

Exercises

SC2_07 – Affine transformations.

1. Find the equation of a reflection over the origin of \mathbf{R}^2 (automorphism), and that of a reflection over a generic center (affinity) not at origin.
2. Verify if the following transformations are linear or affine, and if they are invertible. For an invertible map, find its inverse and display, with MATLAB, how the map and its inverse transform a unitary square.

- $\phi : x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \longrightarrow y = \begin{pmatrix} 2x_1 - 2x_2 + 1 \\ -x_1 + x_2 - 1 \end{pmatrix} \in \mathbb{R}^2$

- $\phi : x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \in \mathbb{R}^2 \longrightarrow y = \begin{pmatrix} 2x_1 - x_2 \\ -x_1 + x_2 - 1 \end{pmatrix} \in \mathbb{R}^2$

- $\phi : x = \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} \in \mathbb{R}^3 \longrightarrow y = \begin{pmatrix} 9 & 3 & -2 \\ 3 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} x \in \mathbb{R}^3$

- $\phi : x = \begin{pmatrix} x_1 \\ x_2 \\ 1 \end{pmatrix} \in \mathbb{R}^3 \longrightarrow y = \begin{pmatrix} -1 & 0 & -2 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix} x \in \mathbb{R}^3$

3. Find the affine map that transforms the triangle with vertices (0,1), (1,2), (2,0) into the triangle with vertices (1,2), (0,-1), (3,3).
4. Find the matrix form of the plane rotation, by an angle θ , around a point $C(x_0, y_0)$ that is not the origin. Display its effect on the square with vertices at (0,0), (1,0), (1,1), (0,1).
5. Display the image of the line in \mathbf{R}^2 , whose equation is $y = -4x + 3$, by means of the transformation induced by the matrix: $A = \begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}$.

6. Display the image of the unit square in \mathbf{R}^2 (centered at the origin) by means of the following transformation:

$$\begin{pmatrix} X \\ Y \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 3 & -2 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

What elementary transformations does it contain?

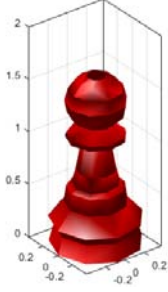
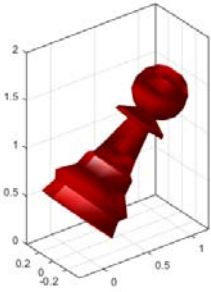
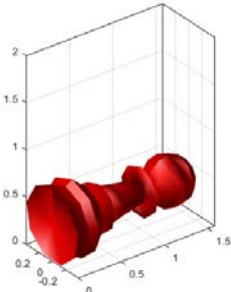
7. Find the matrix form for the 3D rotation by an angle $\theta = -30^\circ$ around the line spanned by the vector (1,1,1). Display its effect on the parallelepiped created and plotted by the following MATLAB code.

```

vert=[0 0 0;2 0 0;2 1 0; 0 1 0; 0 0 1; 2 0 1; 2 1 1;0 1 1];
face=[1 2 3 4; 3 7 8 4; 2 3 7 6; 5 6 7 8];
col=[0 0 0; 1 0 0; 0 0 1; 0 1 0];
patch('Faces',face,'Vertices',vert,'FaceVertexCData',col,'FaceColor','flat', ...
      'FaceAlpha',0.5)
view(3); axis equal; axis tight
```

8. Find the matrix form for the 3D rotation by an angle $\theta = 60^\circ$ around a line passing through a point $P_0(3,2,-1)$ and parallel to the vector (1,1,1). [Hint: normalize the axis vector]

9. In \mathbf{R}^3 a shear, parallel to x - y plane and by a factor k , maps a point (x,y,z) to the point $(x+kz,y+kz,z)$. Find the matrix of such a transformation and display its action on the parallelepiped in exercise 6.
10. The following MATLAB code displays a 3D object with some light effects.

<pre> %% dowload pedone.mat file from the eLearning platform load pedone % load data from the file into MATLAB Workspace patch('Faces',pedone.f.v,'Vertices',pedone.v, 'Facecolor','r','EdgeColor','none') view(3); axis equal; % axis off % to remove axes grid on; box on; zlim([0 2]) camlight('left','infinite'); lighting gouraud x=pedone.v(:,1); y=pedone.v(:,2); z=pedone.v(:,3); </pre>	<p>chess pawn (it: pedone degli scacchi)</p> 
	

The three figures above show the initial, middle and final positions of a pawn falling to the right. Simulate "efficiently" a falling chess-pawn by means of a sequence of transformations (described by suitable matrix products) that rotate the object clockwise by an angle θ around the y -axis, and centered at the point of contact with the ground computed as the lowest and rightmost point (this is the new origin of rotation).