# Exercises

SC2_08 – Conformal transformations.

1.  The following MATLAB code displays an arc of parabola together with some tangent segments. It uses symbolic computations emphasized in red below. Change the symbolic code into numerical code, by approximating derivatives by means of difference quotients or by means of other finite differences, and produce a similar plot.

```
syms t real;  z=@(t) t+i*t^2;
N=197; tt=linspace(-3,3,N);  zt=subs(z(t),tt);
zprime=(diff(z(t)));  zprimet=subs(zprime,tt);
rho=linspace(0,1,50)'; j=0; step=7;
for h=1:step:N
    j=j+1; tanz(:,j)=zt(h)+rho*zprimet(h);
end
plot(real(tanz),imag(tanz),real(zt),imag(zt),'k');
axis([-3 3 -1 4]); axis('equal')
```
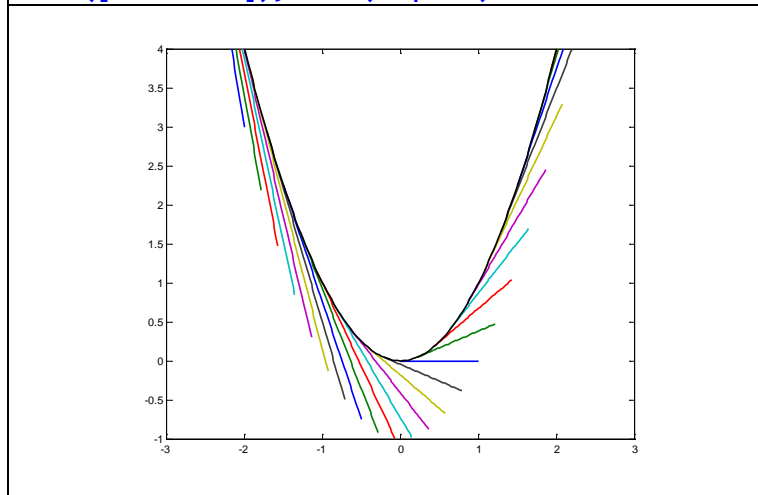


2.  Compute, in the $z$-plane, the angle between the line of complex parametric equation $z_1=t+it$, and the parabola of complex parametric equation $z_2=t+it^2$, at their intersection point P(1,1), and the angle between their image curves in the $w$-plane, by means of the following mappings between complex planes:

    - $w = f(z) = 2z - 1$
    - $w = f(z) = \overline{z}$

3.  Implement, in a MATLAB function, the Steregraphic Projection and its inverse transformation, and display some correspondences between *"generalized circles"*.

4.  Discuss the following Moebius Map $T_M(z)$, highlighting how some *"generalized circles"* are transformed; moreover, find the image of $D = \{z \ : \ |z+1| < 1\}$ under $T_M$. Display your results graphically.

    $$w = T_M(z) = \frac{(1-i)z+2}{(1+i)z+2}$$

5.  Find a Moebius transformation which maps the real axis (together with $\infty$) to itself and which maps the unit circle $|z| = 1$ to the imaginary axis (together with $\infty$). Display some correspondences between *"generalized circles"* and between *"generalized disks"* (internal domains).

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6.  Find a Moebius transformation which maps the disk centered at $i$, and of radius $1/3$, onto the half-plane $\{w = u + iv : u > v\}$. Display this correspondence.

7.  Write a single MATLAB function `W=TM(zk,wk)` that includes the 3 cases:
    *   none among the 6 points ($z_k$ and $w_k$) at $\infty$;
    *   a point $z_k$ at $\infty$;
    *   a point $w_k$ at $\infty$.
    [Hint: `isinf()` and/or `isfinite()` functions]

8.  Implement the algorithm described in the example 1 of Applications of Moebius maps [see: `SC2_08e.pdf` pag. 12], starting from centers and radii of the circles $\Gamma_1$ and $\Gamma_2$ in the $z$-plane:
    *   $\Gamma_1$: center at `c1=0.4`, radius `r1=0.15`;
    *   $\Gamma_2$: center at `c2=0.5`, radius `r2=0.25`;
    Compute also centers and radii of the output circles in the $z$-plane.

9.  How to avoid distorting Side 4 of the transformed rectangle in the example 2 of Applications of Moebius maps [see: `SC2_08e.pdf` pag. 18 - 21].

10. How to avoid distorting Sides 4, 1 and 3 of the transformed rectangle in the example 2 of Applications of Moebius maps [see: `SC2_08e.pdf` pag. 24].

11. Download, from the e-Learning platform, `photo.jpg`, or `photo2.png`, or `photo4.jpg`, or choose another crooked image, and implement the same algorithm as in the previous exercise, to straighten a crooked image.

12. Show graphically, with MATLAB, that the complex function $w = f(z) = z + 1/z$ maps the region in the following image to the upper half-plane $\text{Im}(w) \geq 0$ of the $w$-plane.
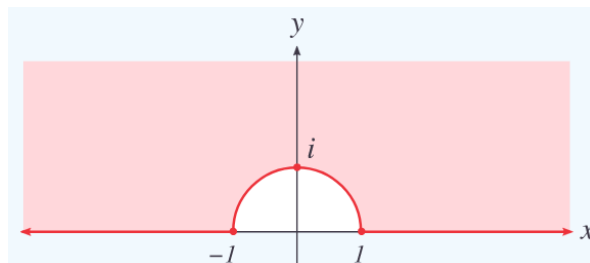


[Hint: NaN values are not displayed in MATLAB figures]