Course of
"Automatic Control Systems"
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Control requirements: Transient performance

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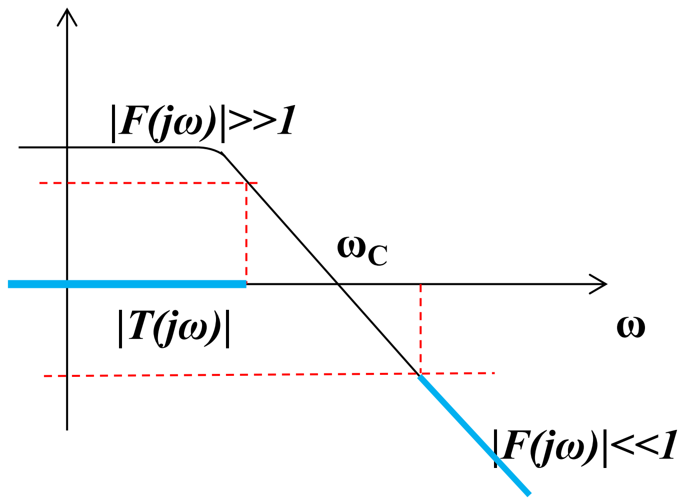
Transient performance

- We have introduced two parameters in the frequency domain related to the transient behavior
  - **Bandwidth B_3 of $T(s)$** related to the rise time
  - **Resonant peak M_p of $T(s)$** related to the overshoot
- We have also assumed to refer to regularly stable open loop functions such that:
  - at low frequencies $F(s) \gg 1 \;\rightarrow\; T(s) \cong 1$
  - at high frequencies $F(s) \ll 1 \;\rightarrow\; T(s) \cong F(s)$



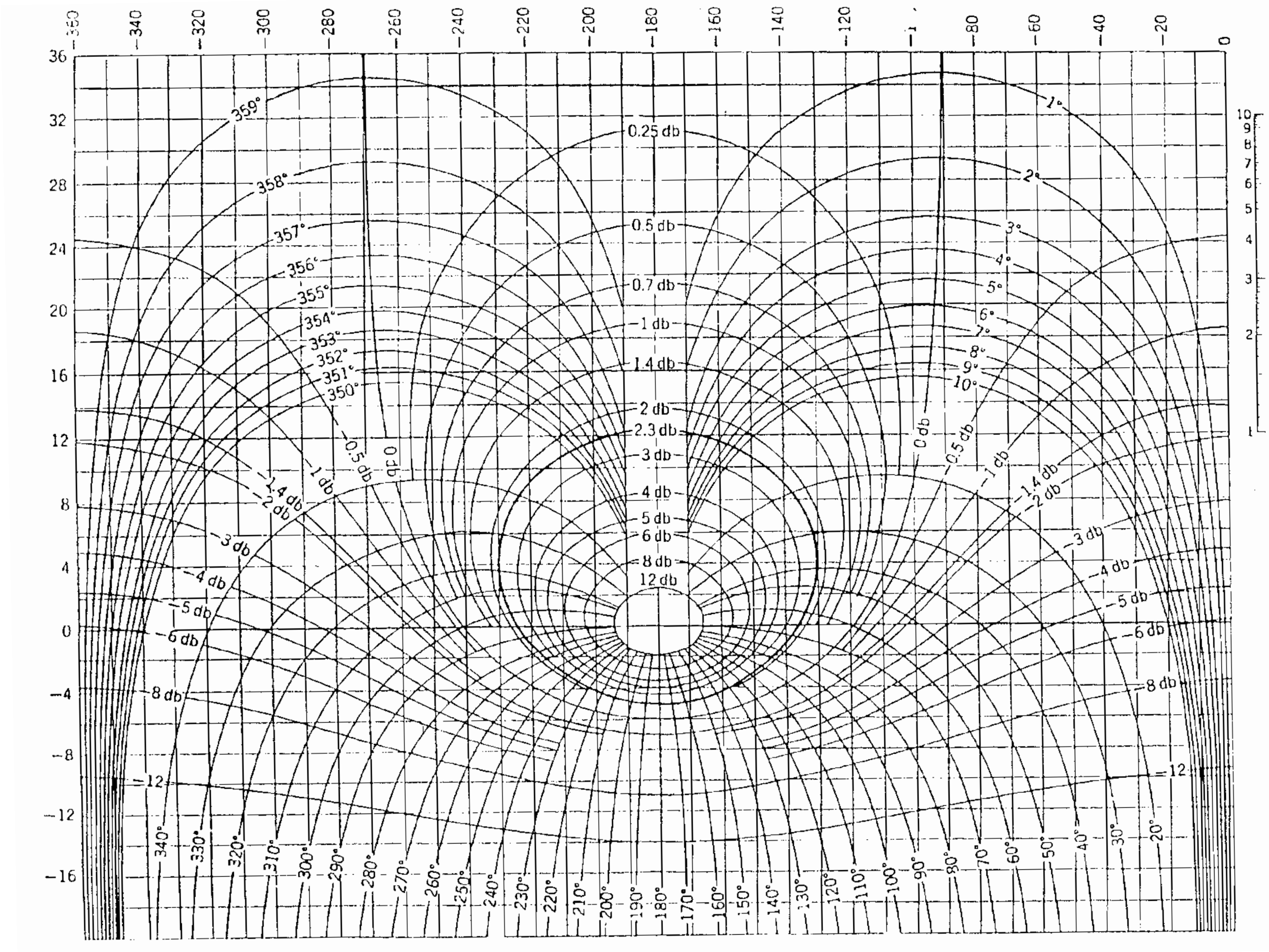
Transient performance

- Hence
  - the bandwidth B_3 of $T(s)$ is the first frequency such that for all frequencies greater than B_3 the magnitude is less than $-3db$
  - the resonant peak M_p of $T(s)$ is the maximum value assumed by the magnitude of $T(s)$

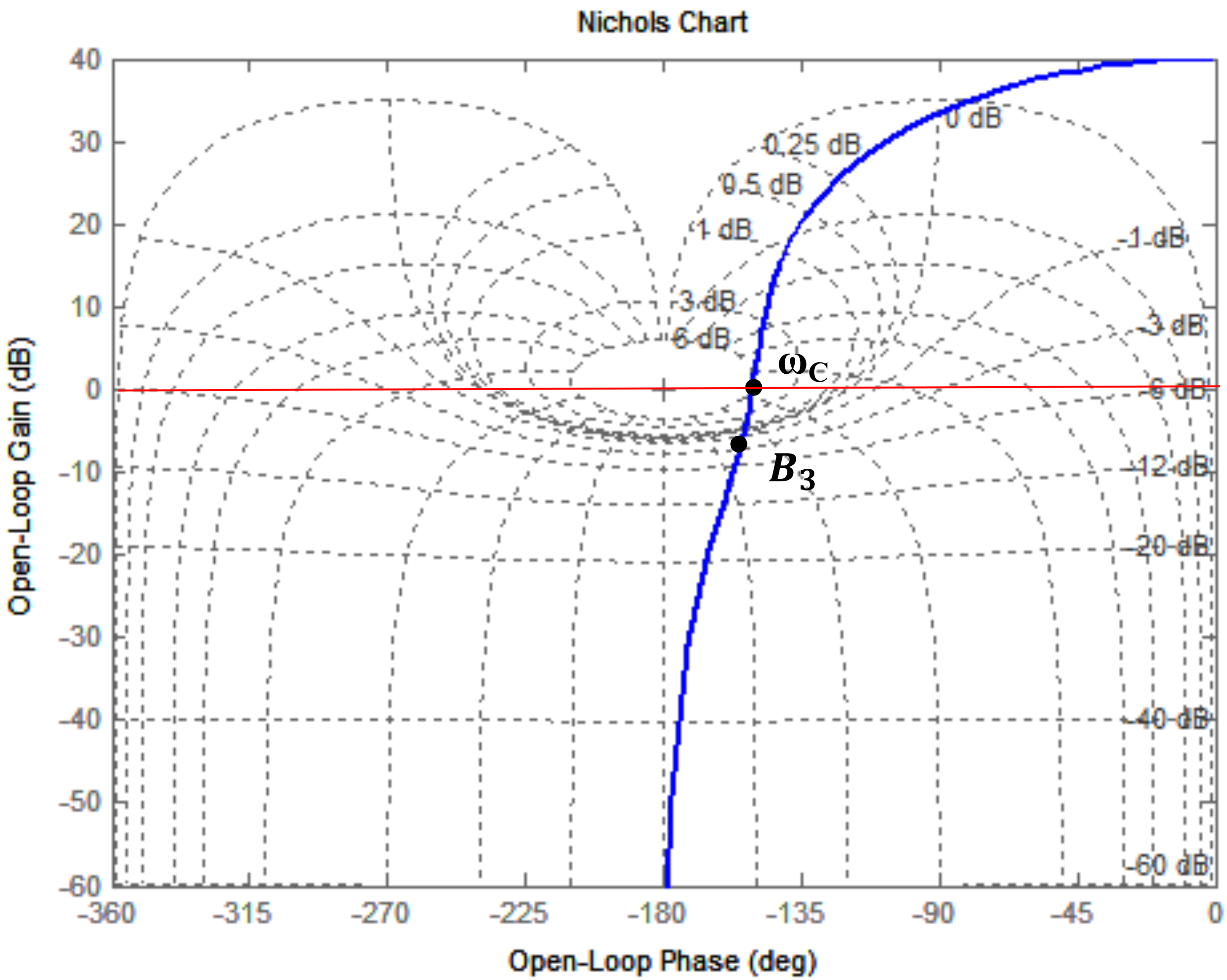
- In order to quantify the bandwidth B_3 and resonant peak M_p of $T(s)$ we need to analyze the behavior of $T(s)$ in the two decades with center ω_c

- To this aim, we have introduced the so called ***Nichols chart*** that relates the magnitude and phase of the open loop function $F(s)$ to the the magnitude and phase of the closed loop function $T(s)$

Nichols chart



- In order to quantify the bandwidth B_3 of $T(s)$, let us consider a regularly stable open loop function $F(s)$ on the Nichols chart.

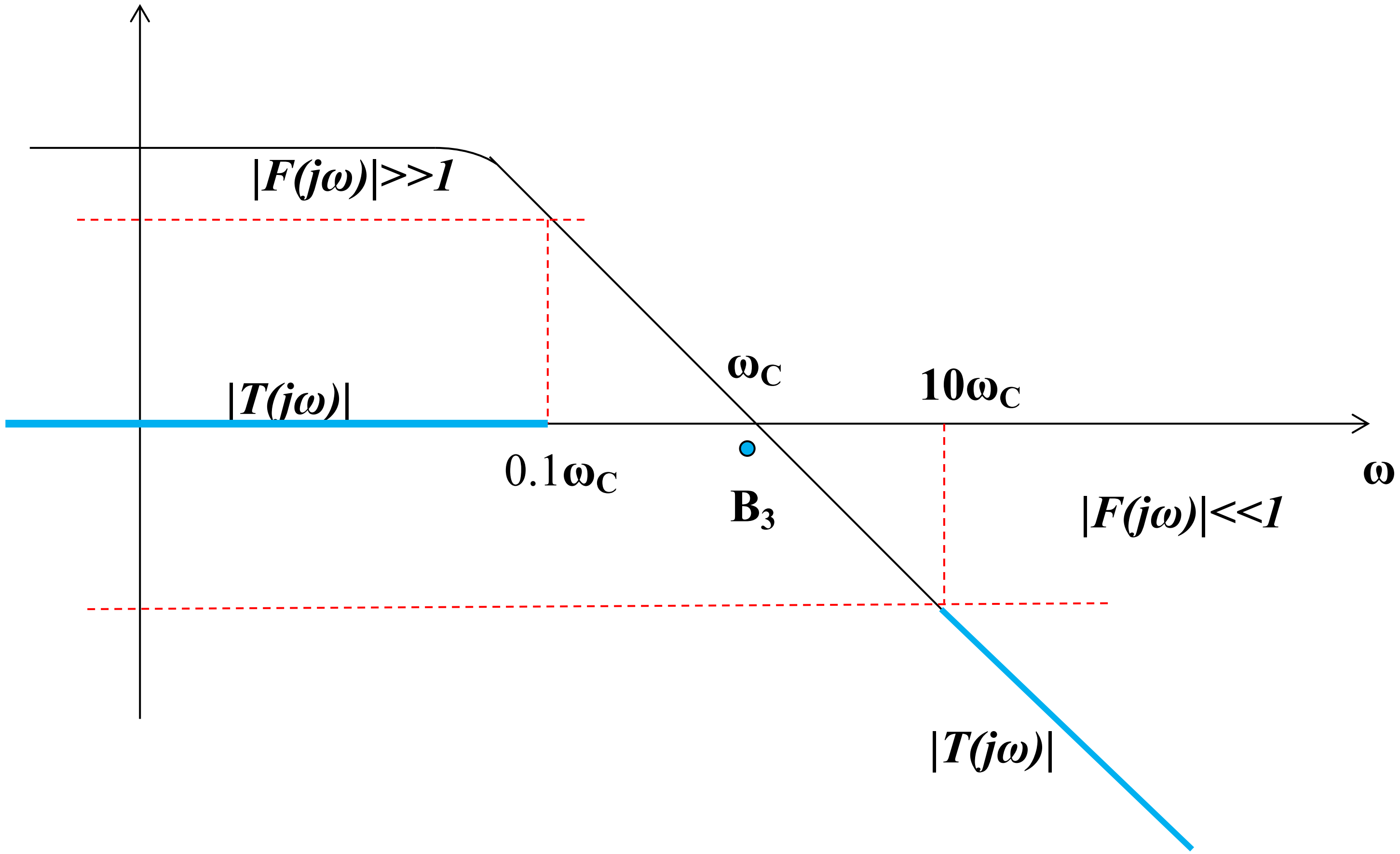


$|T(j0)|_{db} \cong 0$

The distance between ω_c and B_3 is very small

Bandwidth B_3 of $T(s)$

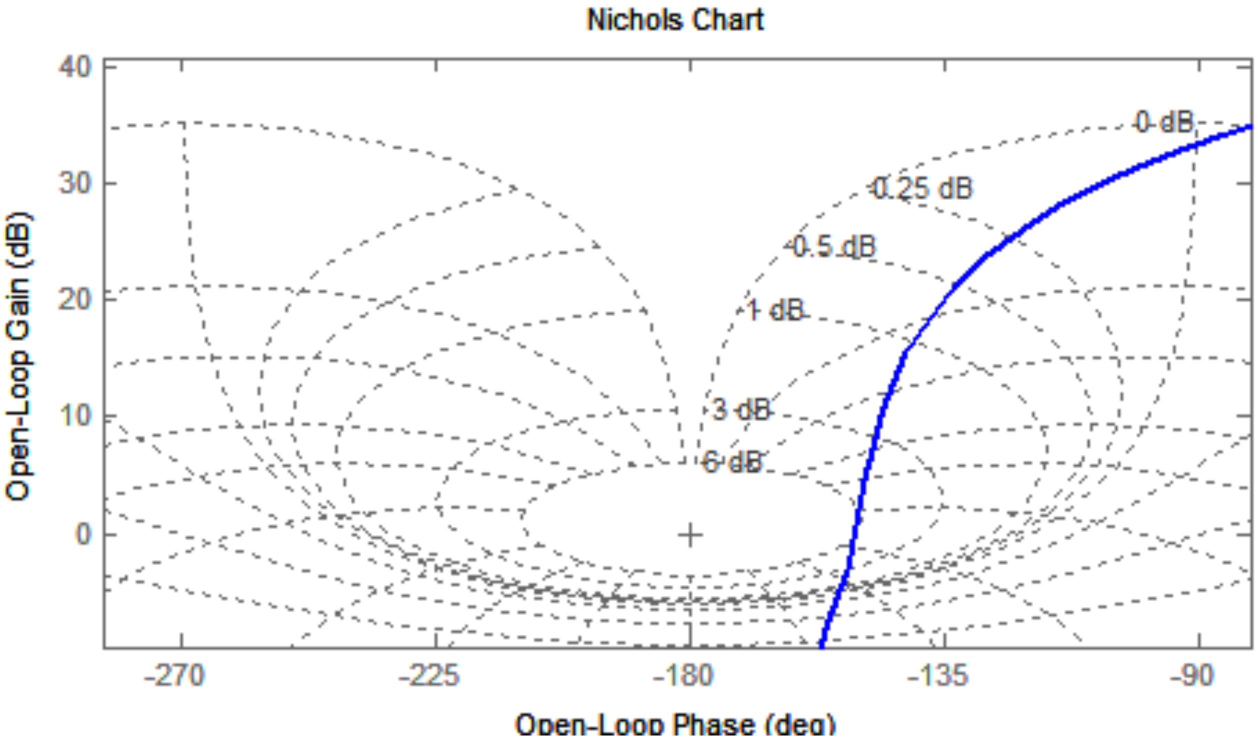
- It implies that we can approximate the bandwidth B_3 with the crossing frequency ω_c



Resonant peak M_p of $T(s)$

- Making use of the Nichols charts, the resonant peak M_p of $T(s)$ corresponds to the magnitude of the smallest of the constant magnitude curves that is the tangent to the $F(s)$ Nichols plot.

- The closed loop function has a resonant peak only if the Nichols plot of $F(s)$ intersect the magnitude surface at 0_{db}

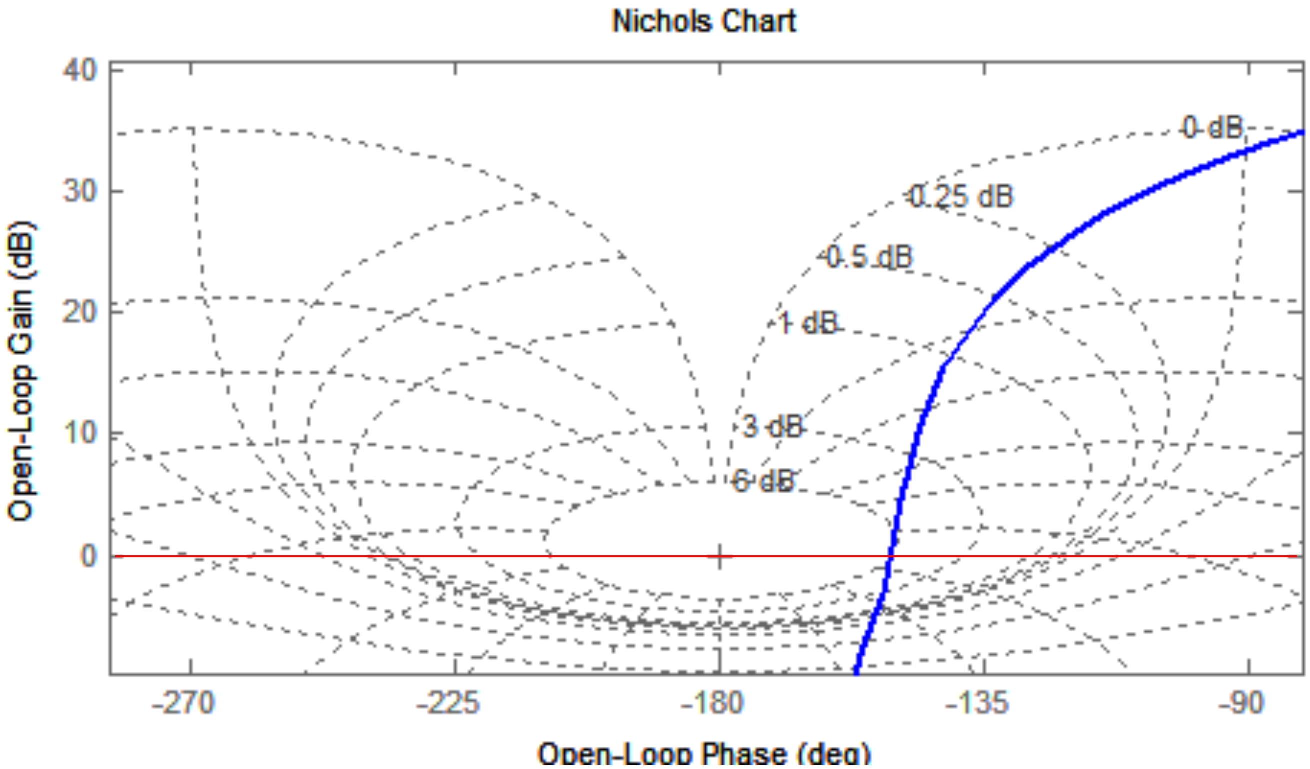


The Nichols plot of $F(s)$ intersect the magnitude surface at 3_{db}

M_p is approximately 6_{db}

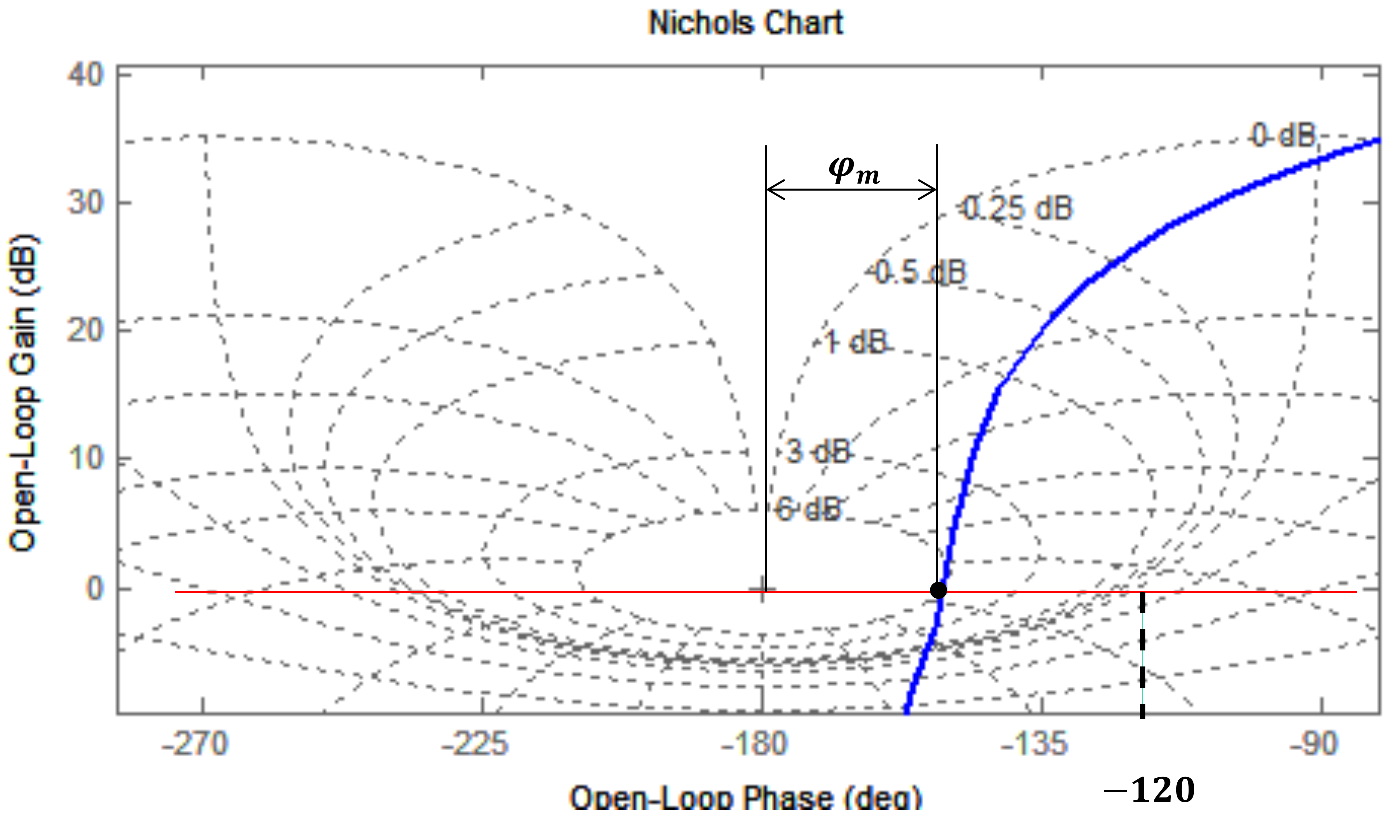
- In order to simplify the evaluation of the resonant peak, it is easy to recognized that:

An approximate value of the resonant peak M_p is given by the value of the constant magnitude curve passing through the intersection of the Nichols plot of $F(s)$ with the open loop 0_{db} axis.



Resonant peak M_p of $T(s)$

- From the previous approximation we can conclude that
  - the resonant peak M_p is strictly related to the phase margin φ_m of $F(s)$
  - The closed loop function has a resonant peak only if $\varphi_m < 60°$



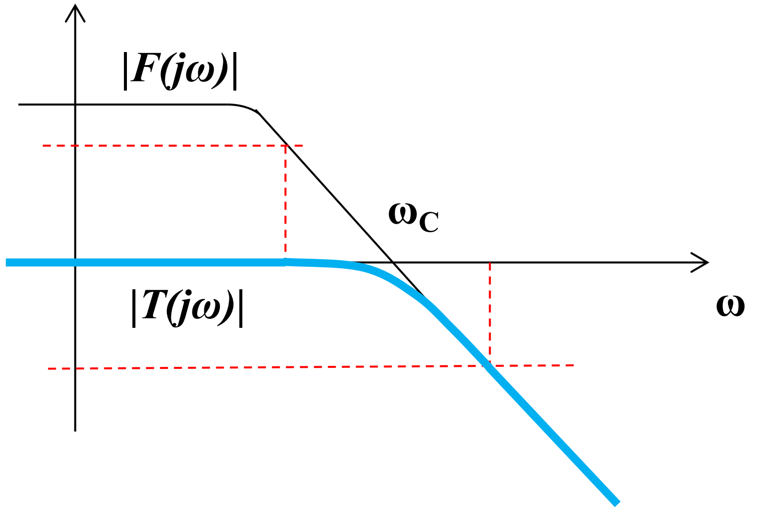
Resonant peak M_p of $T(s)$

- The previous results allows to define to possible approximation $T_a(s)$ of the closed loop function depending on the $F(s)$ phase margin.

CASE 1: $\varphi_m > 60°$

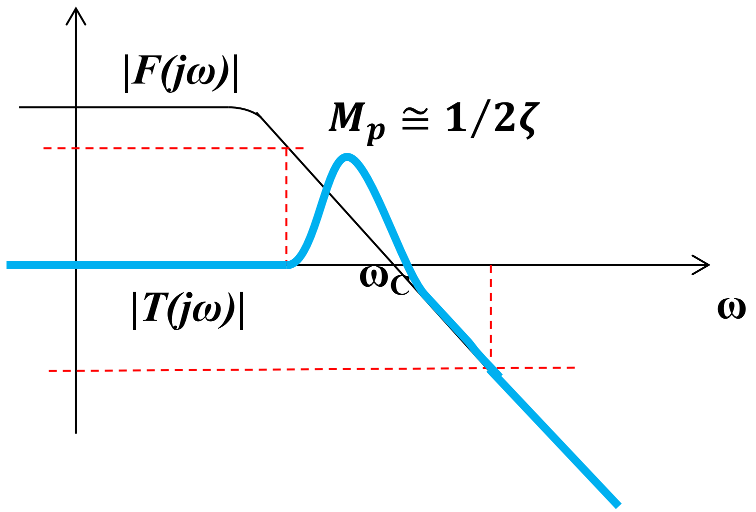
$$T_a(s) = \frac{1}{1 + s/\omega_c}$$

First order approximation



- The previous results allows to define to possible approximation $T_a(s)$ of the closed loop function depending on the $F(s)$ phase margin.

CASE 2: $\varphi_m < 60°$



$$T_a(s) = \frac{1}{1 + 2\zeta s/\omega_c + s^2/\omega_c^2}$$

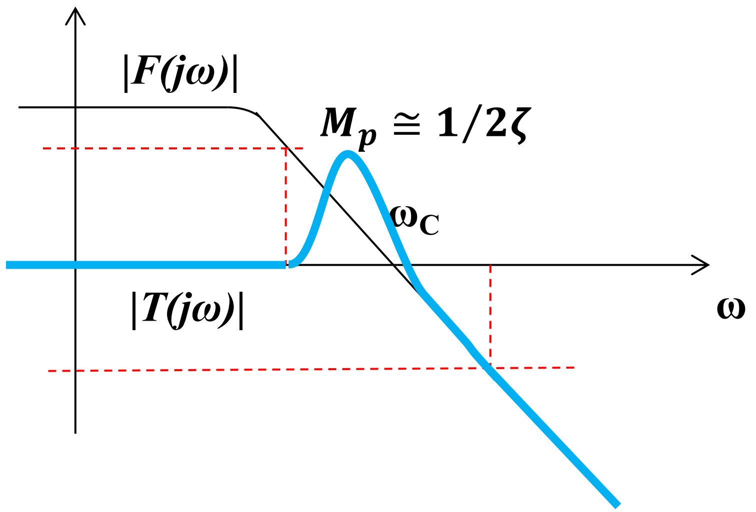
Second order approximation

where, imposing the equality $|T(j\omega_c)| = |T_a(j\omega_c)|$, it is possible to prove that

$$\zeta \cong \frac{\varphi_m}{100}$$

Second order approximation

$$T_a(s) = \frac{1}{1 + 2\zeta s/\omega_c + s^2/\omega_c^2}$$



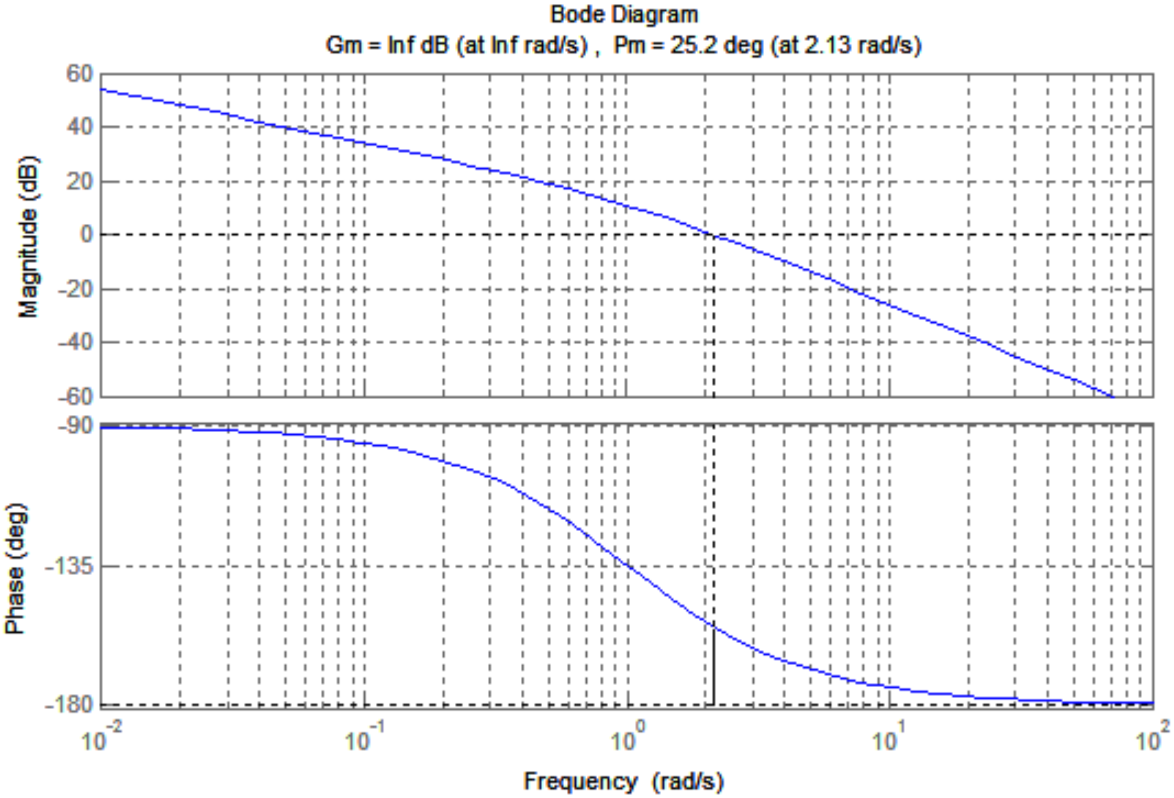
$$\varphi_m = 180° - |\varphi_c|$$

$$\begin{cases} |T(j\omega_c)| = \dfrac{|F(j\omega_c)|}{|1 + F(j\omega_c)|} = \dfrac{1}{|1 + e^{j\varphi_c}|} = \dfrac{1}{2sin\left(\frac{\varphi_m}{2}\right)} \\ |\boldsymbol{T_a}(j\omega)| = \dfrac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_c^2}\right)^2 + \left(\frac{2\zeta\omega}{\omega_c}\right)^2}} \rightarrow |\boldsymbol{T_a}(j\boldsymbol{\omega_c})| = \dfrac{1}{2\zeta} \end{cases}$$

$$\Rightarrow \quad \zeta = sin(\varphi_m/2) \cong \frac{\varphi_m}{2} * \frac{\pi}{180} \cong \frac{\varphi_m}{\mathbf{100}}$$

Example: closed loop approximate function $T_a(s)$

- Let us consider an open loop transfer function $F(s) = \frac{5}{s(1+s)}$



$\omega_c \cong 2.13\ rad/s$

$\varphi_m \cong 25°$

Second order approximation of the closed loop system

Example: closed loop approximate function $T_a(s)$

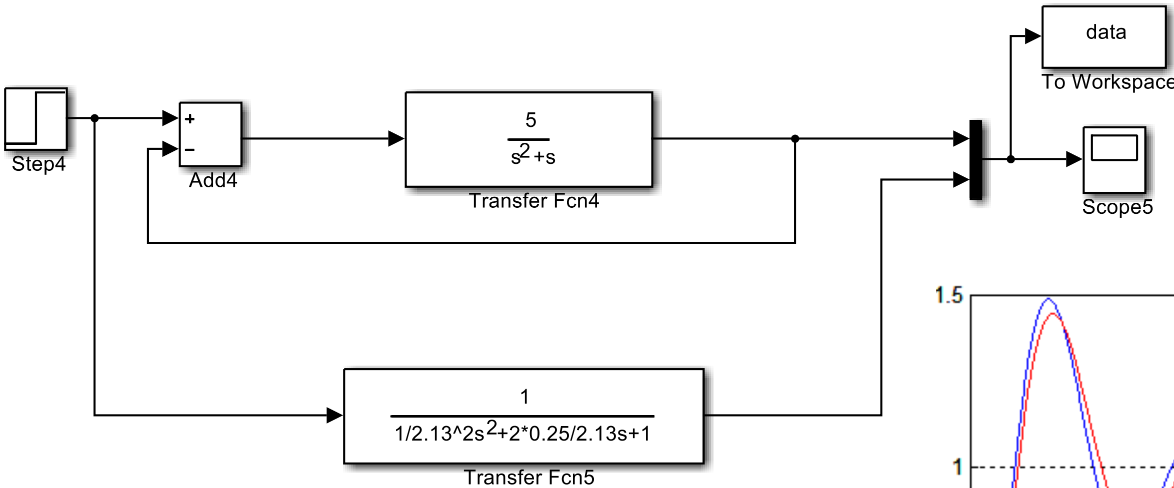
- The second order approximation of the closed loop system is

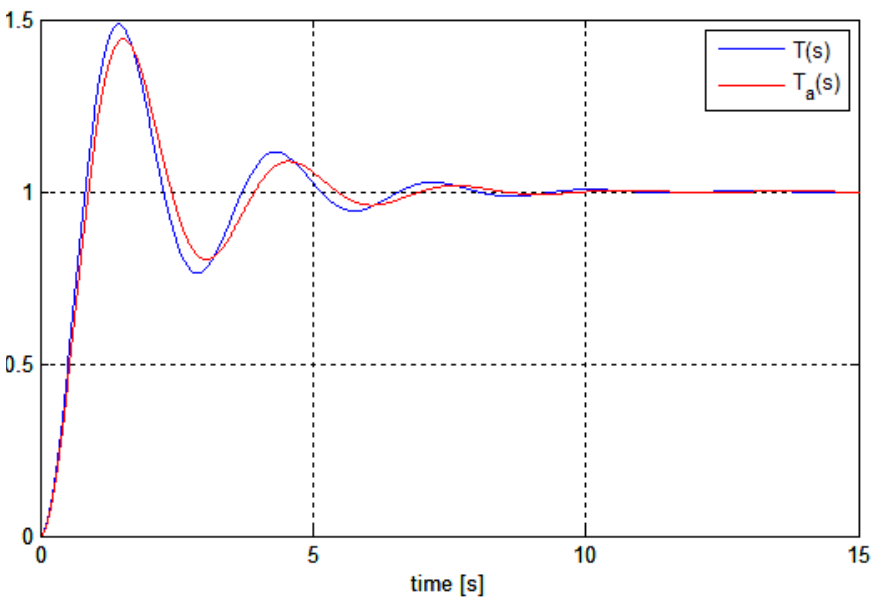
$$T_a(s) = \frac{1}{1 + 2\zeta s/\omega_n + s^2/\omega_n^2}$$

with

- $\zeta \cong \frac{\varphi_m}{100} = 0.25$
- $\omega_n = \omega_c = 2.13$

- In order to verify the effectiveness of the second order approximated model $T_a(s)$, let us compare the step response of $T(s)$and $T_a(s)$.





Rise time and overshoot of $T_a(s)$and $T(s)$ are very similar