1. ***Compute the analytic expression of the step response of the LTI system described by the following transfer function:***

$$F(s)=\frac{(s+24)}{(s^2+3s+45)}.$$

Solution:

$$Y(s)=F(s)U(s)=\frac{(s+24)}{(s^2+3s+45)}\frac{1}{s}.$$

By partial fraction decomposition:

$$Y(s)=\frac{(s+24)}{(s^2+3s+45)}\frac{1}{s}=\frac{A}{s}+\frac{Bs+C}{(s^2+3s+45)}=\frac{(A+B)s^2+(3A+C)s+45A}{s(s^2+3s+45)}\Rightarrow\begin{cases}A+B=0\\3A+C=1\\45A=24\end{cases}$$

$$\Rightarrow\begin{cases}A=8/15\\B=-8/15\\C=-3/5\end{cases}$$

$$Y(s)=\frac{8}{15}\cdot\left[\frac{1}{s}-\frac{(s+9/8)}{(s^2+3s+45)}\right]$$

The polynomials $s^2+3s+45$ and $s+9/8$ can be rewritten as it follows:

$$s^2+3s+45=(s+3/2)^2+(3\sqrt{19}/2)^2$$

$$s+9/8=s+3/2-3/8$$

$$Y(s)=\frac{8}{15}\cdot\left[\frac{1}{s}-\frac{(s+3/2)}{(s+3/2)^2+(3\sqrt{19}/2)^2}+\frac{1}{4\sqrt{19}}\frac{3\sqrt{19}/2}{(s+3/2)^2+(3\sqrt{19}/2)^2}\right].$$

By Laplace antitransform, we achieve the analytic expression of *y(t)*:

$$y(t)=\frac{8}{15}\left[1-e^{-\frac{3}{2}t}\cos\left(\frac{3\sqrt{19}}{2}t\right)+\frac{1}{4\sqrt{19}}e^{-\frac{3}{2}t}\sin\left(\frac{3\sqrt{19}}{2}t\right)\right]1(t).$$

Matlab code to verify the computation:

```
% define a symbolic variable s
syms s
% define Y(s)
Y_s=(s+24)/(s^2+3*s+45)/s;
% compute y(t) by using ilaplace function (inverse Laplace transform)
y_t=ilaplace(Y_s);
```

Compute the parameters for drawing the step response:

$$y(0) = \lim_{s\to\infty} sY(s) = 0$$

$$y'(0) = \lim_{s\to\infty} s^2Y(s) = 1$$

$$y(\infty) = \lim_{s\to 0} sY(s) = \frac{8}{15}$$

Rewrite the trinomial term of the denominator of F:

$$\left(1+\frac{2\varsigma}{\omega_n}s+\frac{s^2}{\omega_n}\right)=\left(1+\frac{1}{15}s+\frac{1}{45}s^2\right)$$

$$\begin{cases}\dfrac{2\xi}{\omega_n}=\dfrac{1}{15}\\ \omega_n=\sqrt{45}\end{cases} \Rightarrow \begin{cases}\xi=0{,}224\\ \omega_n=6{,}71\end{cases}$$

$$T_{a1}=\frac{4{,}6}{\xi\omega_n}\cong 3{,}1 \quad \text{sec}$$

Number of oscillations = 1/(2ζ) ≅ 2.2

$$T_{\max}=\frac{\pi}{\omega_n\sqrt{1-\xi^2}}\cong 0{,}48\,\text{sec}$$

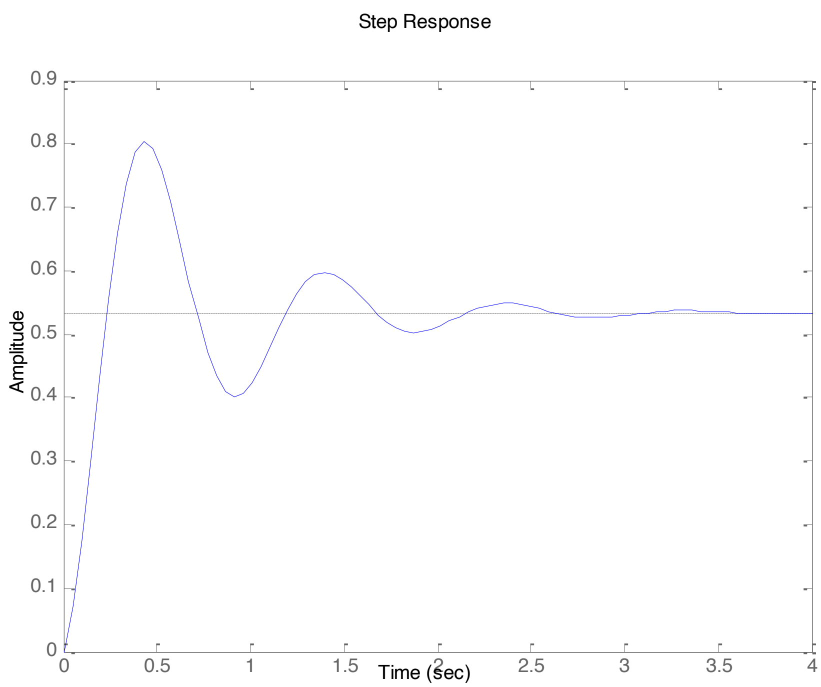
$$s\% = 100e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}\cong 49\%$$

$$y_{\max} = y_{\infty}\left(1 + e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}\right) \cong 0{,}79$$

T_r, oscillation period, $= \frac{2\pi}{\omega} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}}$ =0.96 s

Note the parameters are computed without taking into account the presence of the zero at numerator of *F*.

By using Matlab, we can achieve the plot of *y* (define the system transfer function by the function ***tf*** and then use the command ***step*** for achieving the plot) and verify how the estimated parameters (computed above) are close to the real ones (from the Matlab plot).



2. Compute the analytic expression of the step response of the LTI system described by the following transfer function:

$$F(s) = \frac{(s+15)}{(s^2+9s+20)}.$$

Solution:

$$Y(s) = F(s)U(s) = \frac{(s+15)}{(s^2+9s+20)}\frac{1}{s}$$

By partial fraction decomposition:

$$Y(s) = \frac{(s+15)}{(s^2+9s+20)}\frac{1}{s} = \frac{(s+15)}{(s+4)(s+5)}\frac{1}{s} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+5};$$

$$A = \lim_{s\to 0}\frac{(s+15)}{(s+4)(s+5)} = \frac{3}{4}$$

$$B = \lim_{s\to -4}\frac{(s+15)}{s(s+5)} = -\frac{11}{4}$$

$$C = \lim_{s\to -5}\frac{(s+15)}{s(s+4)} = 2$$

$$Y(s) = \frac{3/4}{s} - \frac{11/4}{s+4} + \frac{2}{s+5}$$

By Laplace antitransform, we achieve the analytic expression of *y*(*t*):

$$y(t) = \left[\frac{3}{4} - \frac{11}{4}e^{-4t} + 2e^{-5t}\right]1(t)$$

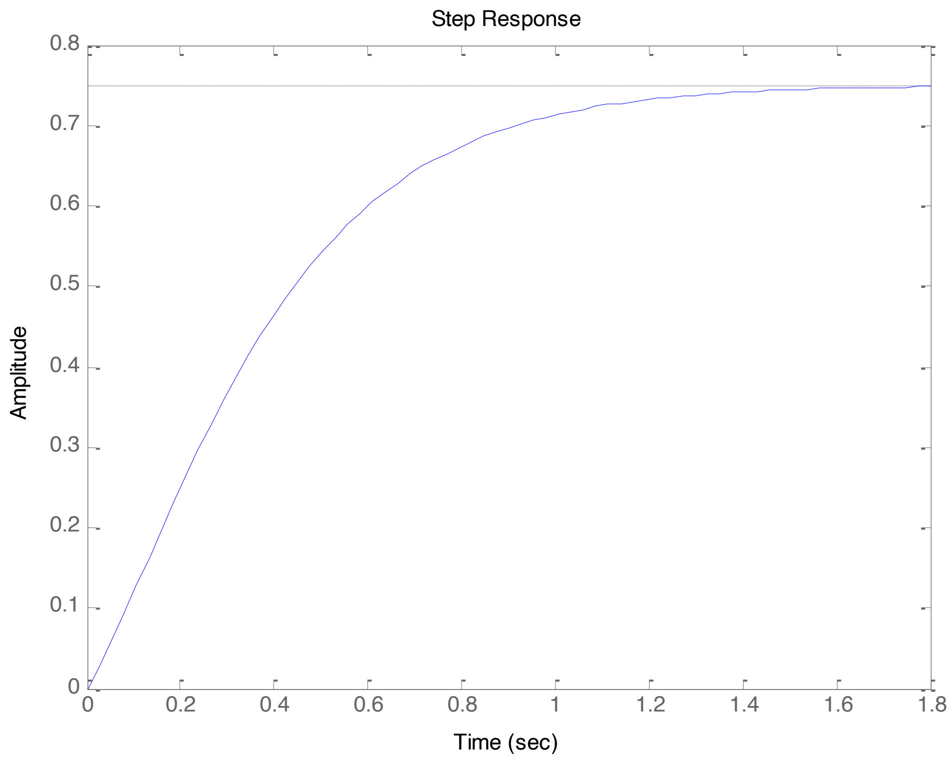
Compute the parameters for drawing the step response *y*(*t*):

$$y(0) = \lim_{s\to\infty} sY(s) = 0$$

$$y'(0) = \lim_{s\to\infty} s^2 Y(s) = 1$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \frac{3}{4}$$

$T_{a1} \cong 4{,}6\tau = 1$ sec, where τ is determined by the pole with lower absolute value (τ=−1/(−4)).


Step Response
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Amplitude
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Time (sec)