Course of
"Automatic Control Systems"
2022/23

Procedure for the controller design

Prof. Francesco Montefusco

Department of Economics, Law, Cybersecurity, and Sports Sciences

Università degli Studi di Napoli Parthenope

francesco.montefusco@uniparthenope.it

Team code: **uxbsz19**

Procedure for the controller design

- The procedure for the controller design consists in three main steps:
  - **STEP 1**

    Convert the closed loop requirements in requirements on the open loop transfer function $F(s)$

  - **STEP 2**

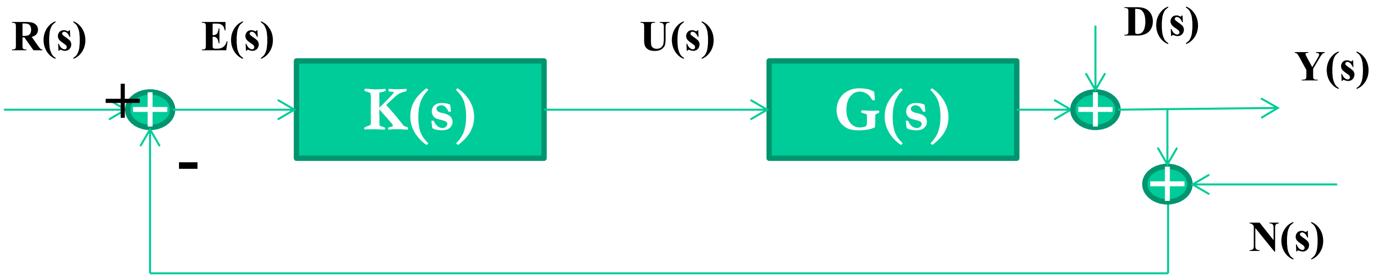
    Design the controller $K(s)$ so that the open loop function $F(s) = K(s)G(s)$ satisfies the requirements in the STEP 1

  - **STEP 3**

    Verify, with the aid of an appropriate software (for example MATLAB), that the closed loop system satisfies the requirements. Otherwise, go back to the STEP1 using more accurate techniques.

STEP 1: convert the requirements on $T(s)$ into requirements on $F(s)$

- The performance of the closed loop system are evaluated in terms of
  - ***Tracking of the reference input***
  - ***Rejection of the disturbs***
  - ***Insensibility to the noise***



- Assuming that the stability of the C.L. system is guaranteed, the responses of the system can be divided in a transient and a steady-state parts.

- The ***steady-state performance*** cares about the steady-state behavior of the closed loop system while the ***transient performance*** cares about the tracking of the reference signal during the transient phase

STEP 1: Steady-state performance

- The steady-state performance are classified in

- ***Tracking of the reference input $R(s)$***
  - Null or bounded steady-state error to polynomial inputs (step, ramp,…)
  - Null or bounded steady-state error to sinusoidal inputs at fixed frequency

- ***Rejection of the disturbs D(s)***
  - Null or bounded steady-state error to polynomial inputs
  - Bounded steady-state error to multi-frequency sinusoidal inputs

- ***Insensibility to the noise $N(s)$***
  - Bounded steady-state error to multi-frequency sinusoidal inputs

Step1: Steady-state performance

- The requirements concerning **polynomial reference and/or disturbs** define the steady-state part of the controller

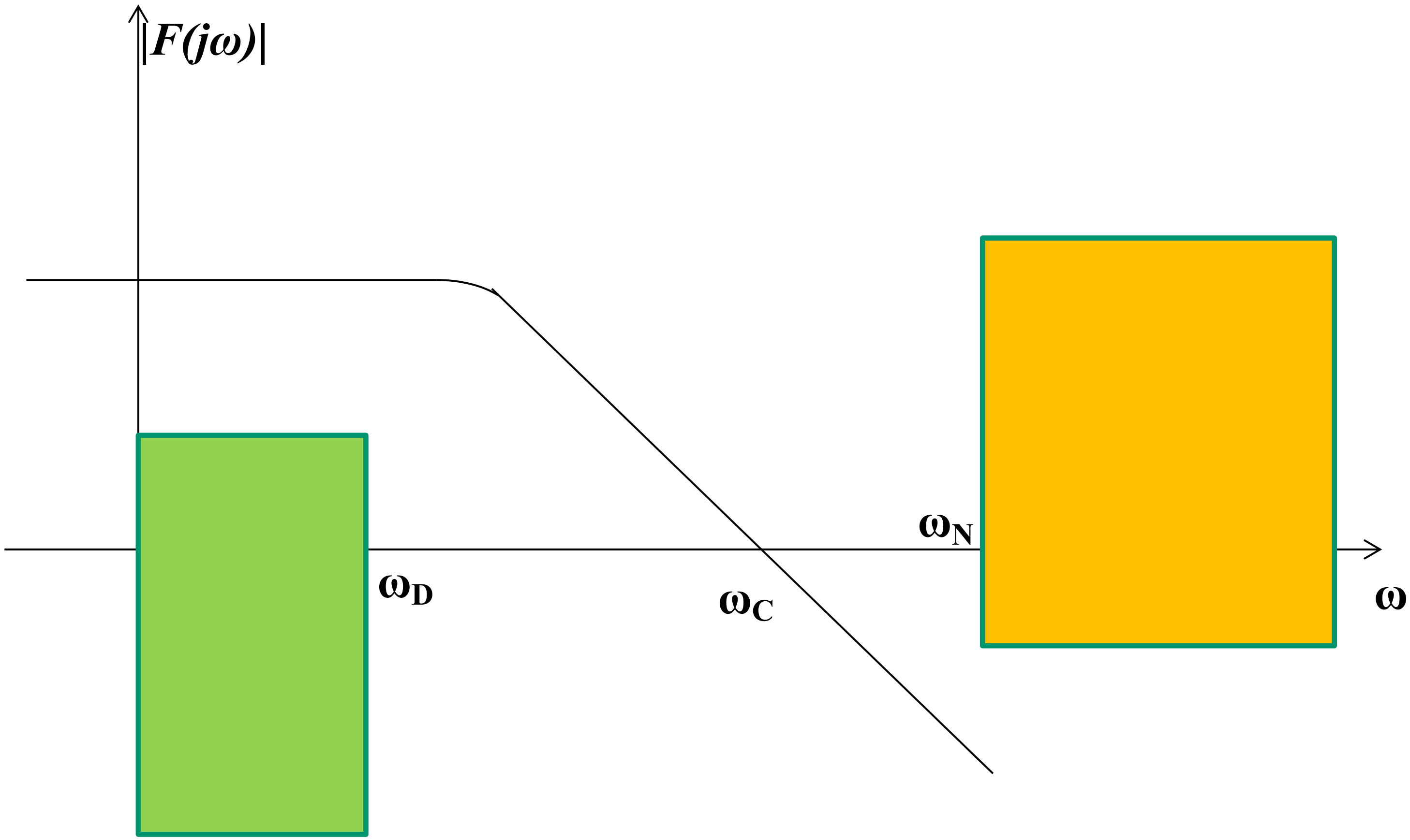
$$K'(s) = \frac{k_0}{s^{\mu}}$$

where k_0 and μ depends on

- the order of the polynomial input
- bounded or null error requirement
- structure of the plan transfer function (gain and poles in the origin)

Step1: Steady-state performance

- The requirements concerning ***multi-frequency noise and disturbs*** defines the set of the forbidden zones for the magnitude Bode diagram of $F(s)$



Step 1: Transient performance

- The ***transient performance*** are usually expressed in terms of tracking properties of a polynomial reference of order 0 (step)

- The transient performance concerns
  - ***Dynamic precision*** (overshoot) ⟶ ***Phase margin*** $\boldsymbol{\varphi_m^*}$

$$\begin{cases} T_a(s) = \dfrac{1}{1+\tau s} & \varphi_m^* > 60° \\ T_a(s) = \dfrac{1}{1+2\zeta s/\omega_n + s^2/\omega_n^2} & \varphi_m^* < 60° \end{cases}$$

  - ***Time response*** (settling time) ⟶ ***Crossing frequency*** $\boldsymbol{\omega_c^*}$

$$\begin{cases} \omega_c^* = \dfrac{1}{\tau} & \varphi_m^* > 60° \\ \omega_c^* = \omega_n & \varphi_m^* < 60° \end{cases}$$

Example

Let us consider the following set of requirements:

1. $e_\infty = 0$ for a reference signal $r(t) = R_0 1(t)$
2. Attenuation $\geq 20_{db}$ for multi-frequency disturbs in the range $[0 \quad 0.01]$ rad/s
3. Attenuation $\geq 80\%$ for multi-frequency noise in the range $[10 \quad 100]$ rad/s
4. Overshoot $s \leq 10\%$
5. Settling time $t_{s5\%} \leq 10s$

Example: steady-state spec. for polynomial reference signal

1. $\boldsymbol{e_\infty = 0}$ **for a reference signal** $\boldsymbol{r(t) = R_0 1(t)}$

- Steady-state requirement for polynomial reference signal.
- To assure a null steady state error for a polynomial signal of order $\mathbf{0}$ it is necessary that $F(s)$ is of type 1, that is $F(s)$ has a pole in the origin.
- Assuming that the plant transfer function $G(s)$ doesn't contain poles in the origin, the steady-state part of the controller is

$$\boldsymbol{K'(s) = \frac{k_0}{s}}$$

where k_0 is a free parameter

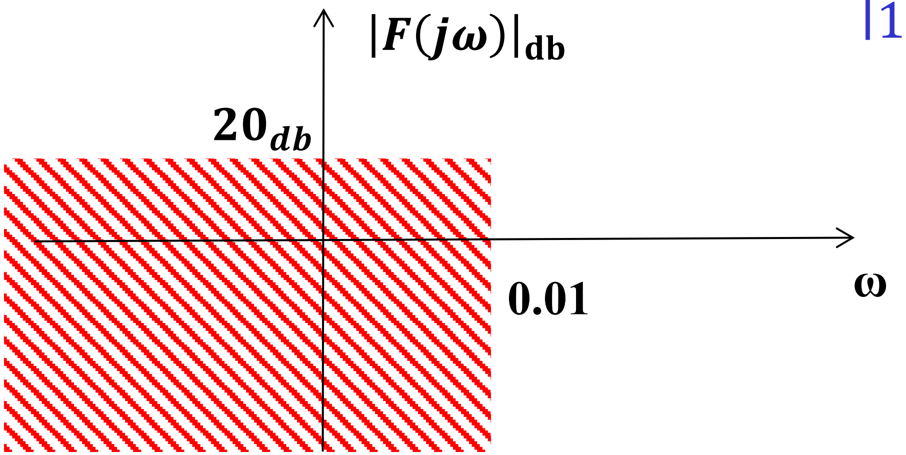
Example: steady-state spec. for multi-frequency disturbs

2. Attenuation $\geq 20_{db}$ for multi-frequency disturbs in the range $[0 \quad 0.01]$ rad/s

- Steady-state requirement for multi-frequency disturb.
- It implies that, in the range $[\mathbf{0} \quad \mathbf{0.01}]$ **rad/s**,

$$|S(s)|_{db} = \left|\frac{1}{1+F(s)}\right|_{db} \leq -20 \quad \rightarrow$$

$$\left|\frac{1}{1+F(s)}\right| \leq 0.1 \quad \rightarrow \quad |F(s)| \geq 10$$

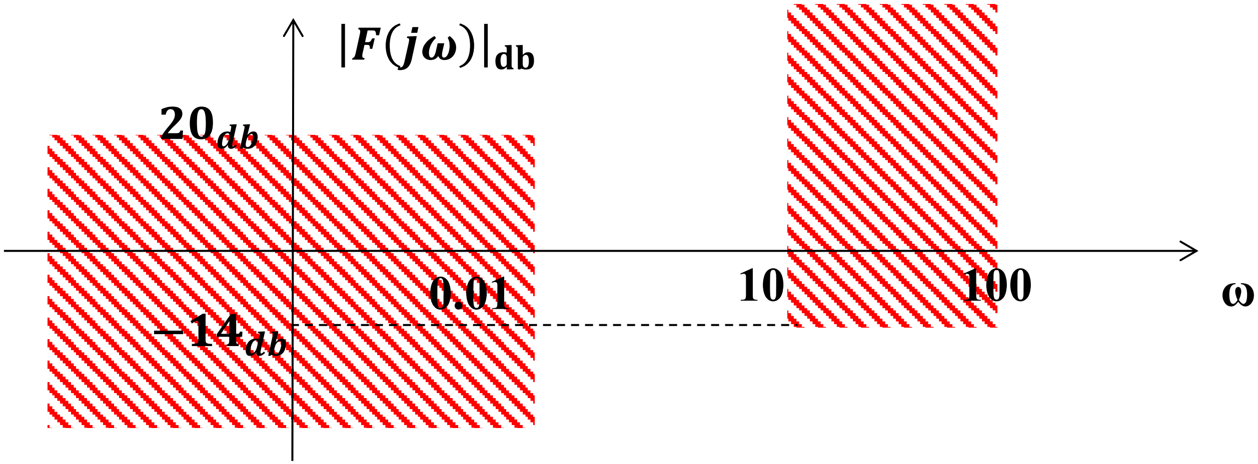


Example: steady-state spec. for multi-frequency noise

3. Attenuation $\geq$ 80% for multi-frequency noise in the range $[10 \quad 100]$ rad/s

- Steady-state requirement for multi-frequency noise
- It implies that, in the range **$[10 \quad 100]$ rad/s**,

$$|T(s)| = \left|\frac{F(s)}{1+F(s)}\right| \leq 0.2 \quad \rightarrow \quad |F(s)| \leq 0.2 \quad \rightarrow \quad |F(s)|_{db} \leq -14$$



Example: transient spec. on the overshoot

4. Overshoot $s \leq 10\%$

- Transient requirement on the overshoot

- Taking into account that $s = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$, we have that

$$s \leq 10\% \quad \rightarrow \quad \zeta \geq 0.6 \;\rightarrow \quad \varphi_m \cong 100\zeta \geq 60°$$

- Hence the complementary sensitivity function can be approximated by a first order system

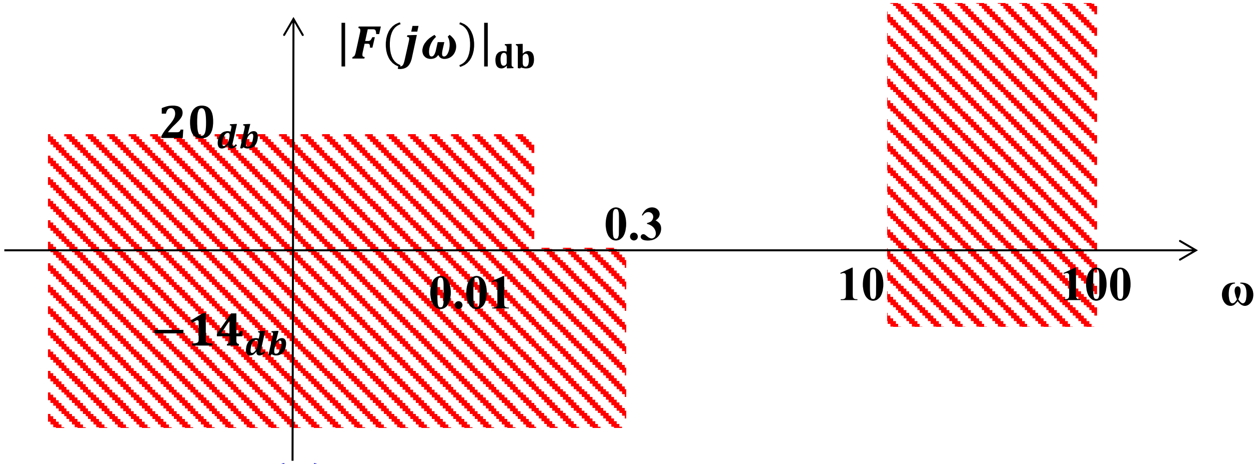
$$T_a(s) = \frac{1}{1 + \tau s}$$

where τ depends on the settling time requirement

5. Settling time $t_{s5\%} \leq 10s$

- Transient requirement on the settling time
- Taking into account that the settling time at 5% for a first order system is defined as $t_{s5\%} \cong 3\tau$, we have that

$$t_{s5\%} \leq 10 \quad \rightarrow \quad \tau \leq \frac{10}{3} \rightarrow \quad \omega_c \geq \frac{3}{10} = 0.3$$



- **The transfer function $F(s)$ should have a crossing frequency $\omega_c > 0.3$ and a phase margin $\varphi_m > 60°$.**

Step2: Controller design

- Design the controller $K(s)$ so that the open loop function $F(s) = K(s)G(s)$ satisfies the requirements in STEP 1

- The controller will be in the form

$$\boldsymbol{K(s) = K'(s) \cdot K''(s)}$$

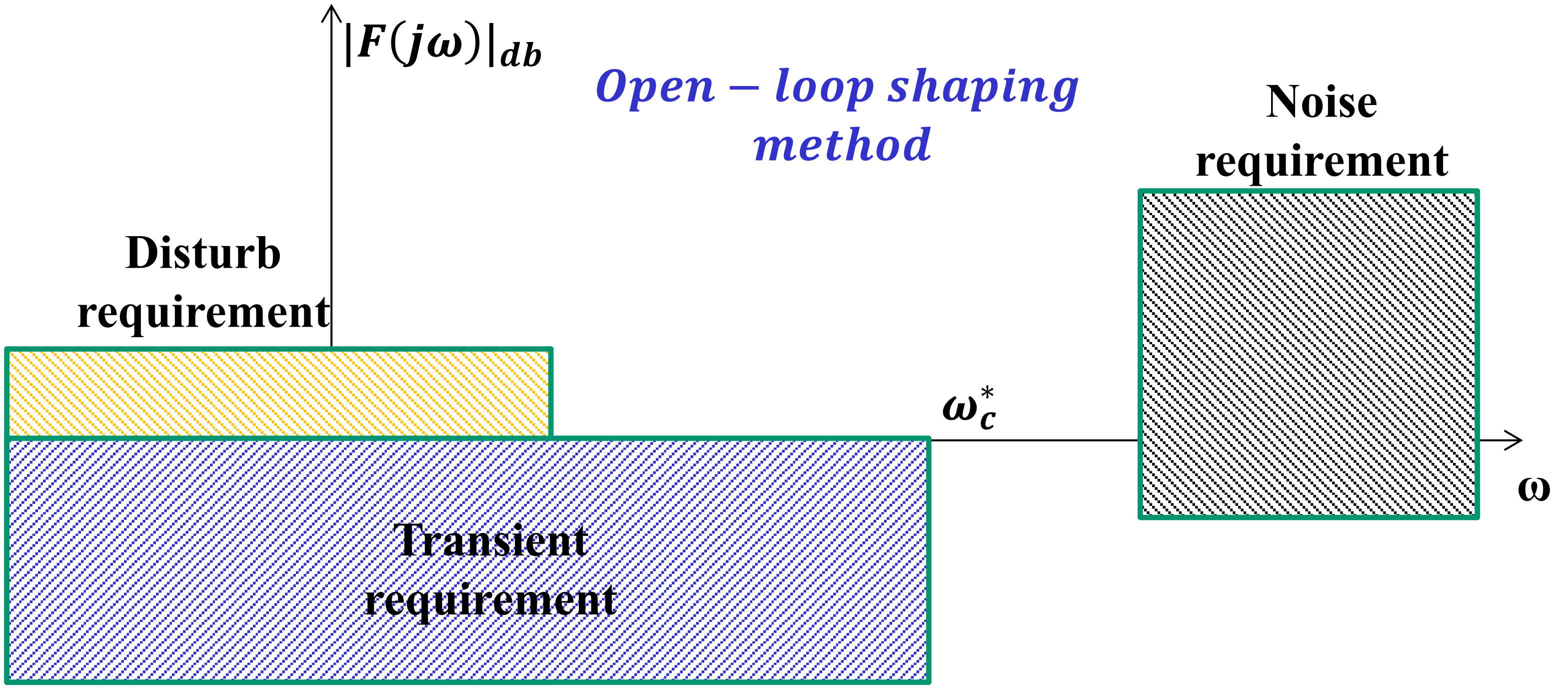
where

- $K'(s)$ have been designed according to the steady-state requirements concerning polynomial reference and/or disturbs

$$\boldsymbol{K'(s) = \frac{k_0}{s^{\mu}}}$$

- $K''(s)$ have to be designed according to the steady-state multi-frequency requirements and the transient requirements

- The control part $K''(s)$ is usually designed so that $F(s)$ doesn't intersect the forbidden zones



$$with\ \angle F(j\omega_c^*) = \varphi_m^* - 180°$$

Step3: Validation of the controller

- Verify, with the aid of an appropriate software (MATLAB, OCTAVE,…), that the closed loop system satisfies the requirements.

- If some of the requirements are not satisfied, more accuracy have been added in the design process:
  - Use the real Bode diagram instead of the asymptotic Bode diagram
  - Use the Nichols chart to evaluate the desired phase margin
  - Satisfy the requirements with a greater safety factor
  - ….