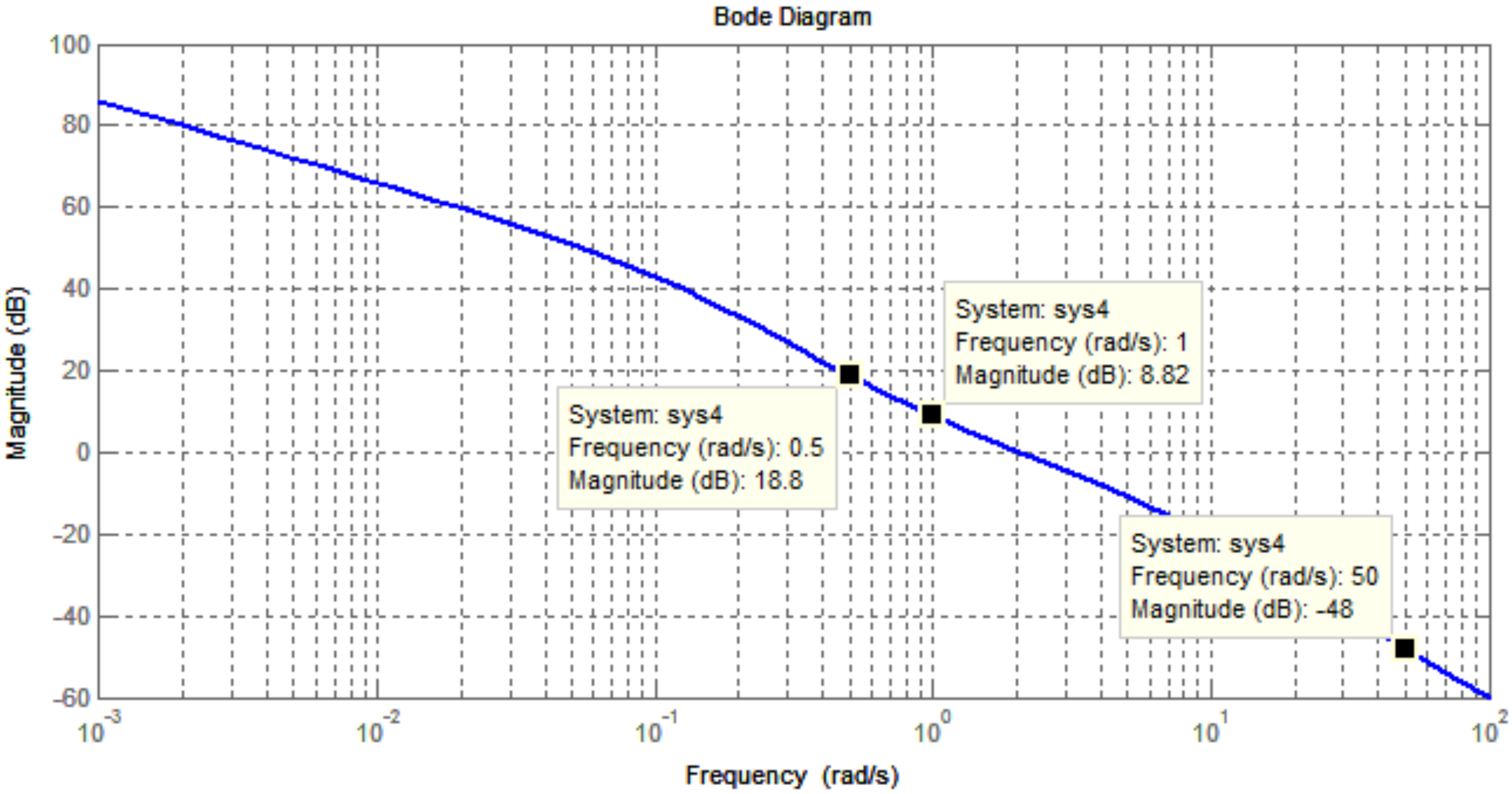
- The transfer function $F'(s)$ contains an uncertain pole whose breaking point moves from 25 rad/s to $+\infty$.

Analysis of the uncertain pole

- Taking into account that the desired crossing frequency $\omega_c \in [10 \quad 20]$ rad/s, the effects of the uncertain pole on $F(s)$ can be summarized as:
  - It doesn't modify the crossing frequency ω_c for any values of τ
  - It causes a phase leg in $\omega = \omega_c$ and hence it reduces the phase margin. The maximum phase lag is due to $\tau = 0.04$.
  - It attenuates the magnitude in the interval $[50 \quad 100]$ rad/s when $\tau \in [0.02 \quad 0.04]$ and hence it makes easier to satisfy the constraint on the multi-frequency noise
- In order to design a robust controller w.r.t. the variation of the uncertain pole, we will assume:
  - $\tau = 0.04$ when we evaluate the phase margin
  - $\tau = 0$ when we evaluate the magnitude Bode plot of $F(s)$

Controller design

- Magnitude Bode plot of $F'(s) = \frac{20(1+s)}{s(1+10s)(1+0.2s)}$



Controller design

- In order to verify the multi-frequency requirements, we need:
  - Amplification $\geq 8_{db}$ in $\omega = 0.5$ rad/s
  - Amplification $\geq 12_{db}$ in $\omega = 1$ rad/s

  With a degree of freedom related to a possible amplification $\leq 28_{db}$ in $\omega = 50$ rad/s

- Assuming a crossing frequency $\omega_c \cong 10$ rad/s, the current phase margin is

$$\varphi_m = 180 + \varphi_c = 180 + (\underset{\substack{\uparrow \\ \textit{Pole in the origin}}}{-90} \underset{\substack{\uparrow \\ \textit{Pole in -0.1}}}{-\,90} \underset{\substack{\uparrow \\ \textit{Pole in -5}}}{-\,60} \underset{\substack{\uparrow \\ \textit{Uncer. pole in -25}}}{-\,30} \underset{\substack{\uparrow \\ \textit{Zero in -1}}}{+\,90}) = 0$$

Controller design

- The controller $K''(s)$ can be easily defined by

$$K''(s) = \frac{(1+10s)}{(1+s)}$$

- In this way we have
  - 13_{db} magnitude amplification in $\omega = 0.5$ rad/s
  - 17_{db} magnitude amplification in $\omega = 1$ rad/s
  - Crossing frequency $\omega_c \cong 10$ rad/s
  - NO phase anticipation in $\omega = 10$ rad/s $(\varphi_m \cong 0)$
  - 20_{db} magnitude amplification in $\omega = 50$ rad/s
- Finally, we can add a zero with a breaking frequency $\omega = 20$ rad/s to assume a phase margin $\varphi_m \cong 30$ without intersecting the noise forbidden zone.

Controller design

- The controller is in the form

$$K(s) = K'(s)K''(s) = \frac{100(1+10s)(1+0.05s)}{s(1+s)}$$

Bode plot of $F(s)$