## Exercises and Laboratories

SC2_01c - Solve the following exercises by means of the MATLAB Symbolic Math Toolbox:

1. For the functions $f(x)$ listed below:

- produce their plot in the specified intervals;
- display their possible axis intersections;
- display their possible horizontal, vertical and oblique asymptotes*;
- display their possible local extrema (max and/or min);
- locate the subintervals where the function increases or decreases.

$$
\begin{aligned}
& f(x)=\frac{10}{x^{3}+1}-4+2 x \text { in }[-4,4] \\
& f(x)=\frac{10}{x^{3}+1}+6+2 x \text { in }[-4,4] \\
& f(x)=\cos (x) e^{-x} \text { in }[-2,4]
\end{aligned}
$$

Write a single function according to the following prototype:

```
function study_fun(fun_handle,interval)
```

* Remember that a horizontal asymptote $y=b$ is computed as $b=\lim _{x \rightarrow \pm \infty} f(x)$. For a vertical asymptote of eq. $x=a$, i.e. such that $\lim _{x \rightarrow a} f(x)= \pm \infty$, the finite value of $a$ can be found by solving the equation $\frac{1}{f(x)}=0$. Lastly, an oblique asymptote is a line of eq. $y=m x+q$, where $m=\lim _{x \rightarrow \pm \infty} \frac{f(x)}{x}$ and $q=\lim _{x \rightarrow \pm \infty}[f(x)-m x]$.

2. Display the 2 D plot of the function $f(z)=z^{3}-3 z^{2}-18$, first considered as a real-valued function of a real argument, and then display the 3D surface of the modulus* of the same function $f(z)$, now considered as a complex-valued function of a complex argument. Find the zeros of this function, once in the real field, and again in the complex field. Also add in the two figures the roots as red dots. Example figures are shown below.
[Hint: use the solve() function to find the roots] [* the MATLAB function $\operatorname{abs}(z)$ returns the complex module when its argument is a complex variable].


3. The linear approximation of a twice differentiable real function $f(x)$, at a point $a$, is given by:

$$
P_{1}(x)=f(a)+f^{\prime}(a)(x-a)
$$

while its quadratic approximation is given by:

$$
P_{2}(x)=f(a)+f^{\prime}(a)(x-a)+1 / 2 f^{\prime \prime}(a)(x-a)^{2}
$$

Both approximations come from the truncated Taylor expansion of $f(x)$ at $a$.
Compute $P_{1}$ and $P_{2}$ for $f(x)=\arcsin (x)$ and $a=1 / 2$. Produce the plot of this $f(x)$ in $[-1,1]$ and add the plots of $P_{1}$ and $P_{2}$. An example of figure is shown below.


