Course of
"Automatic Control Systems"
2022/23

Classification of closed loop systems w.r.t a proportional control action

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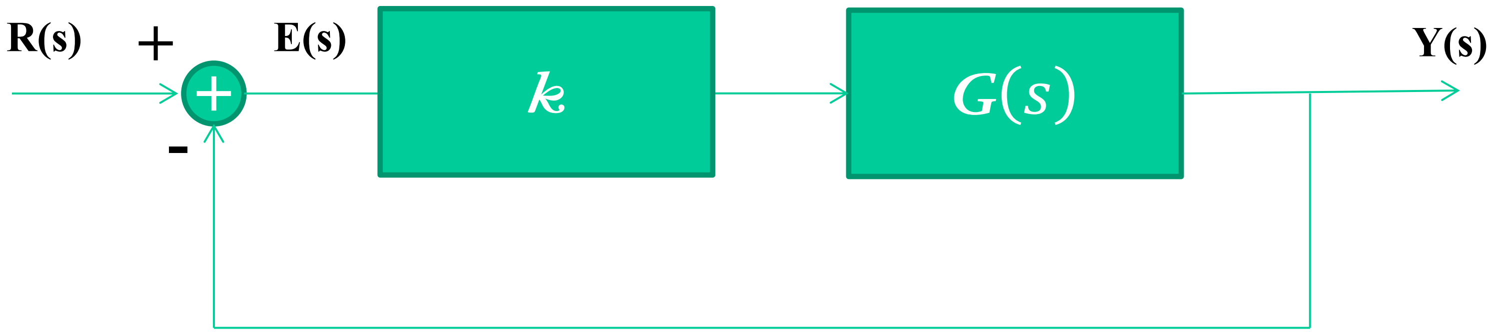
Stability of the closed loop systems

- Let us consider a closed loop system with a proportional control action

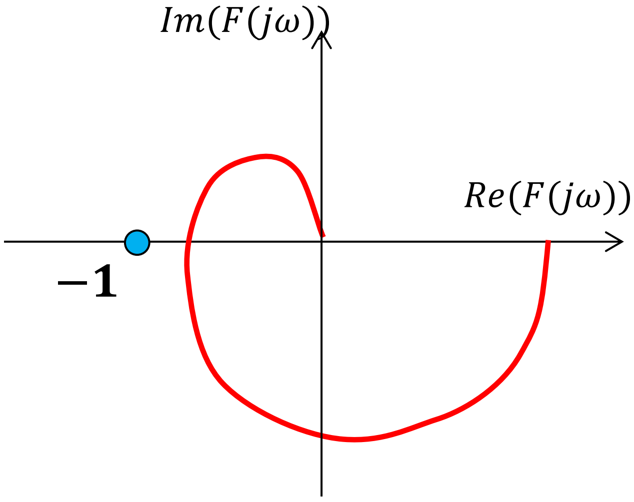


- In the following we will classify the closed loop systems depending on their stability behavior when k varies from 0 to $+\infty$

Regular stability of closed loop systems

- ***The stability*** of a closed loop system $T(s)$ ***is said to be regular*** when
  - T(s) is unstable for high values of the proportional gain k
  - T(s) is asymptotically stable for low values of the proportional gain k

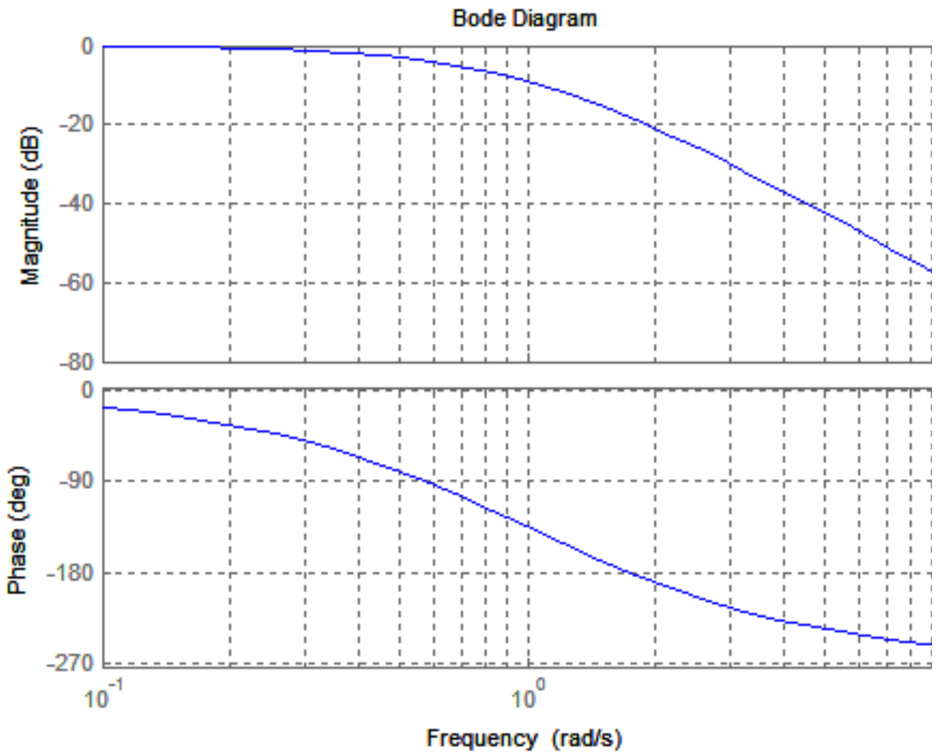
- Closed loop systems with regular stability are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that intersect only once the negative x-axis

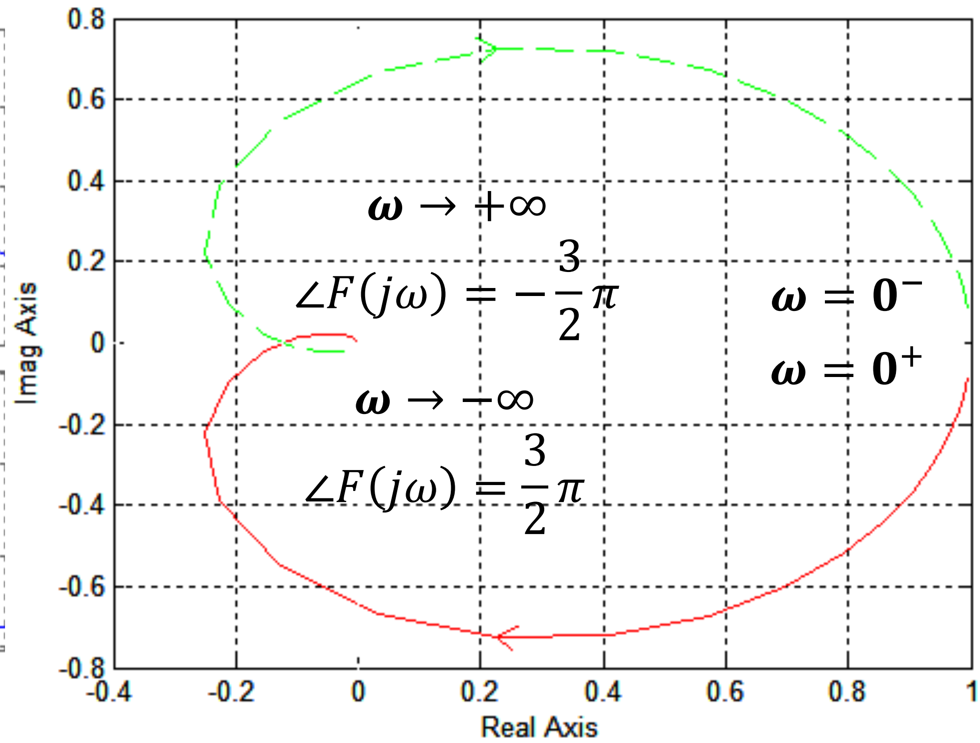


There exists a <u>critical gain $\bar{k}$</u> such that the closed loop system is unstable for <u>$k > \bar{k}$</u>

Regular stability of closed loop systems

- Let us consider the transfer function $F(s) = \frac{1}{(1+s)^3} \rightarrow n_{p^+}(F(s)) = 0$





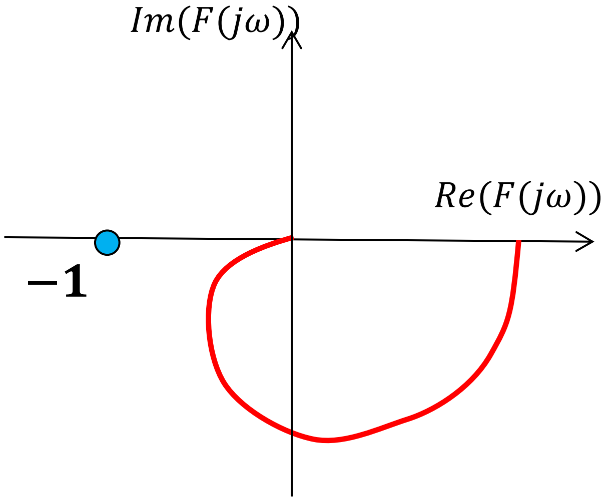
$\overleftarrow{\mathcal{N}} = 0,$ ***the critical gain is*** $\bar{k} \approx 8$

$T(s)$ ***asymptotically stable for*** $k < 8$ $T(s)$ ***unstable for*** $k > 8$

Inherent stability of closed loop systems

- ***The stability*** of a closed loop system $T(s)$ ***is said to be inherent*** when
  - $T(s)$ ***is asymptotically stable for all possible*** $\boldsymbol{k}$

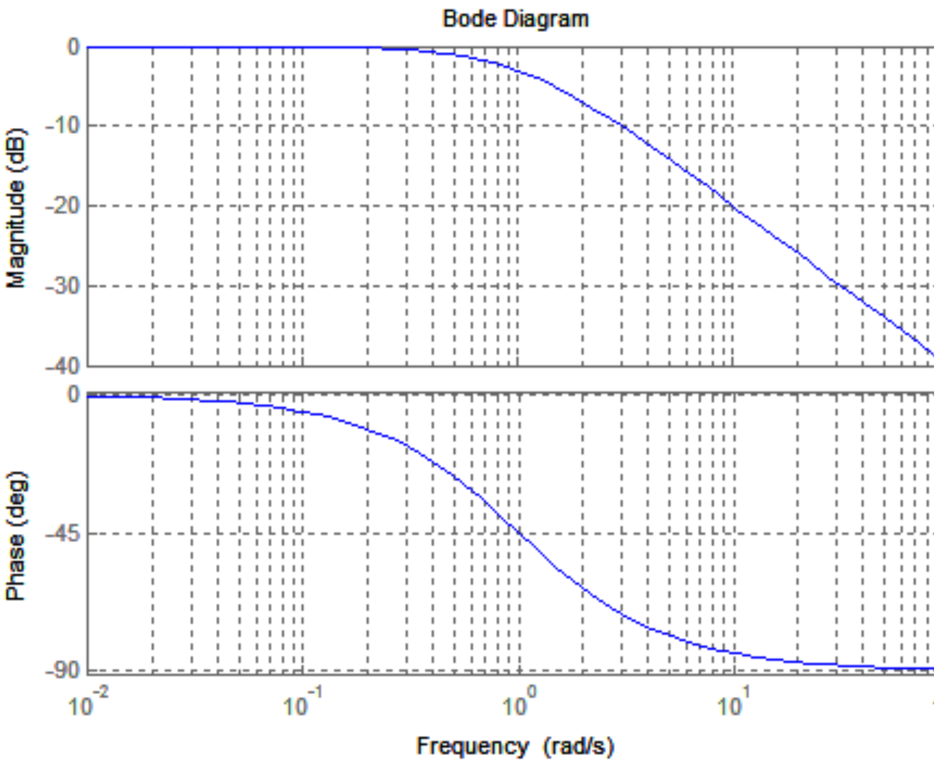
- Inherently stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that doesn't intersect the negative x-axis

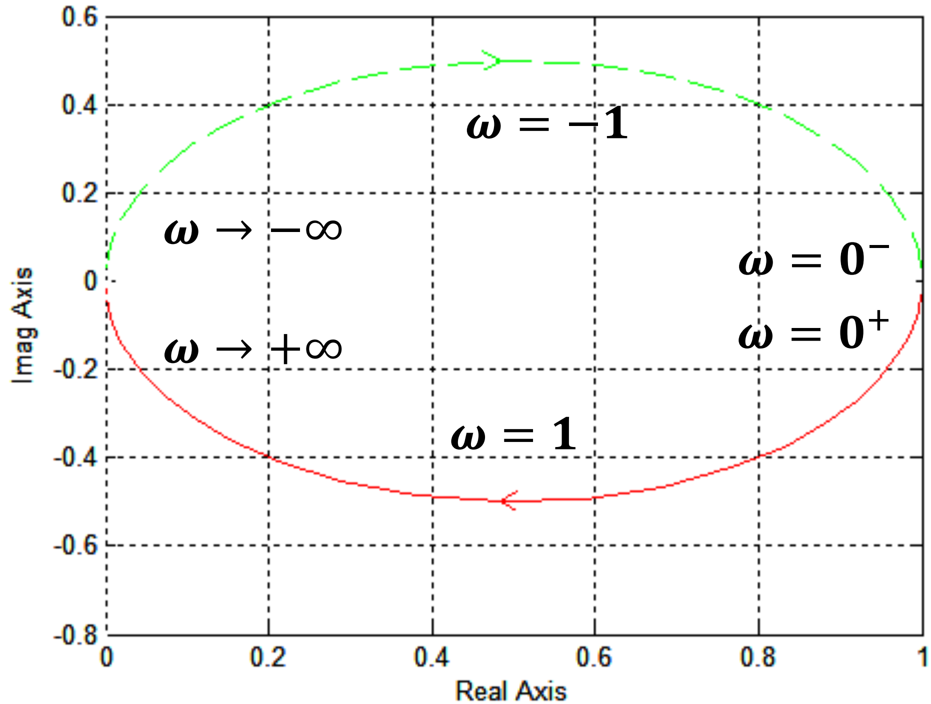


The closed loop system is asymptotically stable regardless the value of the proportional gain

Inherent stability of closed loop systems

- Let us consider the transfer function $F(s) = \frac{1}{1+s} \;\rightarrow\; n_{p^+}\big(F(s)\big) = 0$



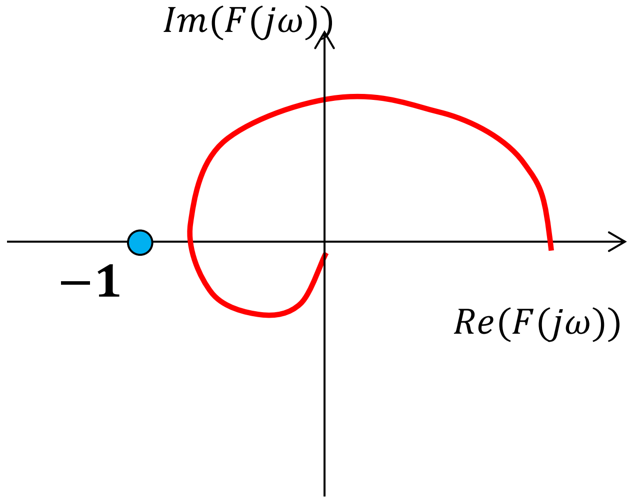


$$\overleftarrow{\mathcal{N}} = 0 \;\; for\ all\ possible\ values\ of\ k$$

Paradoxical stability of closed loop systems

- ***The stability*** of a closed loop system $T(s)$ ***is said to be paradoxical*** when
  - $\boldsymbol{T(s)}$ ***is unstable for low*** values of the proportional gain $\boldsymbol{k}$
  - $\boldsymbol{T(s)}$ ***is asymptotically stable for high*** values of the proportional gain $\boldsymbol{k}$

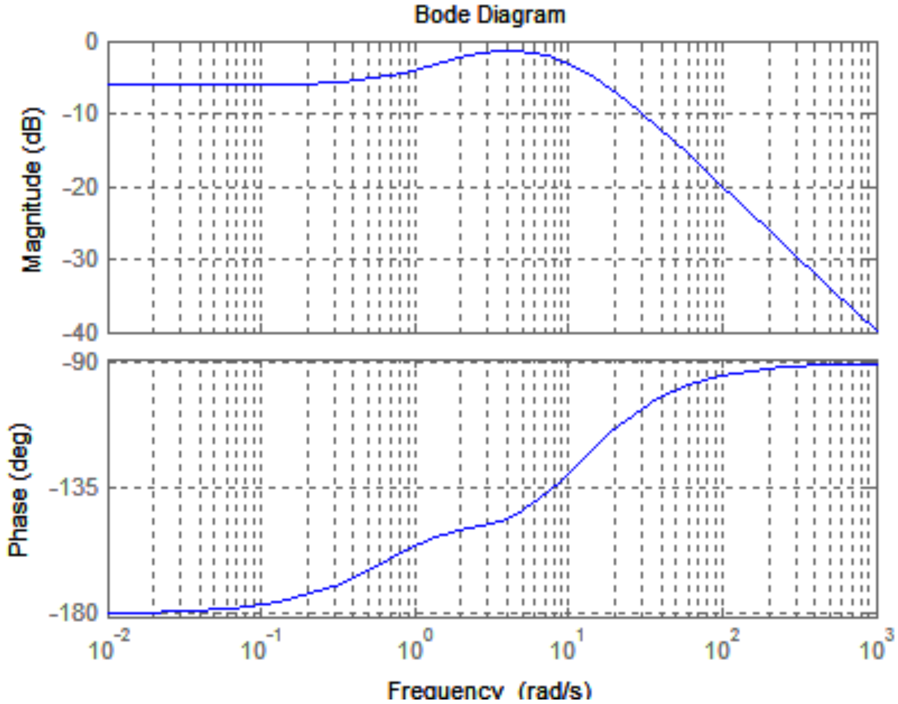
- Paradoxically stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - Poles with positive real part ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} > 0$)
  - A Nyquist plot that intersect only once the negative x-axis

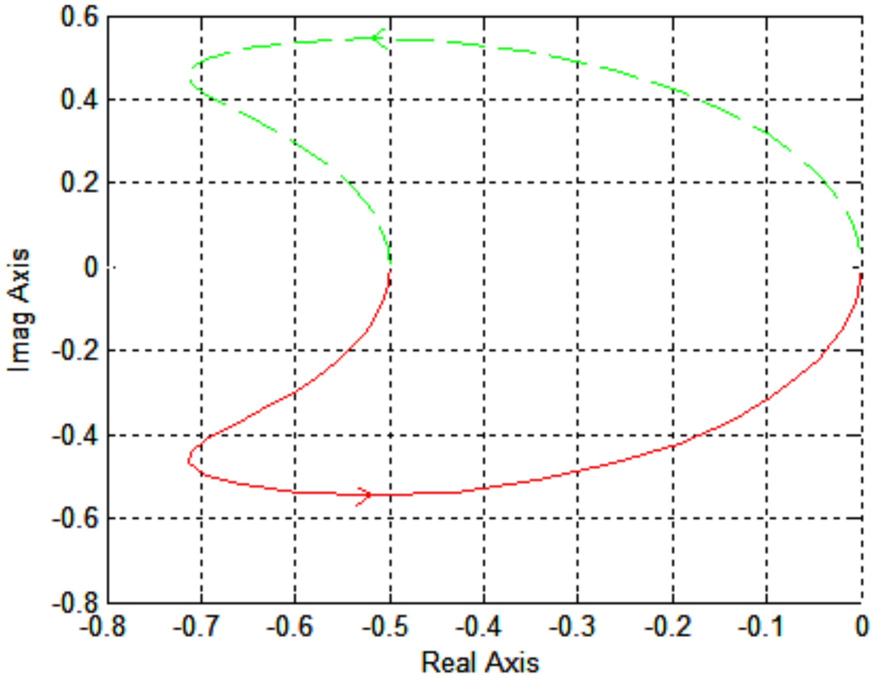


There exists a <u>critical gain $\bar{k}$</u> after which the closed loop system becomes asymptotically stable

Paradoxical stability of closed loop systems

- Let us consider the transfer function $F(s) = \frac{10(1+s)}{(2+s)(s-10)} \rightarrow n_{p^+}(F(s)) = 1$





$\overleftarrow{\mathcal{N}} = 1$ *for k greater that the critical gain* $\bar{k} = 2$

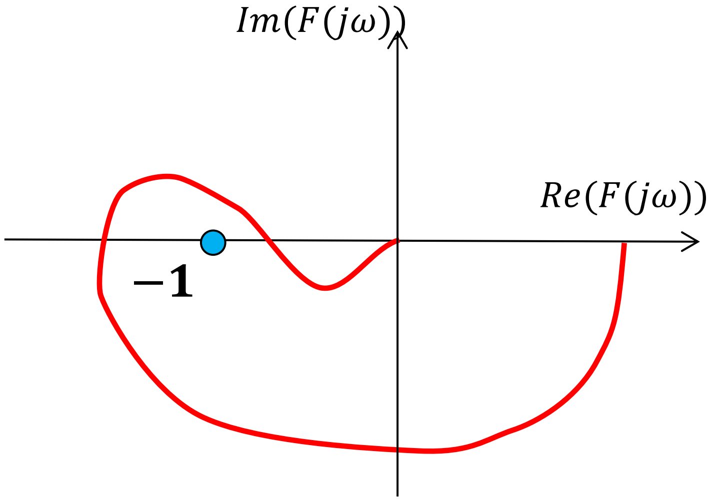
$T(s)$ *asymptotically stable for* $k > 2$

$T(s)$ *unstable for* $k < 2$

Conditional stability of closed loop systems

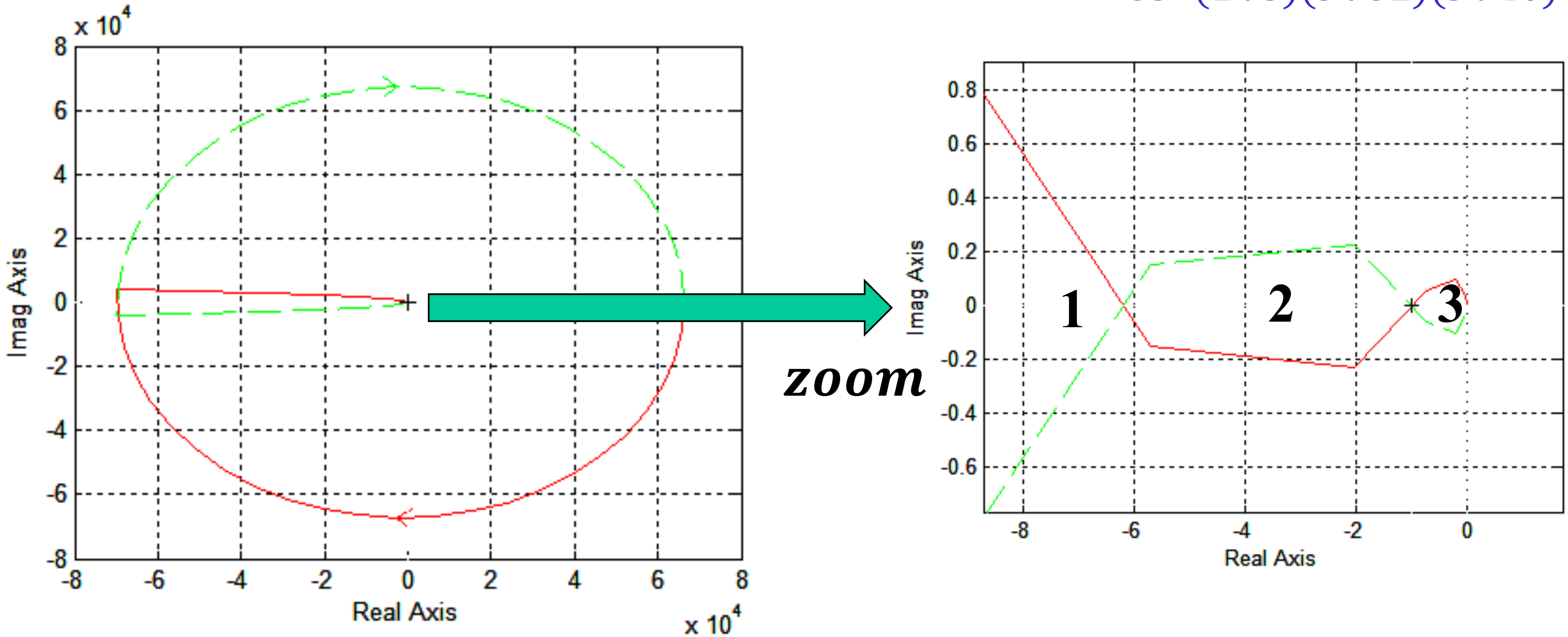
- ***The stability*** of a closed loop system $T(s)$ ***is said to be conditioned*** when
  - $\boldsymbol{T(s)}$ ***is asymptotically stable for a limited interval of*** $\boldsymbol{k}$

- Conditionally stable closed loop systems are usually characterized by an open loop function $F(s)$ with:
  - No poles with positive real part ($T(s)$ asymp. Stable iff $\overleftarrow{\mathcal{N}} = 0$)
  - A Nyquist plot that intersect the negative x-axis more than one time



- Let us consider the transfer function

$$F(s) = \frac{10000(s+3)(s+9)}{3s^2(1+s)(s+32)(s+40)}$$



- For $k \leq k_1$ the critical point moves in the first circle $\overleftarrow{\mathcal{N}} = -2 \rightarrow T(s)$ unstable
- For $k_1 < k < k_2$ the critical point moves in the second circle $\overleftarrow{\mathcal{N}} = 0 \rightarrow T(s)$ asymp. stab.
- For $k > k_2$ the critical point moves in the third circle $\overleftarrow{\mathcal{N}} = -2 \rightarrow T(s)$ unstable

Conditional stability of closed loop systems

- Let us consider the transfer function

$$F(s) = \frac{10000(s+3)(s+9)}{3s^2(1+s)(s+32)(s+40)}$$