Course of
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Control requirements: Steady state performance

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Steady-state performance

- The steady-state performance depends on the class of input signals $R(s), D(s), N(s)$ and the type of polynomial transfer function $F(s)$

- ***Tracking of the reference input $R(s)$***
  - Null or bounded steady-state error to polynomial inputs ***(DONE)***
  - Null or bounded steady-state error to ***sinusoidal inputs*** at fixed frequency

- ***Rejection of the disturbs D(s)***
  - Null or bounded steady-state error to polynomial inputs ***(DONE)***
  - Bounded steady-state error to ***multi-frequency sinusoidal inputs***

- ***Insensibility to the noise $N(s)$***
  - Bounded steady-state error to ***multi-frequency sinusoidal inputs***

Steady-state error to sinusoidal reference at fixed frequency

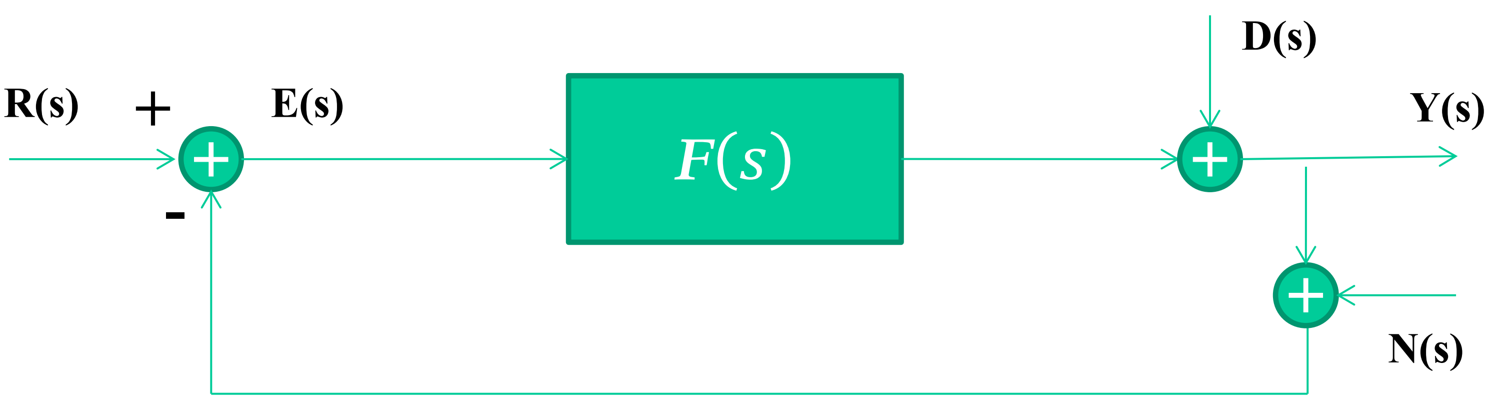
- Let us consider a sinusoidal reference at frequency ω_0

$$r(t) = R_0 sin(\omega_0 t) \quad \text{with Laplace transform} \quad R(s) = \frac{\omega}{s^2+\omega_0^2}$$

- The steady state error to a sinusoidal reference signal can be written as

$$e_{ss}(t) = R_0 \ |S(j\omega_0)| \, sin(\omega_0 t + \angle S(j\omega_0))$$

where $S(s) = \frac{1}{1+F(s)}$ is the sensitivity function.



Steady-state error to sinusoidal reference at fixed frequency

- In order to achieve null steady-state error we need that

$$|S(j\omega_0)| = \left|\frac{1}{1 + F(j\omega_0)}\right| = 0\,.$$

- This can be achieved with a pair of imaginary zeros at the frequency ω_0 of the input signal in the sensitivity function (antiresonance)

- Hence a pair of imaginary poles have to be added in the O.L. function $F(j\omega_0)$

- If the imaginary poles are not included in the plant model $G(s)$, they have to be added in the controller

$$K(s) = \frac{N_K(s)}{(s^2+\omega_0^2)D_K(s)}$$

Bounded steady-state error to multi-frequency sinusoidal disturbs

- The disturbs are sometimes characterized in the frequency domain instead of in the time domain

- ***The requirement on multi-frequency sinusoidal disturb*** is usually expressed by an attenuation factor δ_D for sinusoidal disturbs in a frequency interval Ω_D.

- This requirement is converted in terms of open loop function $F(s)$ constraint taking into account that the steady state error to a sinusoidal disturb $d(t) = D_0 sin(\omega_0 t)$ can be written as

$$e_{ss}(t) = D_0 \ |-S(j\omega_0)| \ sin(\omega_0 t + \angle - S(j\omega_0))$$

where $S(s) = \frac{1}{1+F(s)}$ is the sensitivity function.

Bounded steady-state error to multi-frequency sinusoidal disturbs

- Hence, the requirement on multi-frequency sinusoidal disturb turns out to be

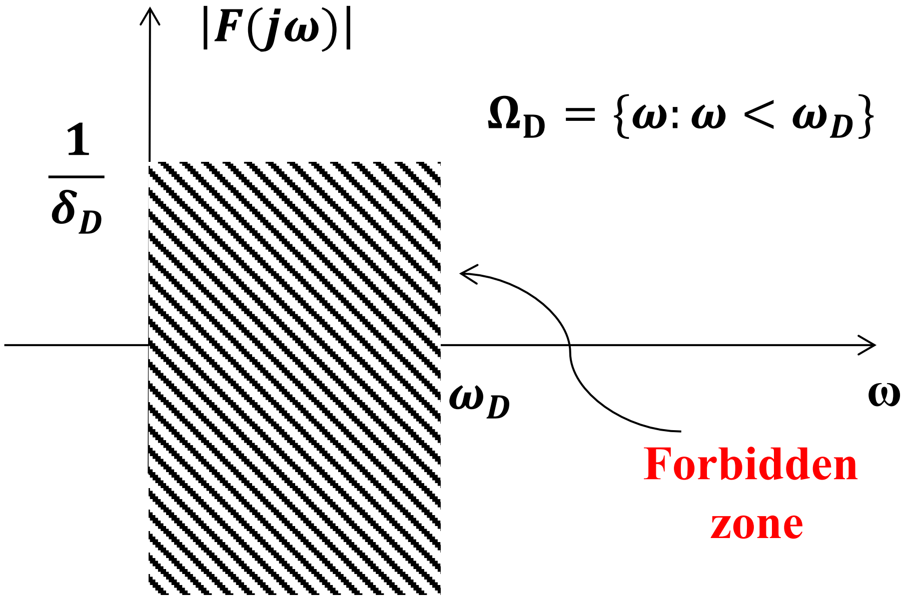
$$|S(j\omega)| < \delta_D \quad \text{for all } \omega \in \Omega_D$$

- That is

$$\left|\frac{1}{1+F(j\omega)}\right| < \delta_D$$

- Taking into account that $\delta_D \ll 1$, the previous inequality is almost equivalent to

$$\left|\frac{1}{F(j\omega)}\right| < \delta_D \quad \rightarrow \quad |F(j\omega)| > 1/\delta_D$$



Bounded steady-state error to multi-frequency sinusoidal noise

- The noise is usually characterized in the frequency domain as a multi-frequency sinusoidal signal

- ***The requirement on multi-frequency sinusoidal noise*** is usually expressed by an attenuation factor δ_N for sinusoidal noise in a frequency interval Ω_N.

- This requirement is expressed in terms of open loop function $F(s)$ taking into account that the steady state controlled output to a sinusoidal noise $n(t) = N_0 sin(\omega_0 t)$ can be written as

$$y_{ss}(t) = N_0 \ |T(j\omega_0)| \ sin(\omega_0 t + \angle T(j\omega_0))$$

where $T(s) = \frac{F(s)}{1+F(s)}$ is the complementary sensitivity function.

Bounded steady-state error to multi-frequency sinusoidal noise

- Hence, the requirement on multi-frequency sinusoidal noise turns out to be

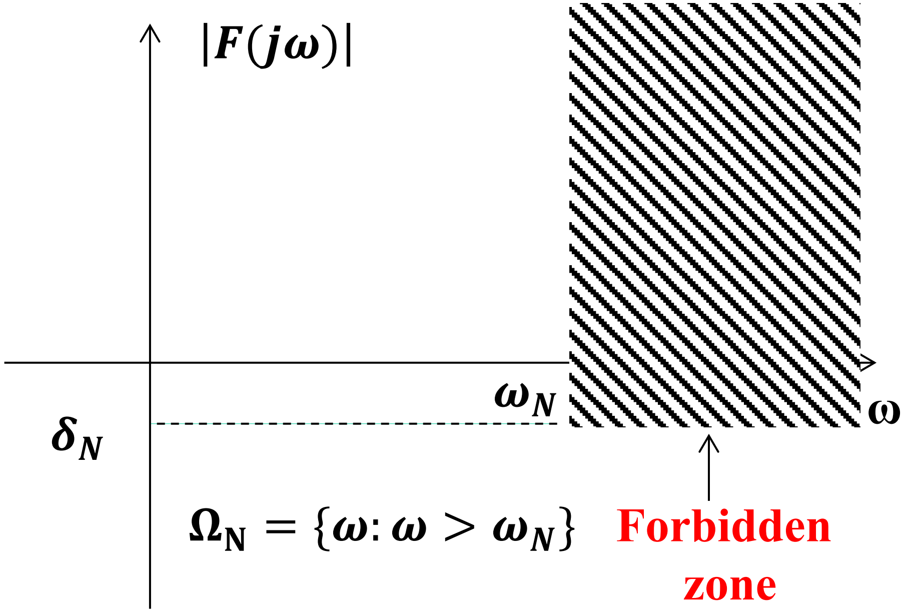
$$|T(j\omega)| < \delta_N \quad \text{for all } \omega \in \Omega_N$$

- That is

$$\left|\frac{F(j\omega)}{1+F(j\omega)}\right| < \delta_N$$

- Taking into account that $\delta_N \ll 1$, the previous inequality is almost equivalent to

$$|F(j\omega)| < \delta_N$$



Rejection of multi-frequency disturbs and noise

- The requirement on multi-frequency sinusoidal disturbs implies that

$$|S(j\omega)| < \delta_D \quad \text{for all } \omega \in \Omega_{\mathrm{D}} \qquad \rightarrow \ |F(j\omega)| > 1/\delta_D$$

- The requirement on multi-frequency sinusoidal noise implies that

$$|T(j\omega)| < \delta_N \quad \text{for all } \omega \in \Omega_{\mathrm{N}} \ \rightarrow \ |F(j\omega)| < \delta_N$$

- The two requirements can be imposed simultaneously only if $\boldsymbol{\Omega_{\mathrm{D}}}$ and $\boldsymbol{\Omega_{\mathrm{N}}}$ are disjoint.

- Usually the multi-frequency disturbs are at low frequencies ($\boldsymbol{\omega < \omega_D}$) while the multi-frequency noise is at high frequencies ($\boldsymbol{\omega > \omega_N}$) .

Rejection of multi-frequency disturbs and noise

- In order to satisfy the requirements on multi-frequency disturbs and noise, the open loop transfer function $F(j\omega)$ should behave similarly to a low pass filter

- $F(j\omega)$ can also contain poles in the origin if it is requested by steady-state requirements on polynomial reference signals and disturbs

Rejection of multi-frequency disturbs and noise