Scuola Interdipartimentale delle **Scienze**, dell'**Ingegneria** e della **Salute**

DiST — Dipartimento di Scienze e Tecnologie — Università Parthenope

L. Magistrale in IA (ML&BD)

Scientific Computing (part 2 – 6 credits)

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Contents

- **Examples of Conformal Mappings as complex functions.**

Example 1: Rotation by 30°

as a complex function

$$w = f(z) = z e^{i\pi/6}$$

$f(z)$ is holomorphic w.r.t. z

conformal map $\forall\, z \in \mathbb{C}$

$$f'(z) = e^{i\pi/6} = \left[1,\ \frac{\pi}{6}\right]$$

no critical point

$$\ell^* = \ell \left| f'(z) \right| = \ell\ 1$$

homothety with factor **1**

identity

$$\theta^* = \theta + \arg f'(z) = \theta + \frac{\pi}{6}$$

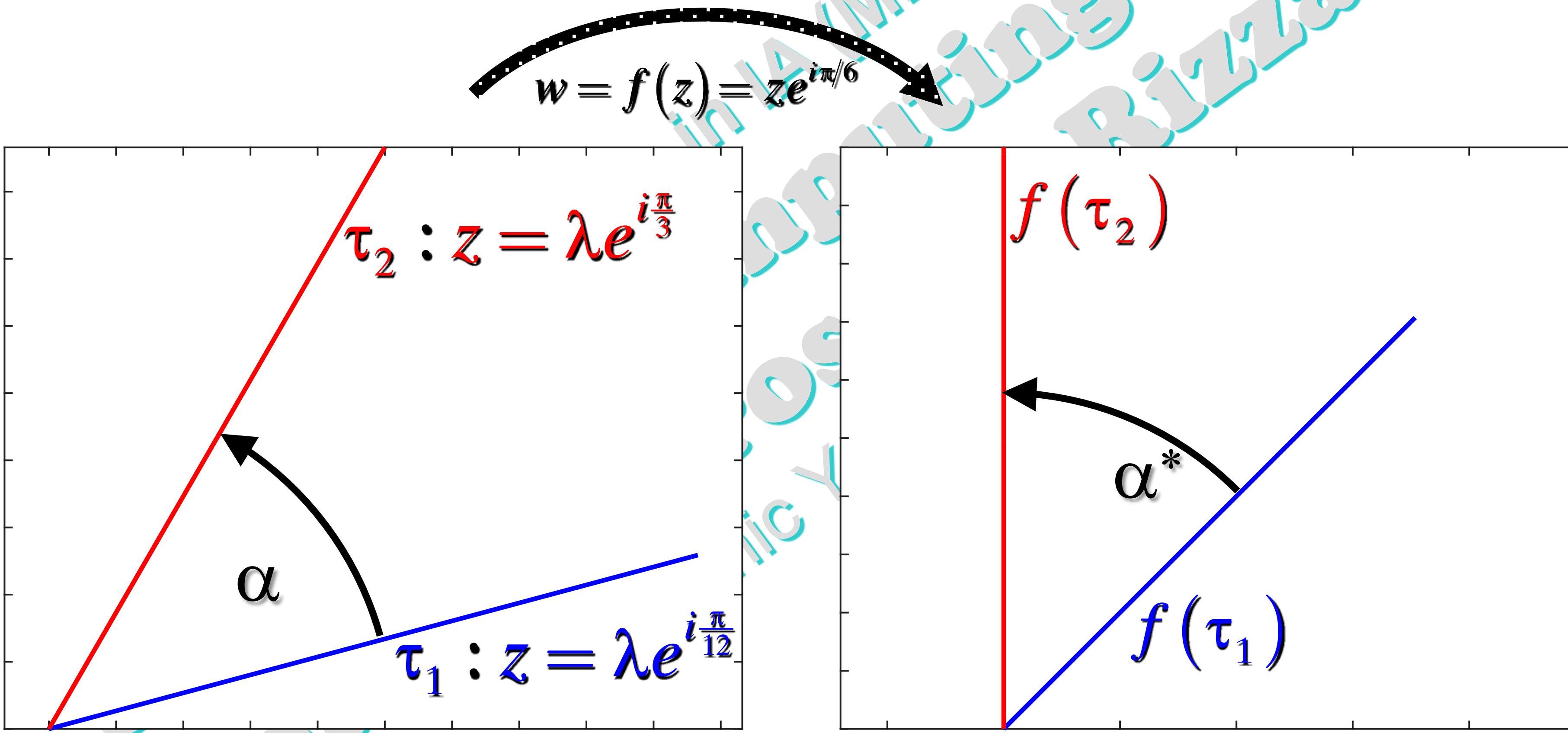
rotation by an angle **π/6**

... locally

a rotation behaves locally in the same way as globally

Example 1: Rotation by 30° (cont.)

$$w = ze^{i\pi/6}$$

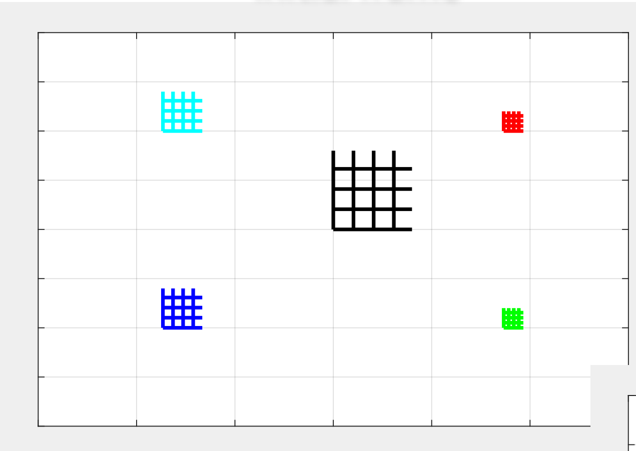


the same magnitude, the same orientation: conformal

Example 1: rotation map

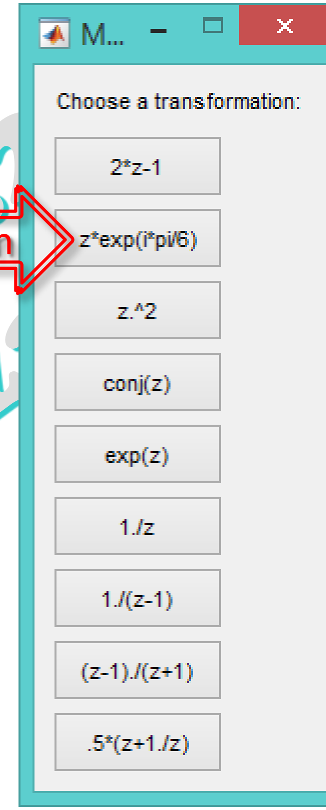
Download conformal.p* and run it

initial frame



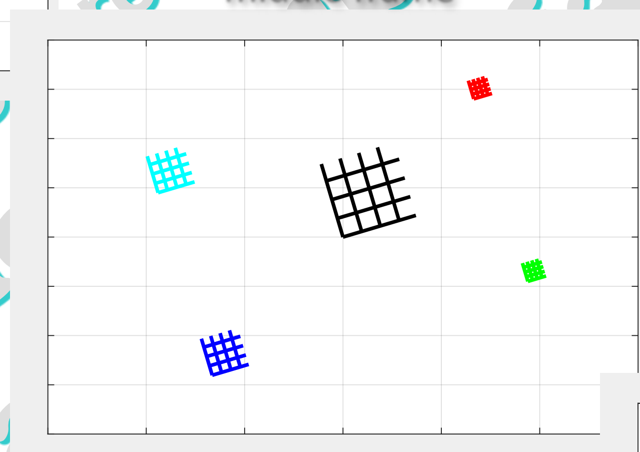
```
>> conformal
```

rotation

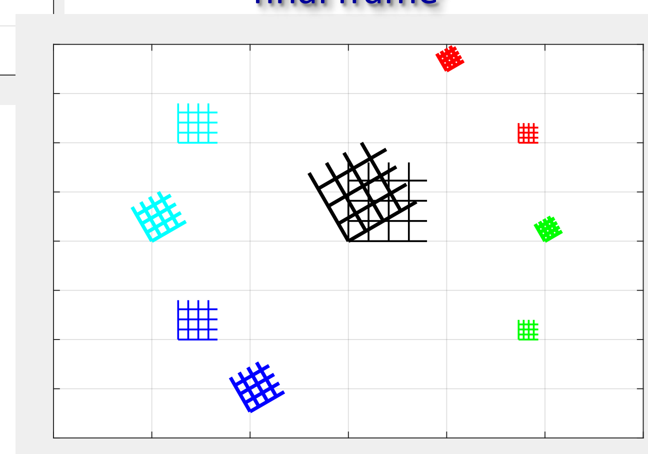


menu()

middle frame



final frame



* Download the p-code from the E-learning platform

Example 2: quadratic map

as a complex function $w = f(z) = z^2$

conformal $\forall z \in \mathbb{C} - \{0\}$ ⇐ $f'(z) = 2z = [2|z|,\ \arg z]$

critical point: $z=0$

locally at: $z \neq 0$

scale factor

$\ell^* = \ell |2z| = \ell\, 2|z|$

angle

$\theta^* = \theta + \arg f'(z) = \theta + \arg z$

homothety

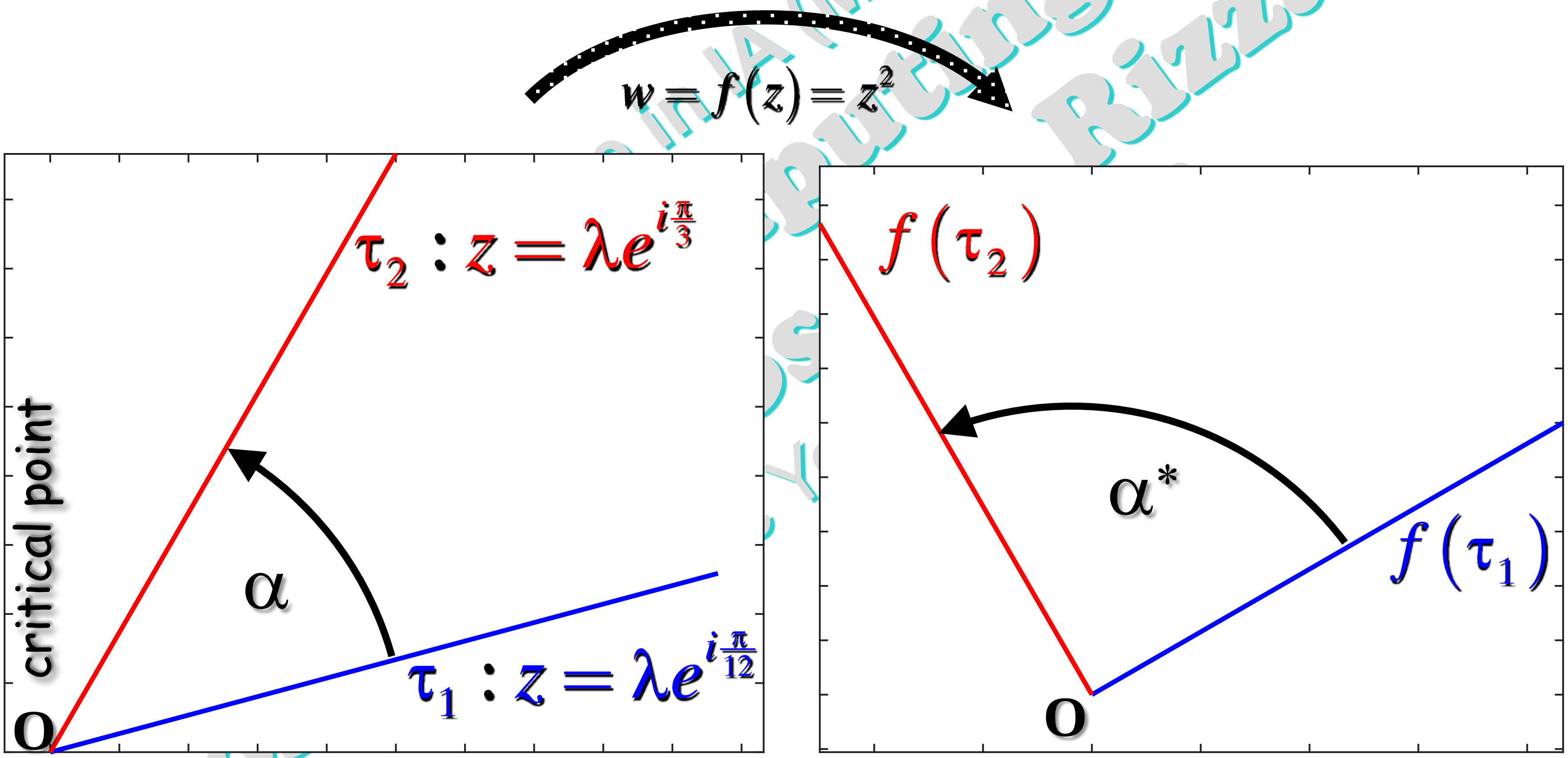
rotation

The local homothety and rotation change at each point of the complex plane

... locally

Example 2: quadratic map (cont.)

critical point: $z=0$

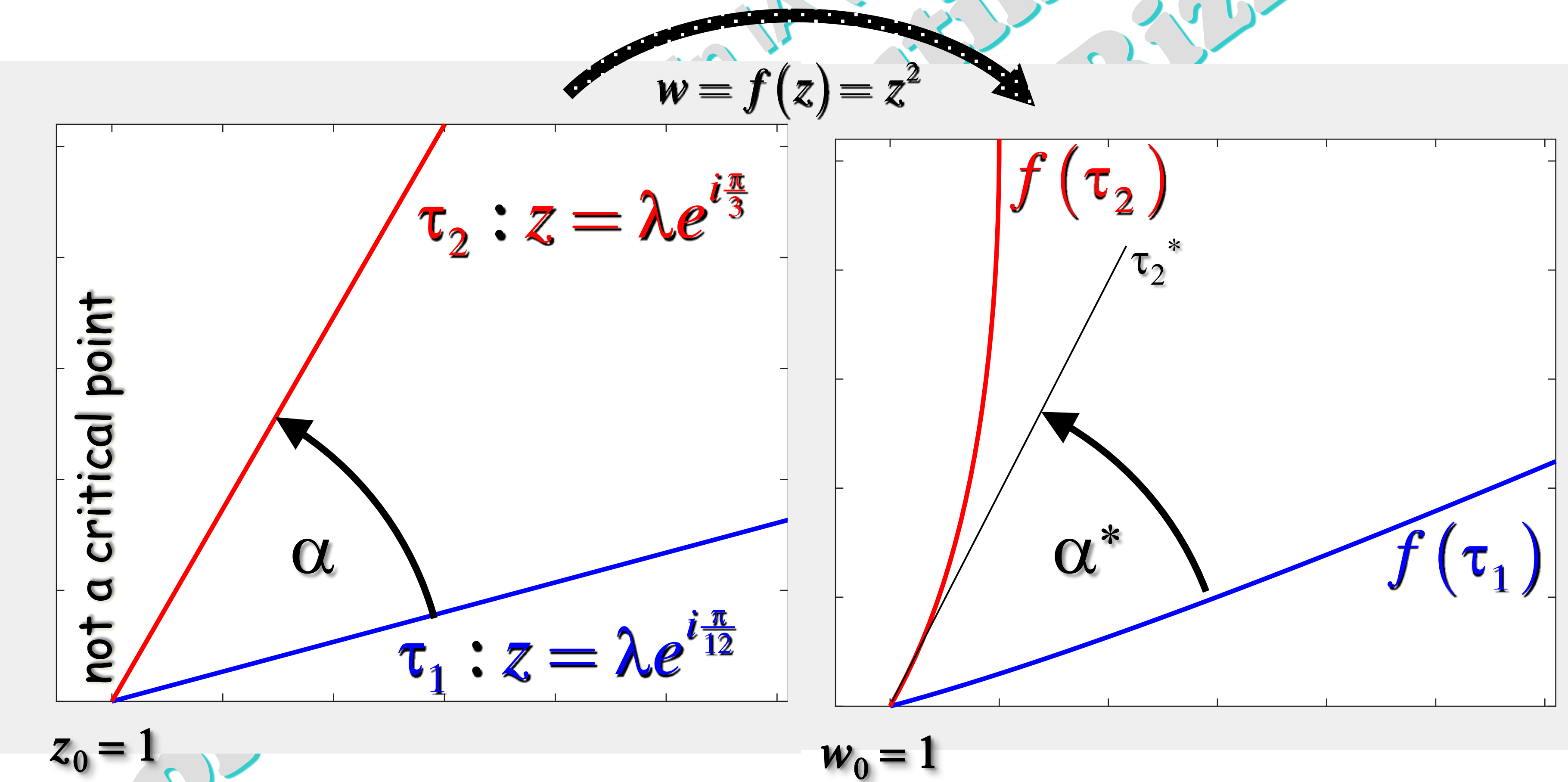


not the same magnitude: non-conformal at O

Example 2: quadratic map (cont.)

$$w = z^2$$

at a point $z_0 \neq 0$



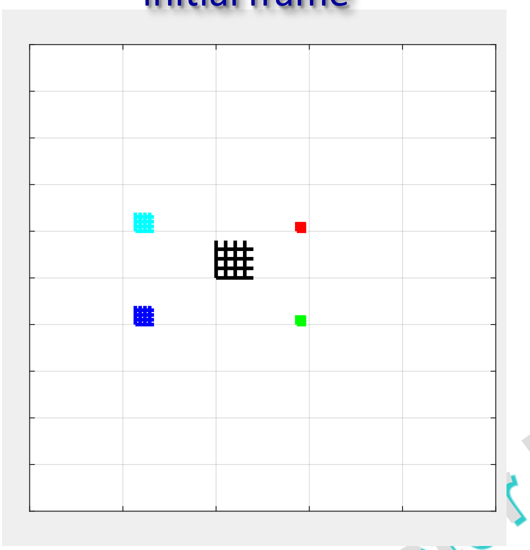
the same magnitude, the same orientation: conformal

(prof. M. Rizzardi)

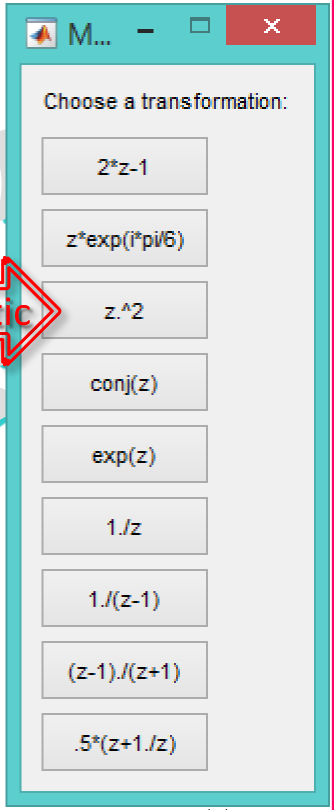
Example 2: quadratic map

Download and run conformal.p

initial frame



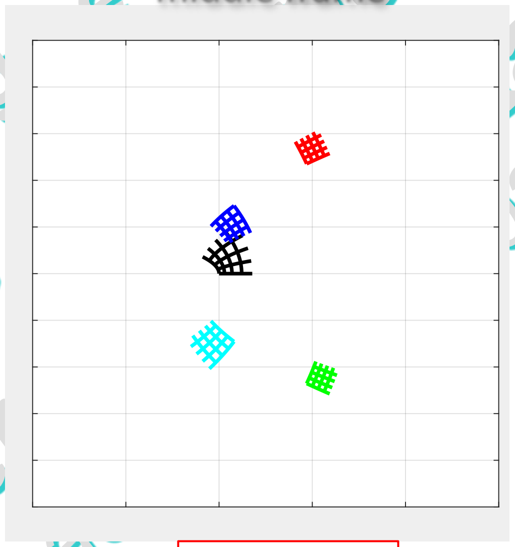
```
>> conformal
```



quadratic

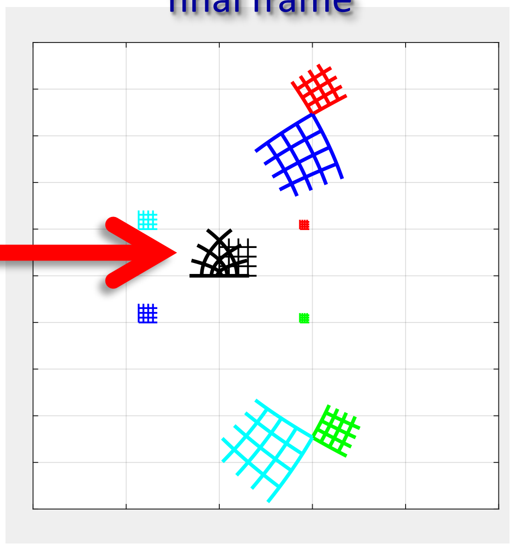
menu()

middle frame



at origin

final frame



Example 3: complex conjugate map

as a complex function $w = f(z) = \overline{z}$

$f(z)$ is not holomorphic w.r.t. z

anticonformal ⇐ $f(z)$ is anti-holomorphic*

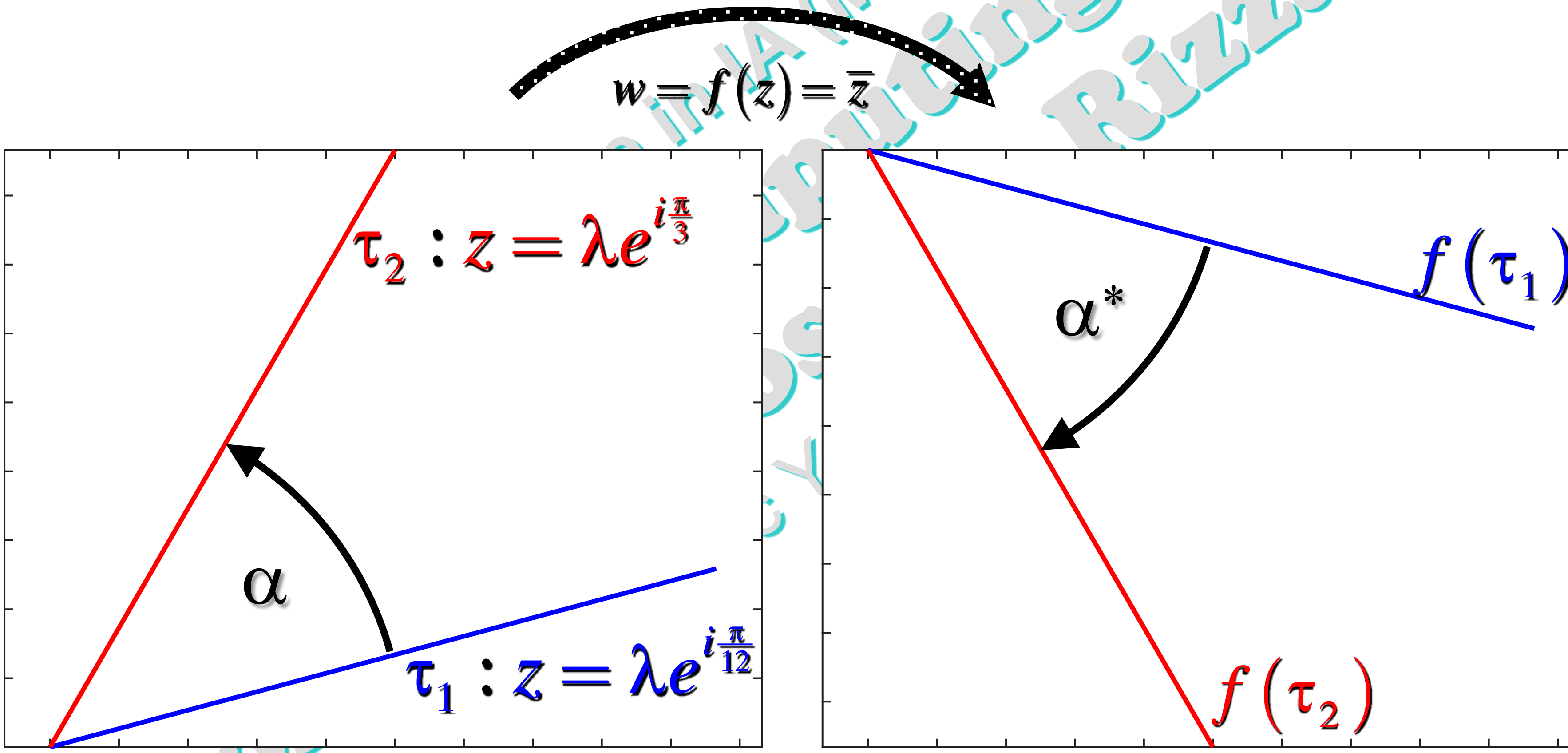
$$w = f(z) = \bar{z} = x - iy \Rightarrow \frac{\partial f}{\partial x} = 1, \quad \frac{\partial f}{\partial y} = -i \Rightarrow \frac{\partial f}{\partial x} - i\frac{\partial f}{\partial y} = 0$$

conjugate Cauchy-Riemann Eqs.

* $f(z)$ is anti-holomorphic, i.e. $f(z)$ is differentiable w.r.t. $\overline{z}$

Example 3: complex conjugate map (cont.)

anticonformal

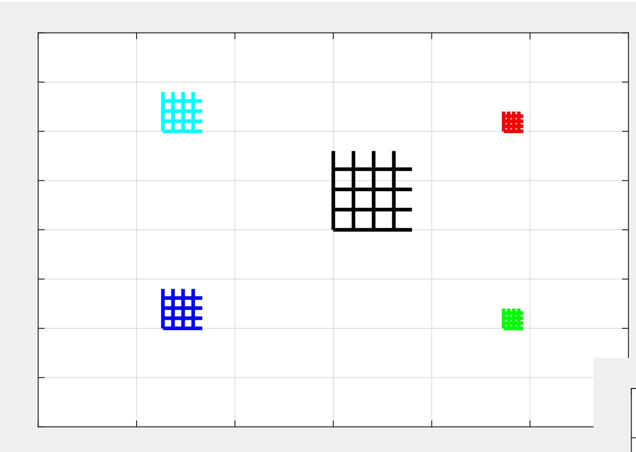


the same magnitude, inverse orientation: anticonformal

Example 3: complex conjugate map

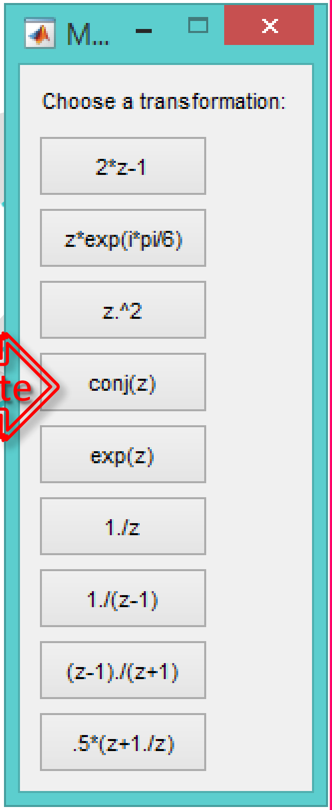
Download and run `conformal.p`

initial frame



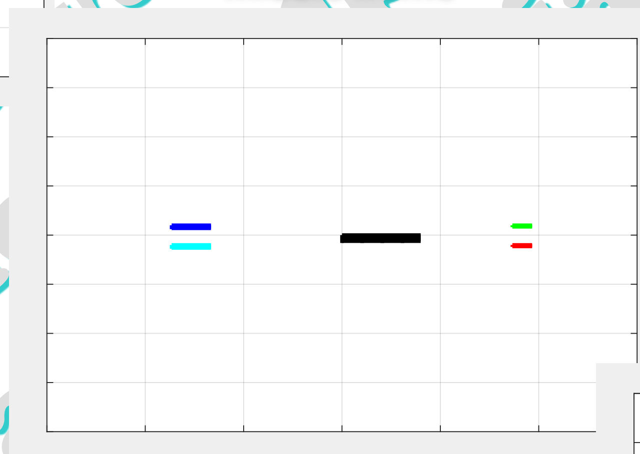
```
>> conformal
```

conjugate

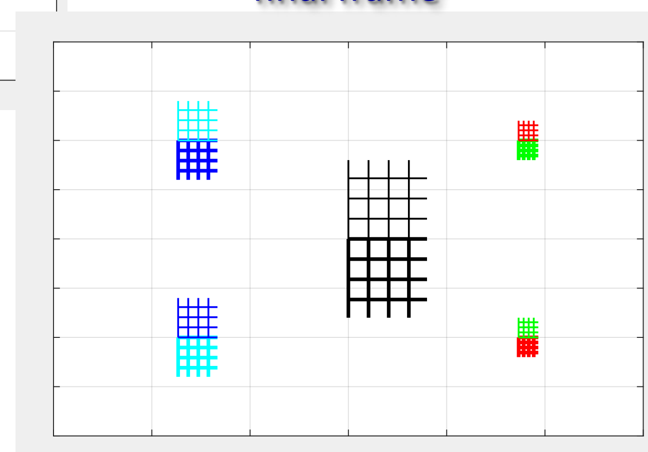


`menu()`

middle frame



final frame



Example 4: exponential map

Download and run **conformal.p**

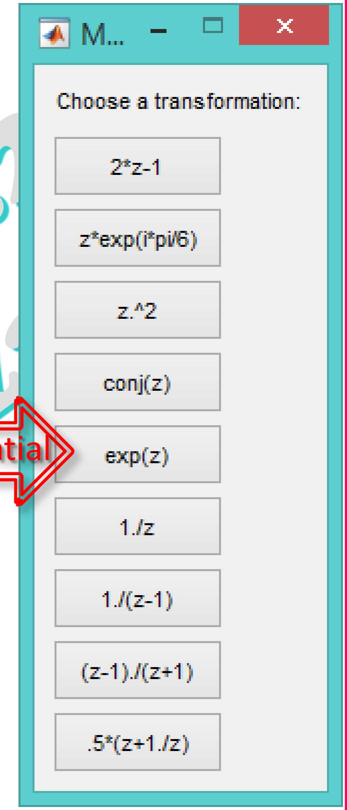
$$w = f(z) = e^z \quad \Rightarrow \quad f'(z) = e^z$$

$$f'(z) = e^z = e^x e^{iy} = [e^x,\ y+2k\pi]$$

$\Rightarrow$

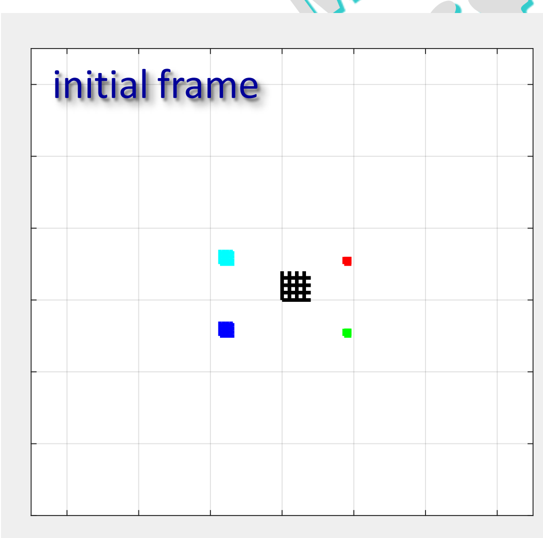
scale factor
$$|f'(z)| = e^x$$
angle
$$\arg[f'(z)] = y$$

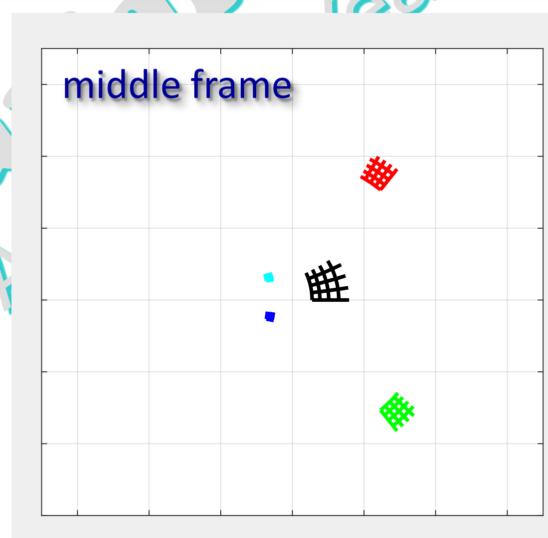
The local homothety and rotation change at each point of the complex plane

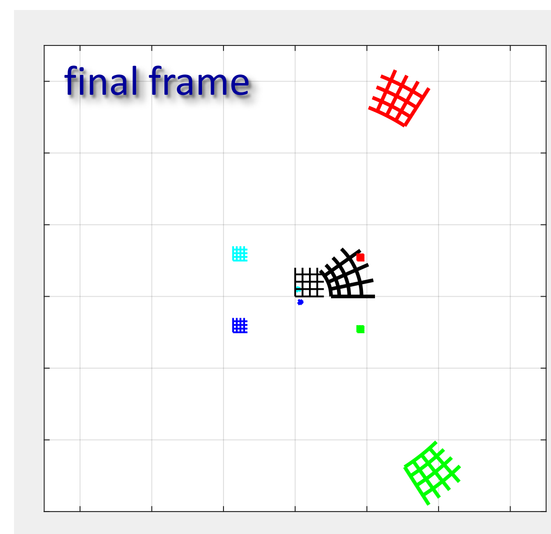


exponential

```
>> conformal
```







Example 5: inversion map

Download and run conformal.p

$$w = f(z) = 1/z \quad \Rightarrow \quad f'(z) = -1/z^2$$

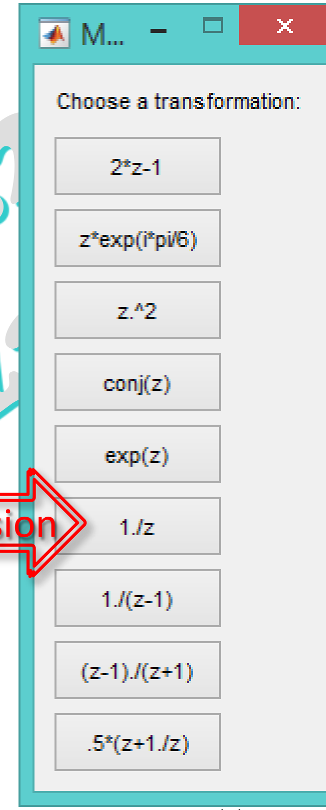
```
syms x y real; z=x+i*y; f=1/z;
disp(diff(f,x)+i*diff(f,y))
0
fprime=diff(f,x)
fprime =
-1/(x + y*1i)^2
```

holomorphic

The local homothety and rotation change at each point of the complex plane

$$\lim_{z\to\infty} f'(z) = 0$$ it is non-conformal at infinity

```
>> conformal
```



inversion

menu()