Exercises

SC2_05 – Affine Spaces and Subspaces.

1. Check which pairs of the following affine subspaces are mutually parallel. Display with MATLAB the subspaces and their direction spaces.

   In $\mathbf{R}^2$:

   $\Sigma_1 : x - 2y + 1 = 0$
   $\Sigma_2 : x - 2y + 3 = 0$
   $\Sigma_3 : 2x + y + 1 = 0$
   $\Sigma_4 : x - y + 1 = 0$

   In $\mathbf{R}^3$:

   $\Sigma_1 : \mathrm{P} = (2\ 1\ 2) + \rho(0\ 0\ 1)$
   $\Sigma_2 : x - y = 0$
   $\Sigma_3 : \mathrm{P} = (1\ 0\ 0) + \rho(1\ -1\ 0)$
   $\Sigma_4 : x - y + z + 1 = 0$

2. In the affine space $\mathbf{R}^3$, let r_1 be the line passing through points $\mathbf{A}(2,-4,-1)$ and $\mathbf{B}(-1,-1,-1)$, and let r_2 be the line whose parametric equations are:

   $$r_2 : \begin{cases} x = 2 + s \\ y = -2 - s \\ z = 1 \end{cases}$$

   check if the two lines are coplanar*, and find the plane π that contains them.

   * Two lines are coplanar if they lie on the same plane, i.e. if they intersect or are parallel.

3. Find the vector parametric equation of the line r passing through point $\mathbf{Q}(2,1,3)$ and orthogonal to the plane π of the previous question.

4. Given the two lines r_1 and r_2:

   $$r_1 : \begin{cases} x = 1 + 3t \\ y = -t \\ z = 1 + 3t \end{cases}$$

   $$r_2 : \begin{cases} x = s \\ y = 2 \\ z = s \end{cases}$$

   find the cartesian equation of the affine plane π containing them.

5. Determine whether the line r and the plane π intersect or are parallel, where:

   $$r : \begin{cases} x - 1 = 0 \\ z = 0 \end{cases}$$

   $$\pi : \quad x + y - z = 0$$

6. Given the following points in $\mathbf{R}^3$, check if they are affinely independent:

   6.1 $\mathbf{A}_0(1,1,1)$, $\mathbf{A}_1(2,1,1)$, $\mathbf{A}_2(1,1,4)$, $\mathbf{A}_3(3,0,1)$.
   6.2 $\mathbf{B}_0(1,-2,0)$, $\mathbf{B}_1(1,-1,0)$, $\mathbf{B}_2(4,-5,0)$, $\mathbf{B}_3(1,0,-1)$.