Exercises and Laboratories

SC2_02 – Linear Spaces and Subspaces.

1. Start with the vectors $\boldsymbol{v}_1=(1,2,0)^T$ and $\boldsymbol{v}_2=(2,3,0)^T$.
   - Are they linearly independent?
   - What space **V** do they span and what is the dimension of **V**?
   - Which matrices $\boldsymbol{A}$ have **V** as their *Column Space*?
   - Which matrices $\boldsymbol{B}$ have **V** as their *Null Space*?

2. Prove that the following sets of real n-tuples are linear subspaces of $\mathbf{R}^n$:
   - $W=\{\boldsymbol{x}\in\mathbf{R}^n : \boldsymbol{x}=(x_1,x_2,\ldots,x_j,\ldots,x_n)^T \wedge x_j=0 \text{ for a fixed } j\}$
   - $W=\{\boldsymbol{x}\in\mathbf{R}^n : \boldsymbol{x}=(x_1,x_2,\ldots,x_n)^T \wedge x_1=x_2=\ldots=x_n\}$
   - $W=\{\boldsymbol{x}\in\mathbf{R}^n : \sum_{k=1,\ldots,n} x_k = 0\}$

   To do this, you can also use the MATLAB *Symbolic Math Toolbox* for a particular value of n.

3. Then, find the dimension and a basis of the previous subspaces.

4. Similarly to what was said about a segment in $\mathbf{R}^2$, let us consider the parametric equation of a plane* in $\mathbf{R}^3$, once three misaligned points P_1, P_2, P_3 are given:

   $$\pi \;:\; P = P_1 + \lambda(P_2 - P_1) + \mu(P_3 - P_1), \qquad \lambda,\mu\in\mathbf{R}$$

   If we limit the real parameters $\lambda,\mu\in[0,1]$, then the previous equation describes the parallelogram having these three points as vertices [`SC2_02b.pdf`].
   - Is this parallelogram a subspace of $\mathbf{R}^3$?
   - Given another point, establish if it belongs to the parallelogram [see `SC2_02b.pdf` for some test points].

   [*Indeed, the previous parametric equation is related to a plane that is an **affine subspace** of the 3D real Affine Space (we'll study these spaces later)]