Course of
"Automatic Control Systems"
2022/23

Control requirements: Transient performance

Prof. Francesco Montefusco

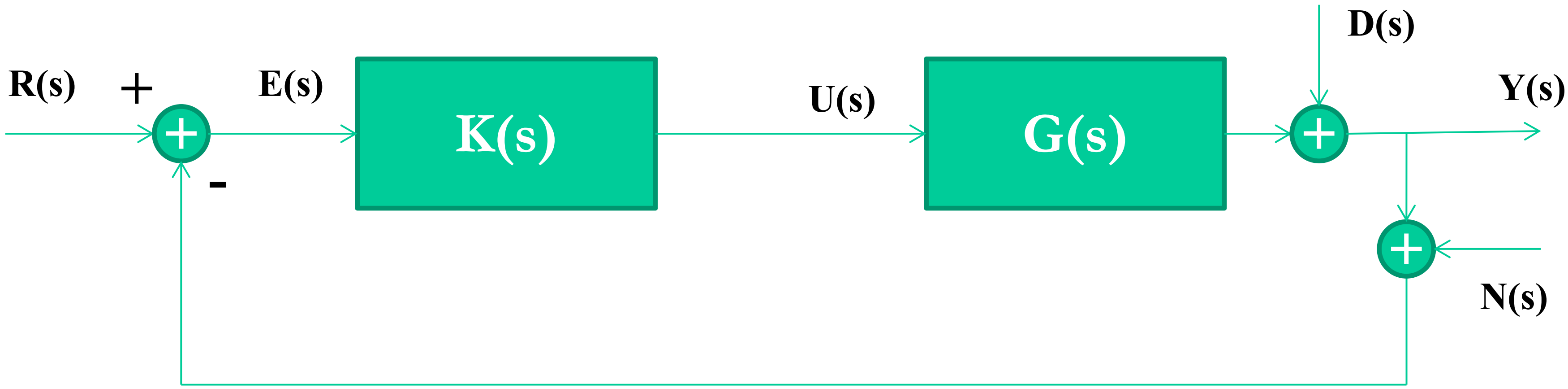
Department of Economics, Law, Cybersecurity, and Sports Sciences

Università degli Studi di Napoli Parthenope

francesco.montefusco@uniparthenope.it

Team code: **uxbsz19**

- A SISO closed loop control system in the Laplace domain can be indicated as



- **G(s) plant to be controlled**
- **K(s) controller**
- **R(s) reference**
- **Y(s) controlled output**
- **U(s) control variable**
- **E(s) tracking error**
- **D(s) disturb**
- **N(s) measurement noise**

Closed loop function

$$T(s) = W(s) = \frac{G(s)K(s)}{1 + G(s)K(s)}$$

Different notations T or W

Control requirements

- The closed loop control requirements can be divided in four classes:
  - ***Stability (DONE)***
  - ***Robust stability (DONE)***
  - ***Steady-state performances (DONE)***
  - ***Transient performances***

Transient performance

- The ***transient performance*** are usually expressed in terms of tracking properties of the reference signal $R(s)$.
- $R(s)$ is usually assumed as a polynomial signal of order 0 (step)
- The transient performance can be divided in
  - ***Dynamic precision*** (overshoot, oscillation period …)
  - ***Time response*** (rise time, peak time, settling time…)

- The rejection of the disturbs is usually not included among the transient performance because the transfer functions $R(s) \rightarrow Y(s)$ and $D(s) \rightarrow Y(s)$ have the same poles (excluding poles-zeros cancellation).

Transient performance

- Let us define two quantities in the frequency domain related to the transient behavior
  - **Bandwidth B_3 of the closed loop system** related to the settling time
  - **Resonant peak M_p of the closed loop system** related to the overshoot

- For the typical magnitude plots encountered so far, we define the **bandwidth B_3** as the first frequency such that for all frequencies greater than B_3 the magnitude is attenuated by a factor greater than $1/\sqrt{2}$ from its value in $\omega = 0$, that is

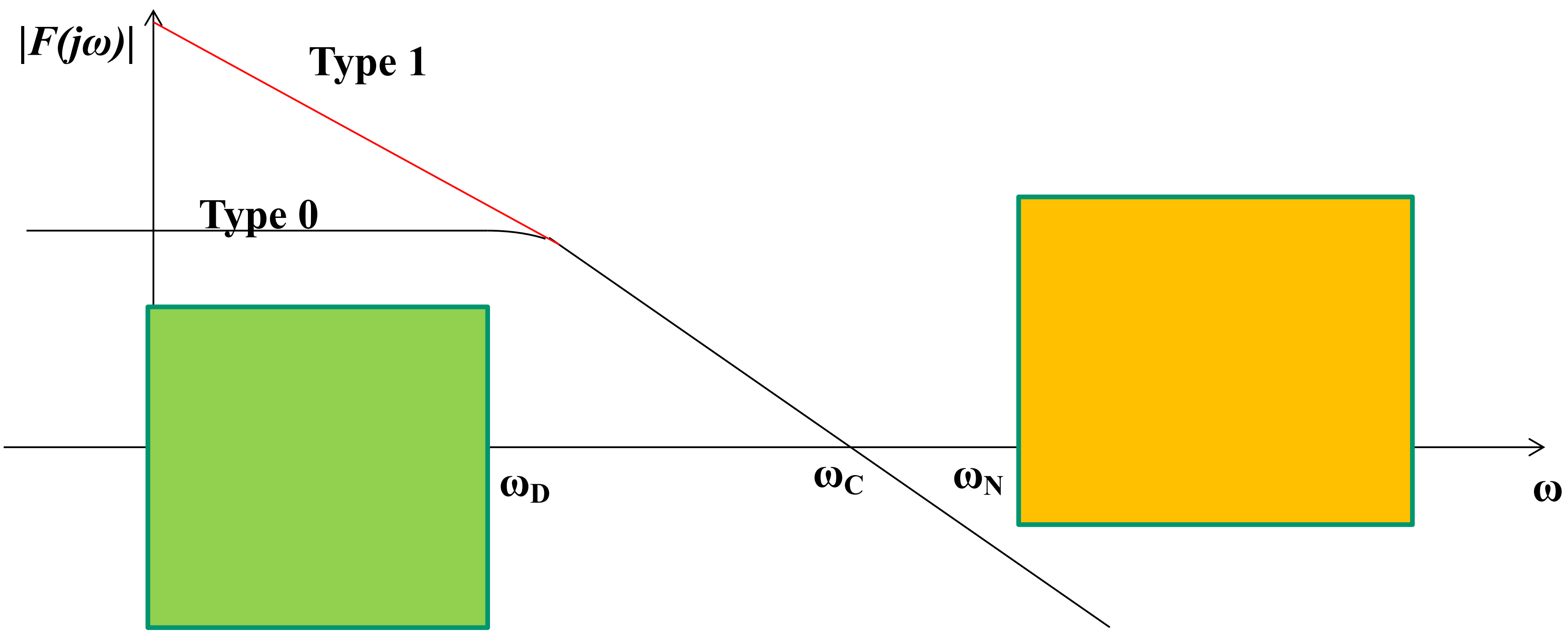
$$\frac{|T(jB_3)|}{|T(j0)|} = \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{|T(j\omega)|}{|T(j0)|} < \frac{1}{\sqrt{2}} \quad for\ all\ \omega > B_3$$

or equivalently in decibel

$$|T(jB_3)|_{\mathrm{db}} = |T(j0)|_{db} - 3_{db}$$

Open and closed loop transfer function

- In order to evaluate the **bandwidth** B_3 and the **resonant peak** M_p of the complementary sensitivity function $T(s)$, it is important to have an idea of the behavior of $T(s)$ assuming a regular stable open loop function $F(s)$.

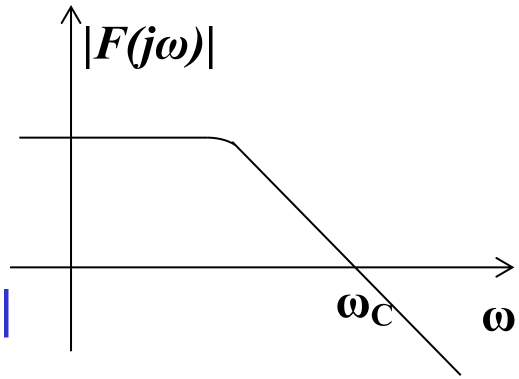


- Taking into account that

$$T(s) = \frac{F(s)}{1 + F(s)},$$

due to the steady state requirements, we usually have

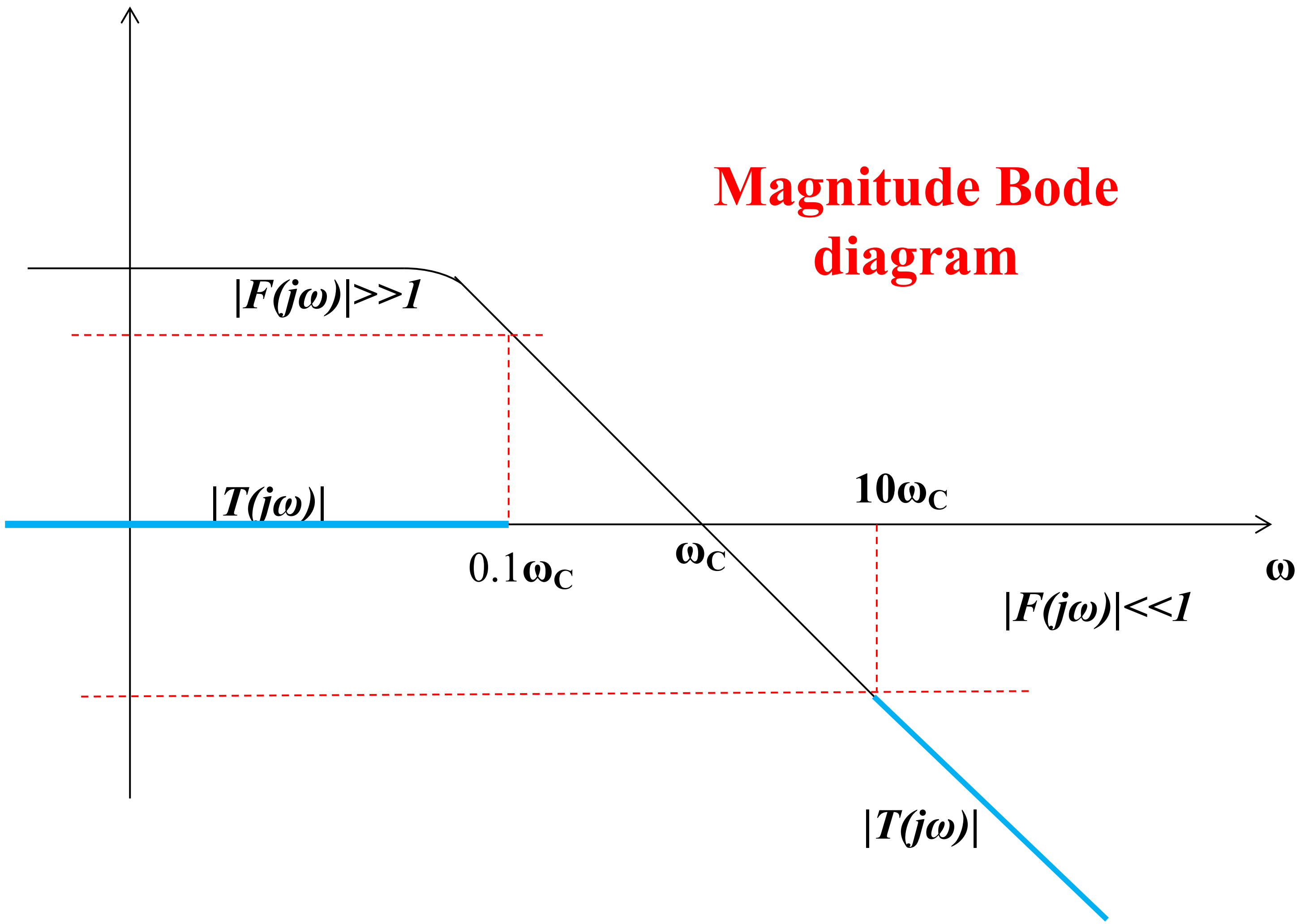
- at low frequencies $|F(s)| \gg 1 \;\rightarrow\; |T(s)| \cong 1$
- at high frequencies $|F(s)| \ll 1 \;\rightarrow |T(s)| \cong |F(s)|$



where we consider

- **Low frequencies** all the ω at least **one decade below the crossing frequency $\boldsymbol{\omega_c}$**
- **High frequencies** all the ω at least **one decade above the crossing frequency $\boldsymbol{\omega_c}$**

Open and closed loop transfer function



Transient performance

- Taking into account that $|T(j0)|_{db} \cong 0$
  - the bandwidth B_3 of $T(s)$ is the first frequency such that for all frequencies greater than B_3 the magnitude is less than $-3db$
  - the resonant peak M_p of $T(s)$ is the maximum value assumed by the magnitude of $T(s)$

- In order to quantify the bandwidth B_3 and resonant peak M_p of $T(s)$ we need to analyze the behavior of $T(s)$ in the two decades with center ω_c

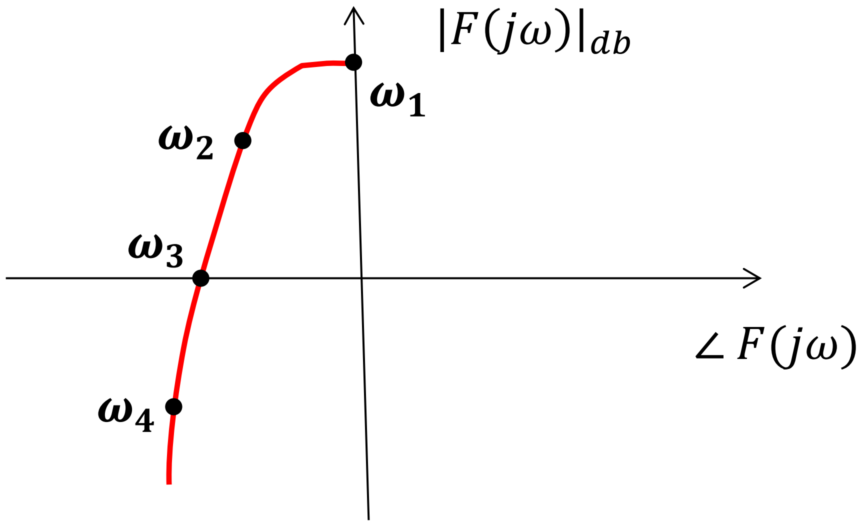
- To do aim, we will introduce the so called ***Nichols chart*** that relates the magnitude and phase of the open loop function $F(s)$ to the the magnitude and phase of the closed loop function $T(s)$

Nichols plots

- The Nichols plots are an **alternative solution to the Bode diagrams** for the representation of the transfer functions with $s = j\omega$.

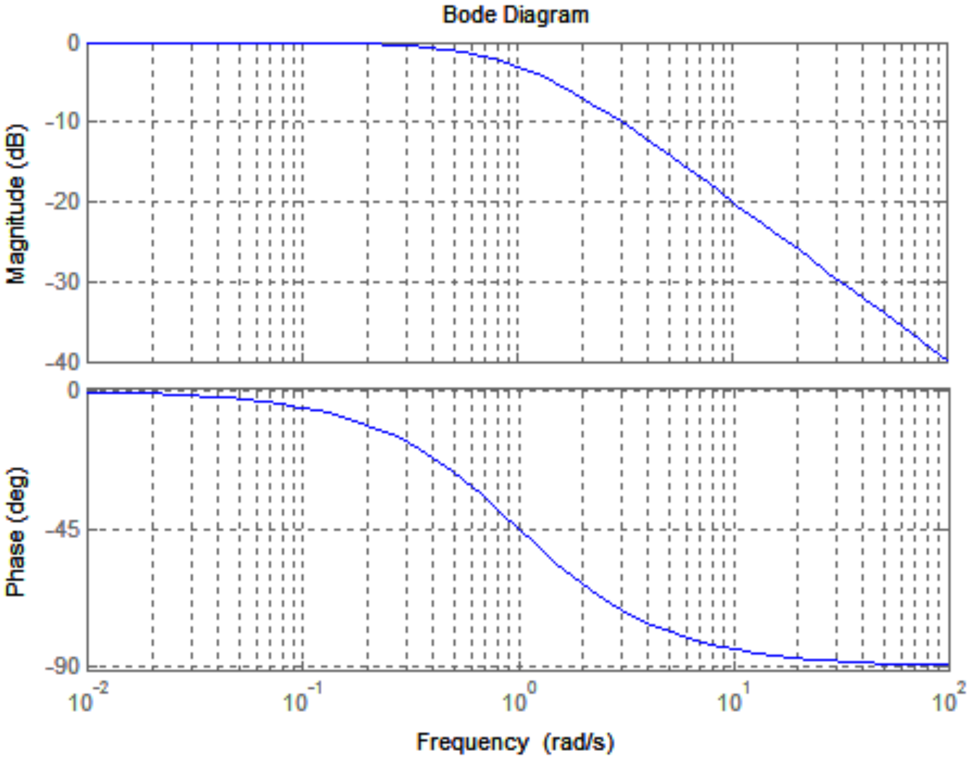
- The Nichols plots are composed by:
  - **x-axis reporting the phase of the transfer function**
  - **y-axis reporting the magnitude in decibel of the transfer function**

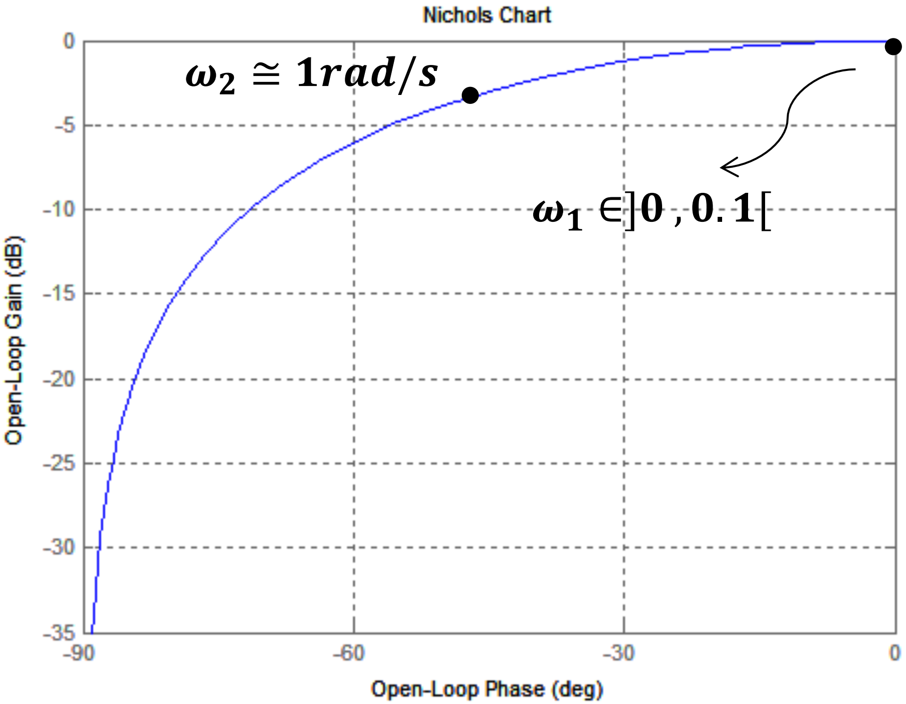
- In a Nichols plot **magnitude and phase of transfer function are represented by a curve parametrized in ω.**



- The Nichols plot can be obtained from the Bode plots of magnitude and phase.

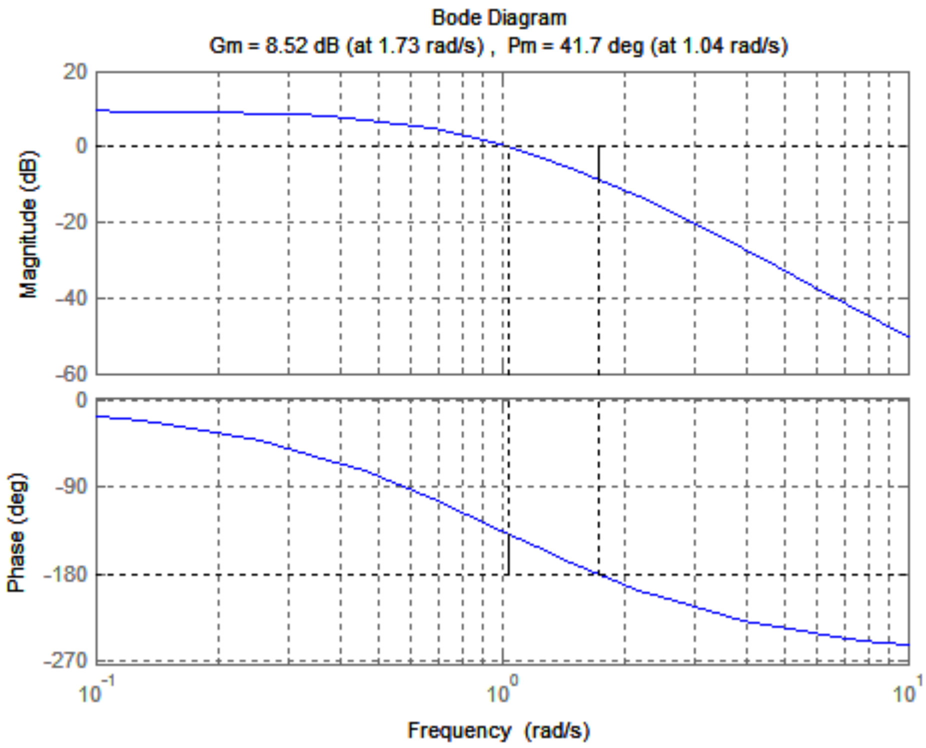
$$F(s) = \frac{1}{1+s}$$

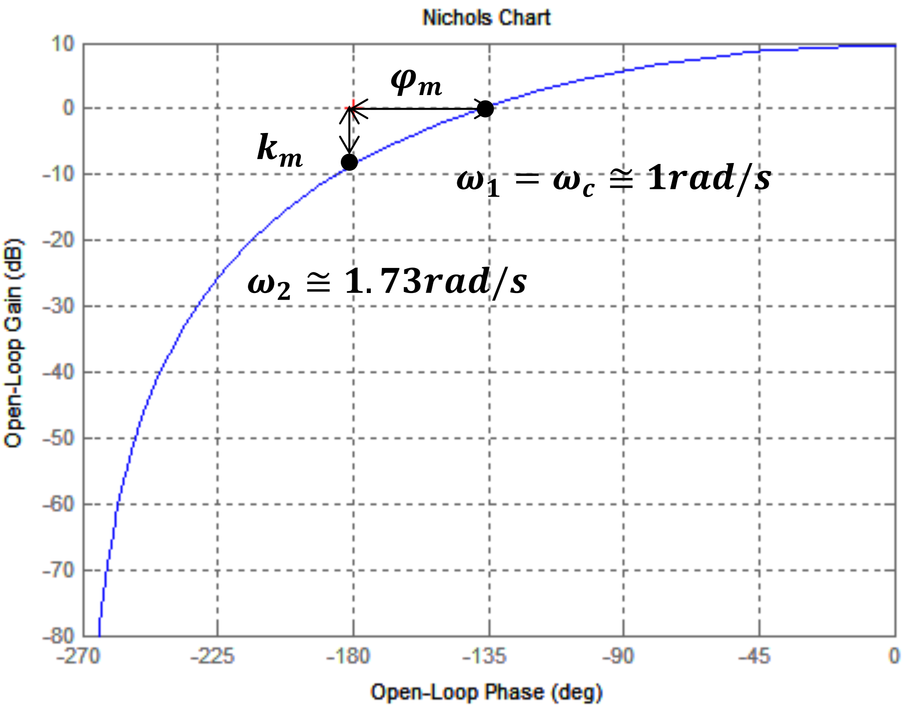




- Note that a point on the Nichols plots can also indicate the value of $F(j\omega)$ in a finite interval of ω.

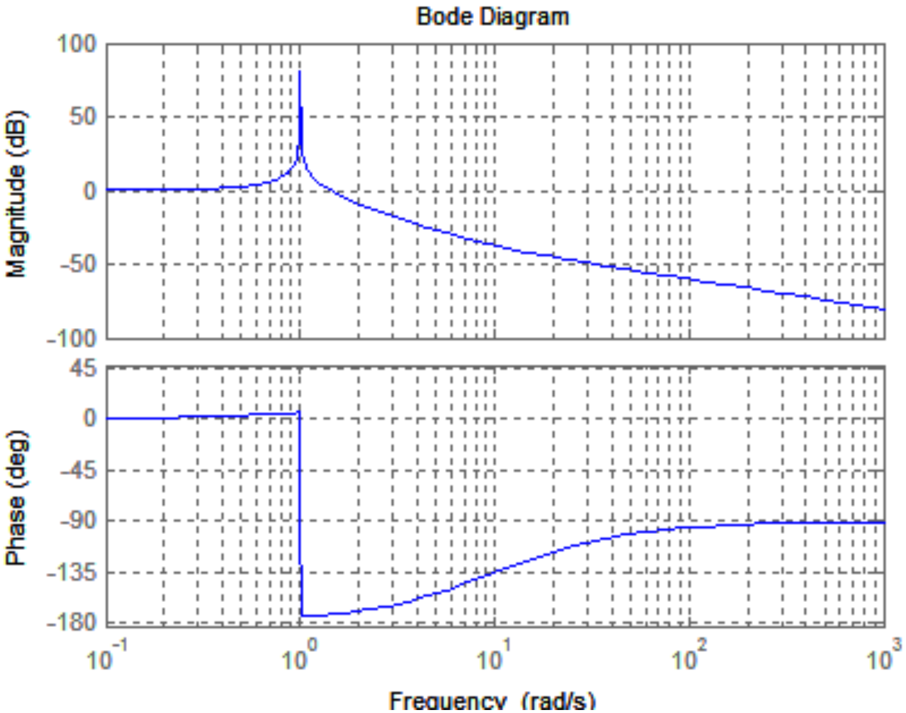
- Let us consider the transfer function $F(s) = \frac{3}{(1+s)^3}$

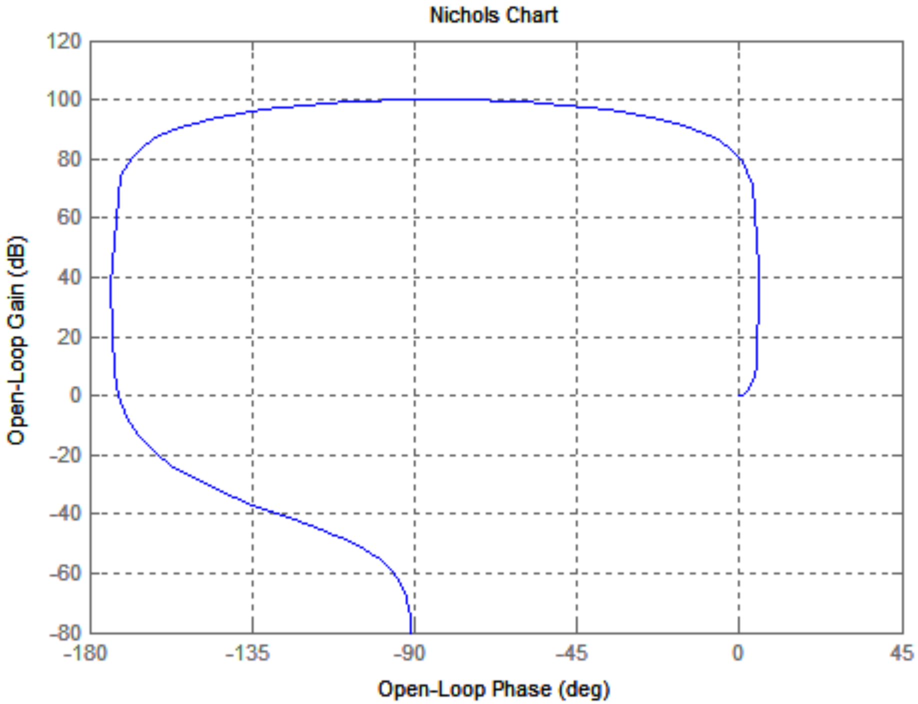




- Frequency function with resonance

$$F(s) = \frac{1 + 0.1s}{1 + s^2}$$





- Nichols charts consist of constant-magnitude loci and constant phase loci of the closed loop system $T(s)$ as a function of the open loop system $F(s)$

- Let us indicate the magnitude and phase of $F(j\omega)$ and $T(j\omega)$ with

$$F(j\omega) = A(j\omega)e^{\alpha(j\omega)} \quad \text{and} \quad T(j\omega) = M(j\omega)e^{\varphi(j\omega)}$$

- Taking into account that

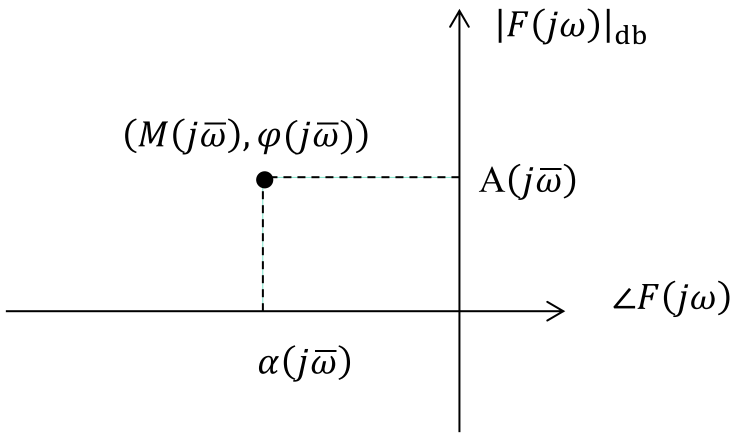
$$T(j\omega) = \frac{F(j\omega)}{1 + F(j\omega)}$$

we have that

$$M(j\omega) = \frac{A(j\omega)}{\sqrt{1 + A(j\omega)^2 + 2A(j\omega)\cos(\alpha(j\omega))}}$$

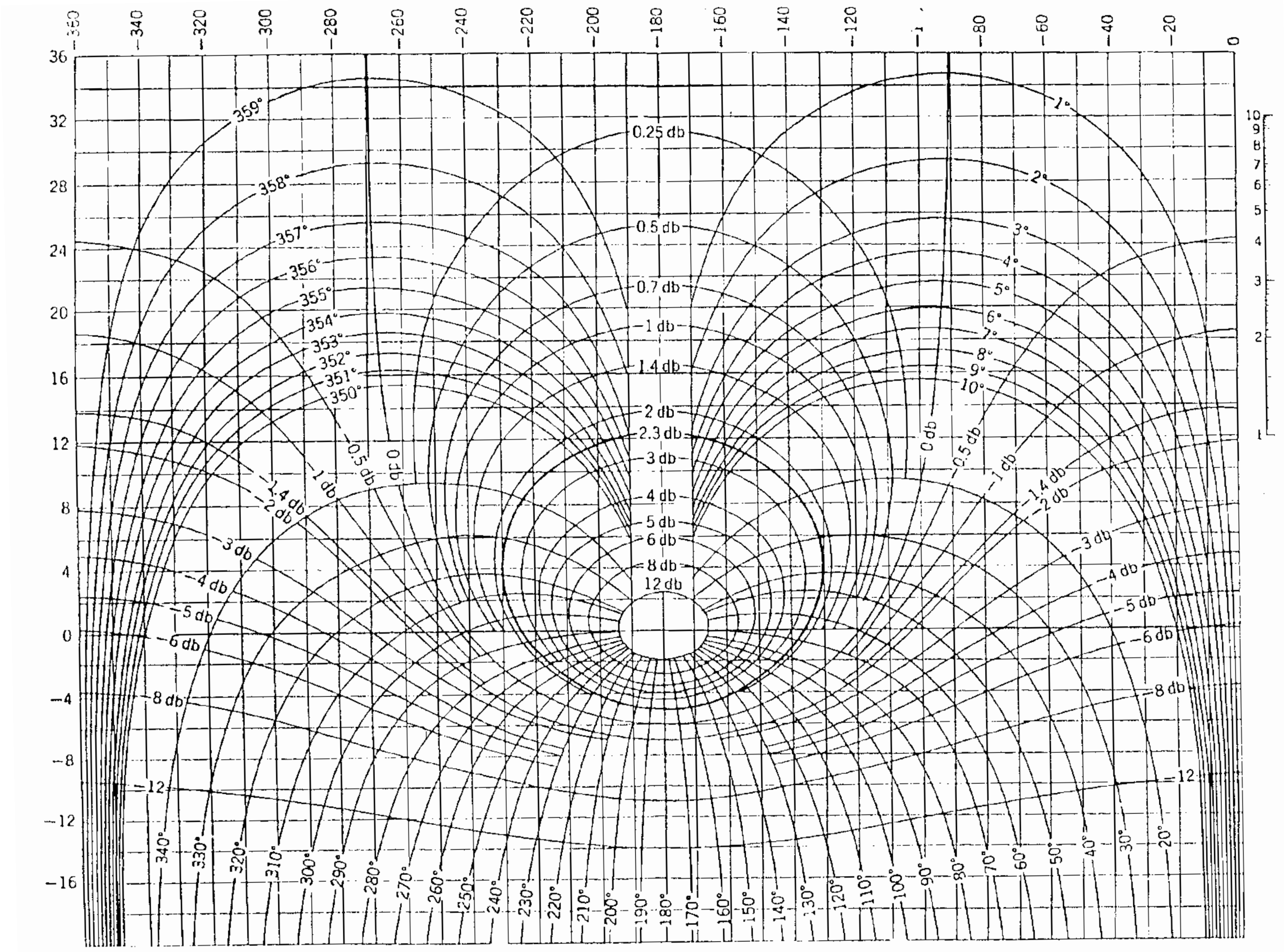
$$\varphi(j\omega) = arctg\left(\frac{\sin(\alpha(j\omega))}{A + \cos(\alpha(j\omega))}\right)$$

- For a fixed value of $\overline{\omega}$, we can
  - identify a point on the Nichols plot of the open loop function $F(j\omega)$
  - associate to this point the corresponding value of magnitude and phase of the closed loop function $T(j\omega)$.



- Repeating this procedure on a grid on points of the Nichols plot of $F(j\omega)$ it will be possible to find the constant-magnitude loci and constant phase loci of the closed loop system $T(s)$

Nichols chart



Nichols chart

- Making use of the previous relations

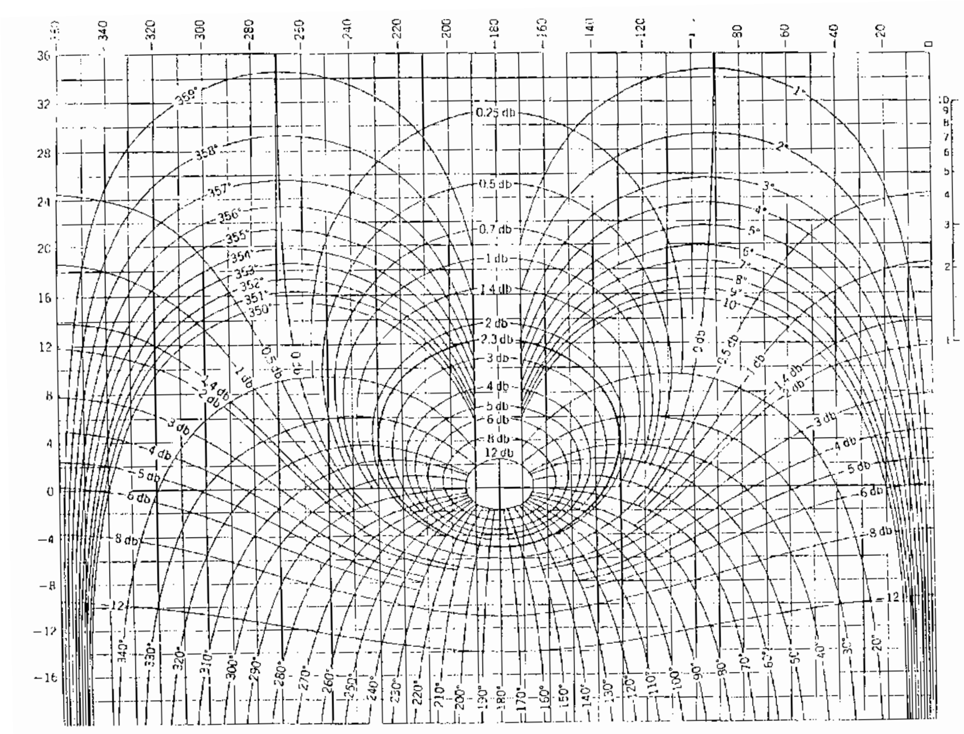
$$M(j\omega) = \frac{A(j\omega)}{\sqrt{1 + A(j\omega)^2 + 2A(j\omega)\cos(\alpha(j\omega))}}$$

$$\varphi(j\omega) = arctg\left(\frac{\sin(\alpha(j\omega))}{A + \cos(\alpha(j\omega))}\right)$$

we can notice that:

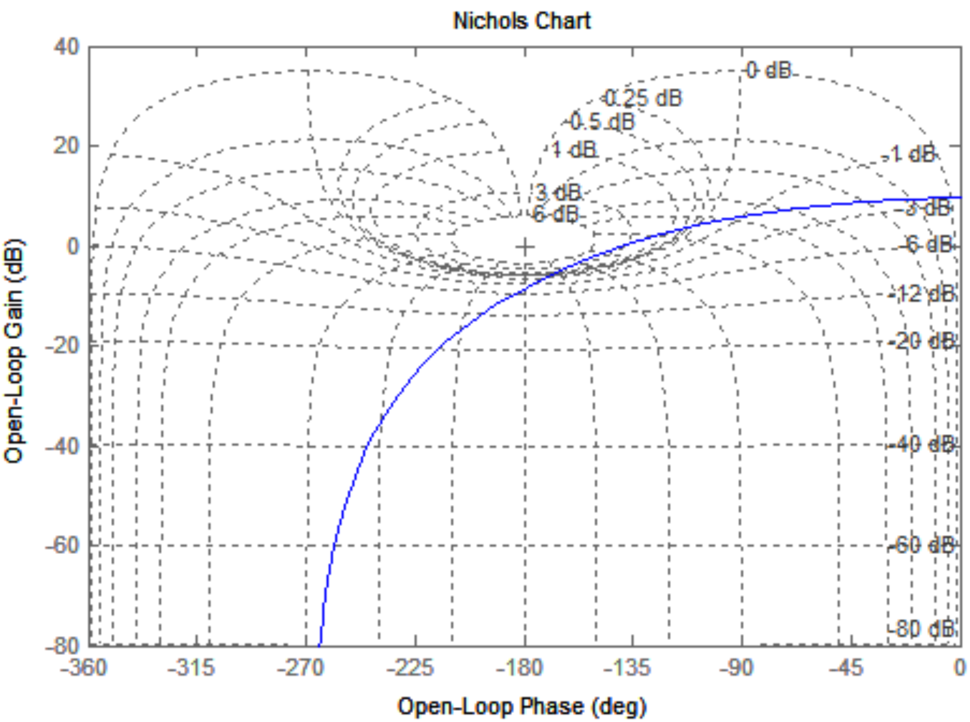
- $A \gg 1 \quad \rightarrow \quad \begin{cases} M \approx 1 \\ \varphi \approx 0 \end{cases}$

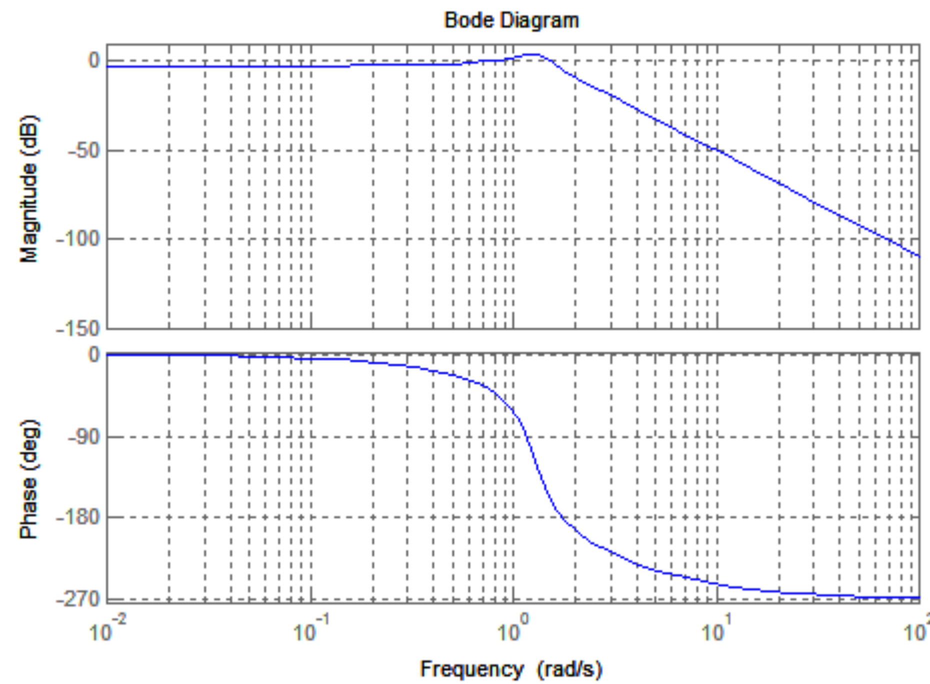
- $A \ll 1 \quad \rightarrow \quad \begin{cases} M \approx A \\ \varphi \approx \alpha \end{cases}$



Nichols chart: example

- Let us consider the transfer function $F(s) = \dfrac{3}{(1+s)^3}$





Open loop function $F(j\omega)$ **Closed loop function $T(j\omega)$**