Laurea Magistrale in IA (ML&BD)

# Scientific Computing (part 2 – 6 credits)

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# Contents

- Symbolic Computations in MATLAB.

# Symbolic Computations in MATLAB

Performing symbolic computations with MATLAB requires you have installed the Symbolic Math Toolbox, which adds a new data type: the symbolic object. A symbolic object must necessarily be **declared** using:

```
syms a
a
a =
a
```

or

```
a=sym('a')
a =
a
```

## Examples

```
a=sym(2), b=2
a =      symbolic 2
2
b =      numerical 2
   2
```

```
a=sym(pi)
a = symbolic π
pi
b=pi
b = numerical π
    3.1416
```

```
syms a
assumptions
ans =
Empty sym: 1-by-0
```

```
syms a real
assumptions
ans =
in(a, 'real')
```

```
syms a positive
assumptions
ans =
0 < a
```

```
v=sym('v',[1 3])
v =  symbolic vector
[v1, v2, v3]
```

```
syms a integer
assumptions
ans =
in(a, 'integer')
```

```
syms a integer positive
assumptions
ans =
[in(a, 'integer'), 1 <= a]
```

```
syms f(x,y)
f  symbolic function
f(x,y) =
f(x,y)
```

```
A=sym('a',[2 3])
A =   symbolic matrix
[a1_1, a1_2, a1_3]
[a2_1, a2_2, a2_3]
```

```
a=sym('a',{'positive','integer'});
assumptions
ans =
[in(a, 'integer'), 1 <= a]
```

# compare

## numerical object

```
a=sqrt(2)
a =
    1.4142
```

```
2/5+1/3
ans =
    0.7333
```

## symbolic object

```
a=sqrt(sym(2))
a =
2^(1/2)
double(a)
ans =
    1.4142
```

the symbolic value is converted in number

```
sym(2)/5+1/sym(3)
ans =
11/15
```

indented

not indented

variables:

numerical

```
a=1;b=-2;c=4;x=1;
p=a*x^2+b*x+c;
disp(p)
     3
```

symbolic

```
syms a b c x
p=a*x^2+b*x+c;
pretty(p)
         2
      a x  + b x + c
```

```
P=matlabFunction(p)
P =
  function_handle with value:
    @(a,b,c,x)c+b.*x+a.*x.^2
```

the symbolic expression is converted into *Anonymous Function*

# A symbolic object is always a "formula"!

```
a=sqrt(sym(2))
a =
2^(1/2)
a=sym('a')
a =
a
syms f(x), f
f(x) =
f(x)
syms x h real
Df=(subs(f,x,x+h) - f) / h
Df =
(f(x+h)-f(x))/h
```

symbolic constant

symbolic variable

symbolic function

substitute, in **f**, **x** with **x+h**

difference quotient

```
p=sym(pi)
p =
pi
syms r
d=2*p*r
d =
2*pi*r
cos(d)
ans =
cos(2*pi*r)
subs(cos(d),r,2)
ans =
1
```

```
a = sym('b')
a =
b
syms b; b
b =
b
```

```
a=sqrt(sym(2)); double(a)
ans =
      1.4142
p=sym(pi); double(p)
ans =
      3.1416
```

**double**: conversion from symbolic to numeric

# Simplify a symbolic expression

```
syms x a b c
f=cos(x)^2-sin(x)^2;
simplify(f)
ans =
cos(2*x)
f=exp(c*log(sqrt(a+b)));
simplify(f)
ans =
(a + b)^(c/2)
```

```
syms x real
e1=((exp(-x*1i)*1i) - (exp(x*1i)*1i));
e2=(exp(-x*1i) + exp(x*1i));
espr=e1/e2;
s1=simplify(e1), s2=simplify(e2)
```

s1 = $2\sin(x)$

s2 = $2\cos(x)$

Download *live script*: **simplify_expressions.mlx**

```
S=simplify(espr)
```

S =

$$-\frac{e^{2xi}\,i-i}{e^{2xi}+1}$$

**Increase to 10 the number of simplification steps**

```
S10=simplify(espr,'Steps',10)
```

S10 =

$$\frac{2\,i}{e^{2xi}+1}-i$$

**Increase to 30 the number of simplification steps**

```
S30=simplify(espr,'Steps',30)
```

S30 =

$$\frac{(\cos(x)-\sin(x)\,i)\,i}{\cos(x)}-i$$

**Increase to 50 the number of simplification steps**

```
S50=simplify(espr,'Steps',50)
```

S50 = $\tan(x)$

# Simplify a symbolic expression



## Increase to 30 the number of simplification steps

```
S30=simplify(espr,'Steps',30)
```

S30 =

$$\frac{(\cos(x) - \sin(x)\,\mathrm{i})\,\mathrm{i}}{\cos(x)} - \mathrm{i}$$

Substitute variables
Applying calculus functions
Computing integral transforms
Converting numbers
Rewriting and simplifying expressions
Solving equations
Copy Ctrl+C
Copy as LaTeX
Copy as MathML

Combine expression
Expand expression
Rewrite expression
Simplify expression
Simplify fractions

## Increase to 50 the number of simplification steps

```
S50=simplify(espr,'Steps',50)
```

S50 = $\tan(x)$

```
S=simplify(espr,'Steps',50,'All',true)
```

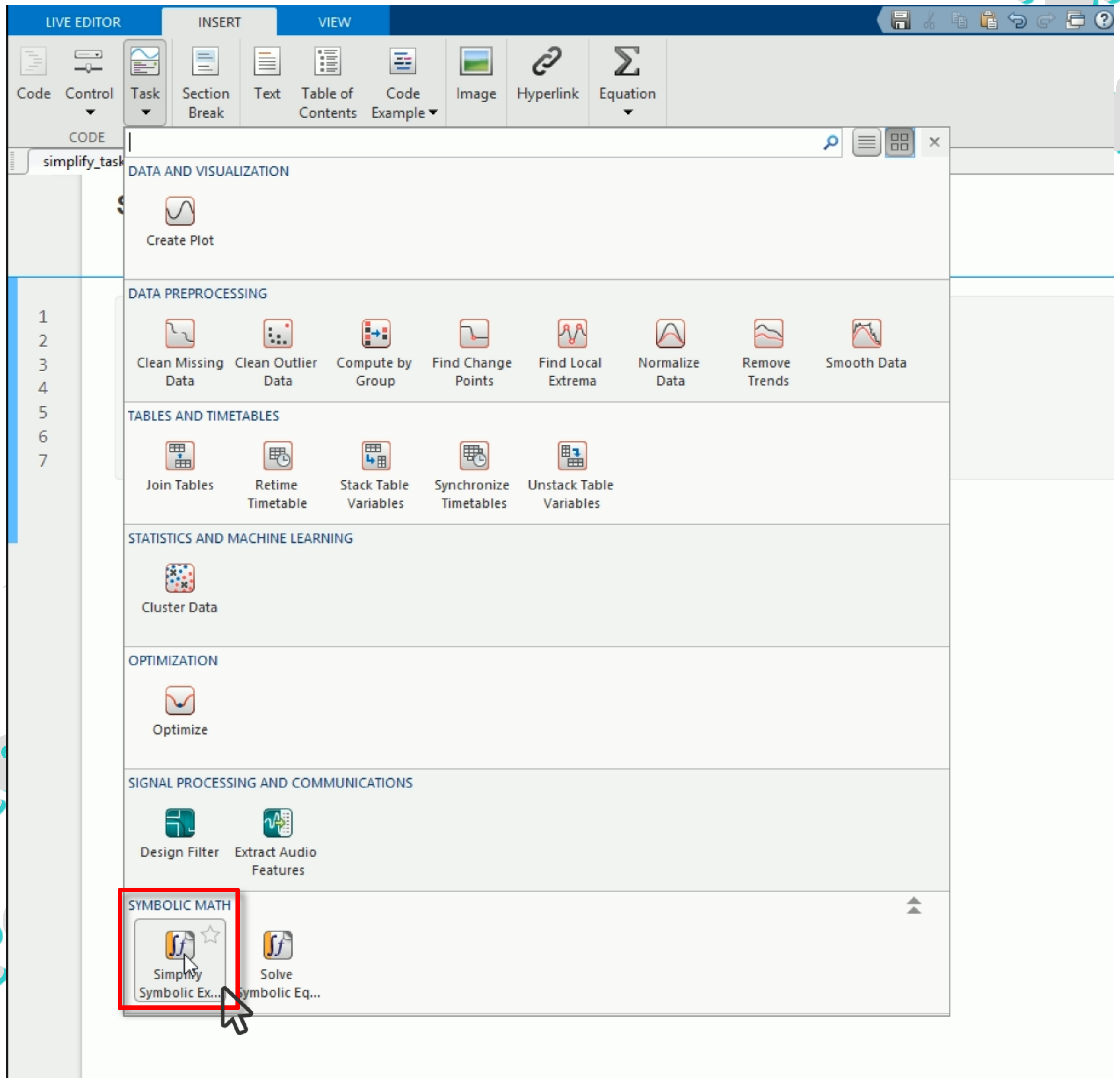
to display all the possible simplifications

S =

$$\left(\begin{array}{c} \tan(x) \\ \dfrac{1}{\cot(x)} \\ \dfrac{\sin(x)}{\cos(x)} \\ \dfrac{(\cos(x) - \sin(x)\,\mathrm{i})\,\mathrm{i}}{\cos(x)} - \mathrm{i} \\ \dfrac{\sigma_2}{\cos(x)} - \mathrm{i} \\ \dfrac{2\,\mathrm{i}}{\sigma_1} - \mathrm{i} \\ (2\,\sigma_3 + \sin(x)\,\mathrm{i} - 1)\,\mathrm{i} \\ \vdots \end{array}\right)$$

Download *live script*: simplify_expressions.mlx

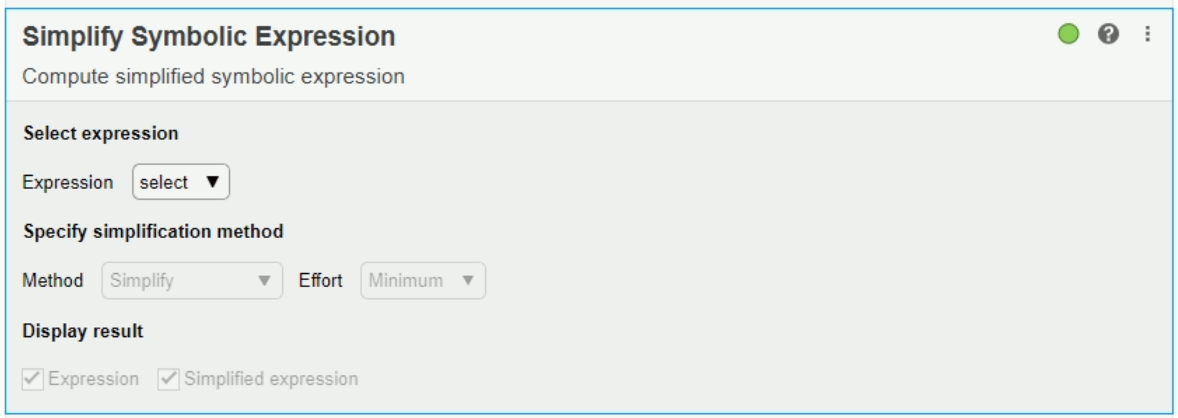
# Simplify a symbolic expression by means of the Live Editor task **Simplify Symbolic Expression**




Download *live script*: simplify_task1.mlx

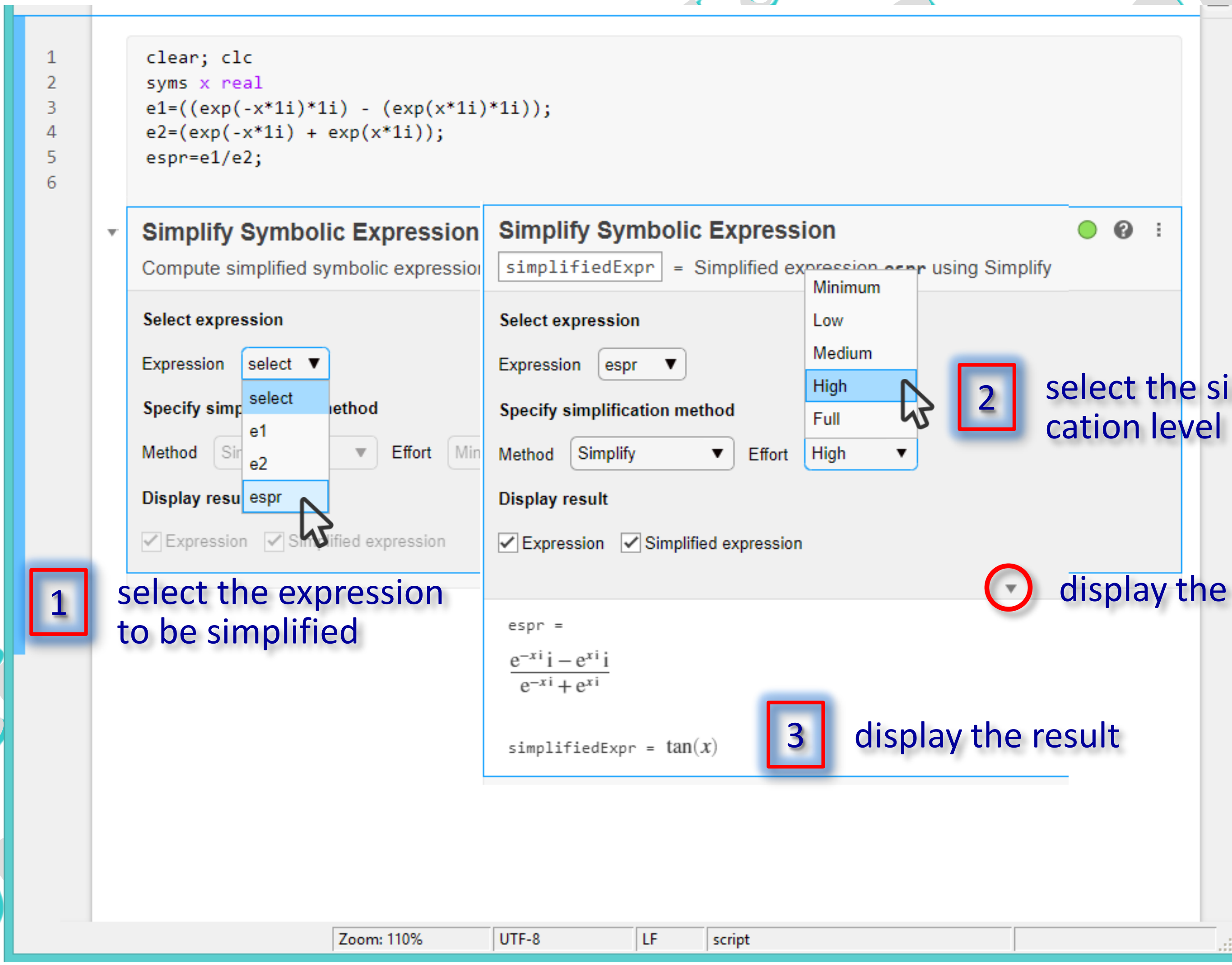
# Simplify a symbolic expression by means of the Live Editor task **Simplify Symbolic Expression**

```
clear; clc
syms x real
e1=((exp(-x*1i)*1i) - (exp(x*1i)*1i));
e2=(exp(-x*1i) + exp(x*1i));
espr=e1/e2;
```




Download *live script*: **simplify_task2.mlx**

# Simplify a symbolic expression by means of the Live Editor task **Simplify Symbolic Expression**




Download *live script*: simplify_task2.mlx

# Symbolic Math Toolbox: solve equations and systems

## a single equation

```
syms a b c x real
eqn=a*x^2+b*x+c == 0
S=solve(eqn)
Warning: Solutions are only valid under certain conditions. To include parameters and
conditions in the solution, specify the 'ReturnConditions' value as 'true'.
> In sym/solve>warnIfParams (line 478)
In sym/solve (line 357)
S =
-(b + (b^2 - 4*a*c)^(1/2))/(2*a)
-(b - (b^2 - 4*a*c)^(1/2))/(2*a)
S=solve(eqn,'ReturnConditions',true)
S =
  struct with fields:
            x: [2×1 sym]
   parameters: [1×0 sym]
   conditions: [2×1 sym]
S.x
ans =
-(b + (b^2 - 4*a*c)^(1/2))/(2*a)
-(b - (b^2 - 4*a*c)^(1/2))/(2*a)
S.conditions
ans =
4*a*c <= b^2 & a ~= 0
4*a*c <= b^2 & a ~= 0
A=solve(eqn,a)
A =
-(c + b*x)/x^2
```

```
syms x
eqn=x^3 == -1
S=solve(eqn,x)
S =
                    -1
1/2 - (3^(1/2)*1i)/2
(3^(1/2)*1i)/2 + 1/2
```

```
syms x real
eqn=x^3 == -1
S=solve(eqn,x)
S =
-1
```

```
syms x
eqn=x^3 == -1
S=solve(eqn,x,'Real',true)
S =
-1
```

## linear system

```
syms u v
eqns=[2*u + v == 0, u - v == 1];
S=solve(eqns,[u v])
S =
  struct with fields:
    u: 1/3
    v: -2/3
```

## non-linear system

```
syms u v
eqns=[2*u^2 + v^2 == 0,u - v == 1];
[U,V]=solve(eqns,[u v])
U =
1/3 - (2^(1/2)*1i)/3
(2^(1/2)*1i)/3 + 1/3
V =
- (2^(1/2)*1i)/3 - 2/3
  (2^(1/2)*1i)/3 - 2/3
```

# Symbolic "Calculus": examples [1]

## limits

$$\lim_{x\to\infty}\frac{1}{x}$$

$$\lim_{x\to 0}\frac{1}{x}$$

$$\lim_{x\to 0^-}\frac{1}{x}$$

```
syms x real; limit(1/x,x,inf)
ans =
0
limit(1/x,x,0)
ans =
NaN
limit(1/x,x,0,'left')
ans =
-Inf
limit(1/x,x,0,'right')
ans =
Inf
syms x real positive
limit(-x/abs(-x),x,0)
ans =
-1
```

```
syms x real positive
limit(1/x,x,0)
ans =
Inf
```

NaN means Not a Number

```
syms x h real
limit((cos(x+h)-cos(x))/h,h,0)
ans =
-sin(x)
```

limit of the difference quotient of cos(x)

## summation of series

```
syms x n
an=x^n/sym('n!');
symsum(an,n,0,inf)
ans =
exp(x)
```

$$e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

## Taylor expansion

```
syms x
T=taylor(exp(x))
T =
x^5/120 + x^4/24 + x^3/6 + x^2/2 + x + 1
```

See MATLAB *Live Script*:
Find Maclaurin Series of Univariate Expressions

# Symbolic "Calculus": examples [2]

## derivatives

**ordinary derivatives**

```
syms x real
f=sin(x)
f =
sin(x)
diff(f)
ans =
cos(x)
diff(f,2)
ans =
-sin(x)
diff(f,3)
ans =
-cos(x)
```

$2^{nd}$ derivative (`diff(f,2)`)

$3^{rd}$ derivative (`diff(f,3)`)

**partial derivatives**

```
syms x y real
f=sin(x)*exp(i*y)
f =
sin(x)*exp(i*y)
diff(f,x)
ans =
exp(y*1i)*cos(x)
diff(f,y)
ans =
exp(y*1i)*sin(x)*1i
diff(f,y,2)
ans =
-exp(y*1i)*sin(x)
```

`diff(f,x)` derivative w.r.t. x

`diff(f,y)` derivative w.r.t. y

`diff(f,y,2)` $2^{nd}$ partial derivative w.r.t. y

**gradient**

$$\nabla_{x,y}(f) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

```
syms x y real
f=sin(x)*exp(i*y);
gradient(f)
ans =
   exp(y*1i)*cos(x)
exp(y*1i)*sin(x)*1i
```

**jacobian matrix**

$$J_{x,y}(u,v) = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}$$

```
syms x y real
u=x*cos(y); v=y*cos(x);
jacobian([u;v],[x y])
ans =
[     cos(y), -x*sin(y)]
[ -y*sin(x),     cos(x)]
```

# Symbolic "Calculus": examples [3]

## integrals

```
syms x real
syms n integer positive
int(x^n)
ans =
x^(n+1)/(n+1)
int(cos(x))
ans =
sin(x)
```

indefinite integrals

```
syms x n real
int(sin(n*x),x)
ans =
-cos(n*x)/n
int(sin(n*x),n)
ans =
-cos(n*x)/x
```

definite integrals

```
syms x n real
int(sin(n*x),x,0,pi/n)
ans =
2/n
```

$$\int_0^{\pi/n} = -\cos(nx)/n \Big|_0^{\pi/n} =$$
$$= -\cos(\pi)/n + \cos(0)/n = 2/n$$

# Symbolic "Calculus": examples [4]

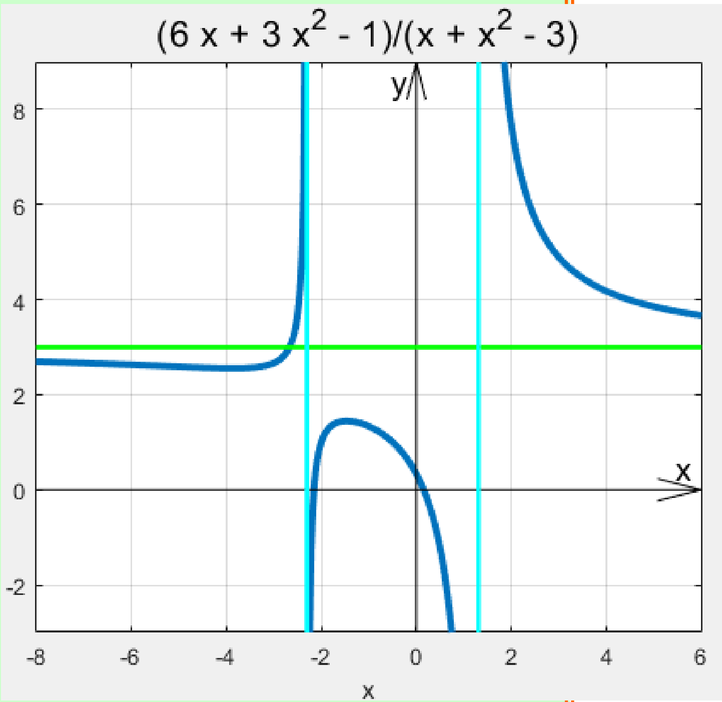
## study of the function: $f(x)=\dfrac{3x^2+6x-1}{x^2+x-3}$

```
syms x real; num=3*x^2+6*x-1; den=x^2+x-3; f=num/den;
pretty(f)
        2
     3 x  + 6 x - 1
     --------------
        2
       x  + x - 3
h=ezplot(f,[-8 6]); % or fplot(f,[...])
h.LineWidth=3; % or set(h,'LineWidth',2)
axis equal; grid on; AX=[-7 7 -3 9]; axis(AX)
hold on
quiver(AX(1),0,1,0,diff(AX(1:2)),'Color','k')
quiver(0,AX(3),0,1,diff(AX(3:4)),'Color','k')
text(AX(2),0,'x ','FontSize',14,'HorizontalAlignment','right')
text(0,AX(4),'y ','FontSize',14,'HorizontalAlignment','right')
asint_O=[limit(f,x,-inf) limit(f,x,inf)]
asint_O =
[ 3, 3]
asint_V=solve(den,x)  % or solve(1/f, x)
asint_V =
- 13^(1/2)/2 - 1/2
  13^(1/2)/2 - 1/2
line(AX(1)*[-1 1],(asint_O(1))*[1 1],'Color','g')
line([asint_V';asint_V'],[AX(3:4);AX(3:4)]','Color','c')
```

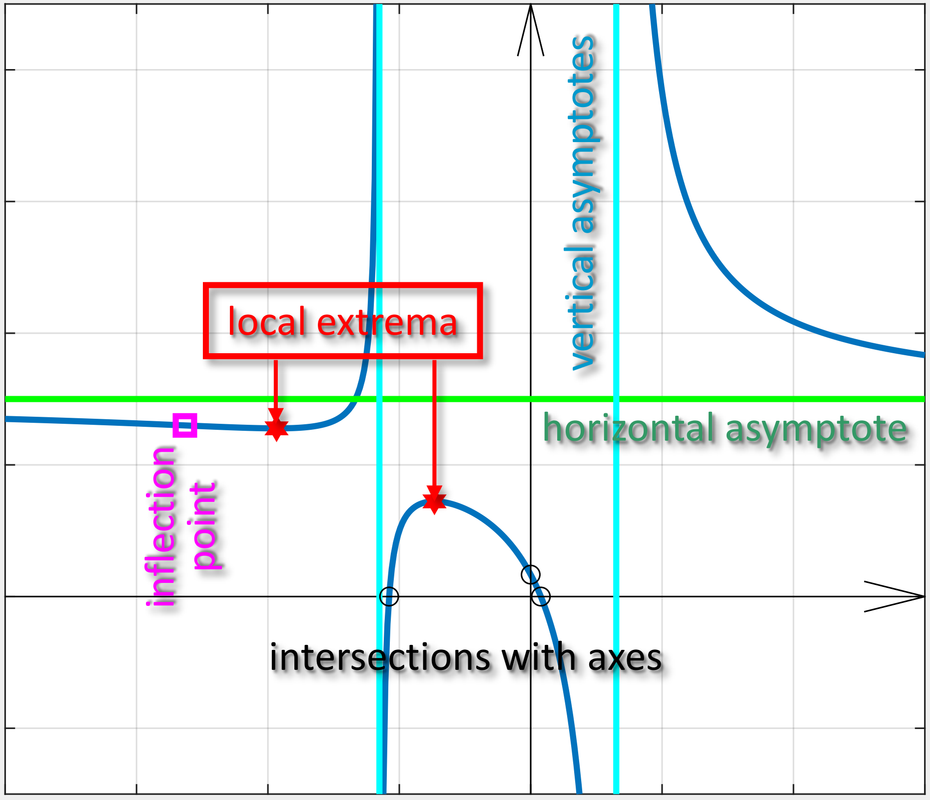
cartesian axes as arrows

horizontal asymptote

vertical asymptotes



```
f0=subs(f,x,0)  % intersection with y-axis
f0 =
1/3
x0=solve(f)       % function zeros
x0 =
- (2*3^(1/2))/3 - 1
  (2*3^(1/2))/3 - 1
plot(0,f0,'ok',x0,zeros(size(x0)),'ok'
f1=simplify(diff(f)) % min and max
f1 =
-(3*x^2+16*x+17)/(x^2+x-3)^2
x_minmax=solve(f1)
x_minmax =
- 13^(1/2)/3 - 8/3
  13^(1/2)/3 - 8/3
y_minmax=simplify(subs(f,x_minmax))
y_minmax =
(2*13^(1/2))/13 + 2    max
2 - (2*13^(1/2))/13    min
```




```
plot(x_minmax,y_minmax,'hr','MarkerFaceColor','r','MarkerSize',8)
f2=simplify(diff(f,2))
f2 =
(2*(3*x^3 + 24*x^2 + 51*x + 41))/(x^2 + x - 3)^3
x_fles=simplify(solve(f2,'Real',true)) % inflections
x_fles = root(z^3 + 8*z^2 + 17*z + 41/3, z, 3)
x_fles=double(x_fles)
x_fles =        -5.2635
y_fles=subs(f,x_fles);
plot(x_fles,y_fles,'sm','LineWidth',2,'MarkerSize',8)
```

all the numerical roots

```
[Num,Den]=numden(f2);
c=sym2poly(Num);
disp(roots(c))
     -5.2635 +        0i
     -1.3682 +   0.85112i
     -1.3682 -   0.85112i
```

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# Symbolic Linear Algebra: examples [1]

rotation matrix by an angle of t radians

```
syms t real
A=[cos(t) -sin(t);sin(t) cos(t)];
det(A)  determinant
ans =
cos(t)^2+sin(t)^2
simplify(det(A))
ans =
1
A^2
ans =
[ cos(t)^2-sin(t)^2, 2*cos(t)*sin(t)]
[-2*cos(t)*sin(t), cos(t)^2-sin(t)^2]
simplify(A^2)
ans =
[cos(2*t), -sin(2*t)]
[sin(2*t),  cos(2*t)]
```

rotation matrix by 2*t

A is an orthogonal matrix

```
A'*A
ans =
[ cos(t)^2+sin(t)^2,                 0]
[                 0, cos(t)^2+sin(t)^2]
simplify(A'*A)
ans =
[1, 0]
[0, 1]
A*A'
ans =
[ cos(t)^2+sin(t)^2,                 0]
[                 0, cos(t)^2+sin(t)^2]
simplify(A*A')
ans =
[1, 0]
[0, 1]
```

The product of 2 rotation matrices gives the rotation of an angle equal to the sum of the 2 angles

# Symbolic Linear Algebra: examples [2]

```
syms a b c d real
A=[a b;c d]; det(A)
ans =
a*d - b*c
v=A(:,1) + 0.5*A(:,2);
rank(A) == rank([A v])
ans =
  logical
   1
A1=inv(A)
ans =
[  d/(a*d - b*c), -b/(a*d - b*c)]
[ -c/(a*d - b*c),  a/(a*d - b*c)]
A1*A
ans =
[(a*d)/(a*d-b*c)-(b*c)/(a*d-b*c),                               0]
[                              0, (a*d)/(a*d-b*c)-(b*c)/(a*d-b*c)]
simplify(A1*A)
ans =
[1, 0]
[0, 1]
```

***Rouché-Capelli Theorem***

if equal, then the system is compatible

***Cramer Rule*** for the inverse matrix

```
syms a b
A=[a a;b b]
A =
[a, a]
[b, b]
colspace(A)
ans =
[   1]
[ b/a]
null(A)
ans =
[ -1]
[  1]
```

2 columns are equal

Column Space

Null Space

$\mathcal{N}(A)$: Null Space of $A(m,n)$:
$$\mathcal{N}(A) = \{x \in \mathbb{R}^n : Ax = \underline{0}\}$$

$\mathcal{R}(A)$: Column Space of $A(m,n)$:
$$\mathcal{R}(A) = \{u \in \mathbb{R}^m : \exists x : u = Ax\}$$

```
syms p q; y=[p;q]; x=A\y
x =
[  (d*p-q*b)/(a*d-b*c)]
[ (-c*p+a*q)/(a*d-b*c)]
disp(simplify(A*x))
p
q
```

solve the system

check the solution





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# Symbolic Linear Algebra: examples [3]

$A(n\times n)$

$\lambda$ ***eigenvalue*** $\overset{def}{\Longleftrightarrow}$ $\exists\, x\neq \underline{0} : Ax=\lambda x$

$v$ ***eigenvector*** w.r.t. $\lambda$ $\overset{def}{\Longleftrightarrow}$ $Av=\lambda v$

```
syms a b c real
A=[a,b,c; b,c,a; c,a,b]
A =
[ a, b, c]
[ b, c, a]
[ c, a, b]
[v,lambda]=eig(A);
```

∀k λ(k,k) = eigenvalue and v(:,k) = related eigenvector of A

```
lambda=simplify(lambda)
lambda =
[(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2),                                  0,     0]
[                               0, -(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2),     0]
[                               0,                                  0, a+b+c]
v=simplify(v)
v =
[-(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2)/(a-c)-(a-b)/(a-c),  (a^2-a*b-a*c+b^2-b*c+c^2)^(1/2)/(a-c)-(a-b)/(a-c), 1]
[ (a^2-a*b-a*c+b^2-b*c+c^2)^(1/2)/(a-c)-(b-c)/(a-c), -(a^2-a*b-a*c+b^2-b*c+c^2)^(1/2)/(a-c)-(b-c)/(a-c), 1]
[                                                 1,                                                  1, 1]
disp(simplify(A*v(:,1)-lambda(1,1)*v(:,1)))
0
0
0
```

check the definition of $\lambda$ and $v$

```
disp(all(simplify(A*v(:,1) == lambda(1,1)*v(:,1))))
1
```

true  **all()**: universal quantifier

```
disp(simplify(A*v(:,2)-lambda(2,2)*v(:,2)))
0
0
0
```

```
disp(all(simplify(A*v(:,2) == lambda(2,2)*v(:,2))))
1
```

true

```
disp(simplify(A*v(:,3)-lambda(3,3)*v(:,3)))
0
0
0
```

```
disp(all(simplify(A*v(:,3) == lambda(3,3)*v(:,3))))
1
```

true

# Symbolic Differential Equations: examples [1]

```
syms y(t)
Y=dsolve(diff(y,t) == 1+y^2)
Y =
tan(C1 + t)
          1i
         -1i
Y1=diff(Y)
Y1 =
tan(C1 + t)^2 + 1
                  0
                  0
Y=dsolve(diff(y,t) == 1+y^2, y(0)==1)
Y =
tan(t+pi/4)
Y1=diff(Y)
Y1 =
tan(t + pi/4)^2 + 1
h=ezplot(Y);
grid on; hold on
plot(0,1,'ok','MarkerFace','k','MarkerSize',5)
```

general solution dependent on an arbitrary constant

check the solution

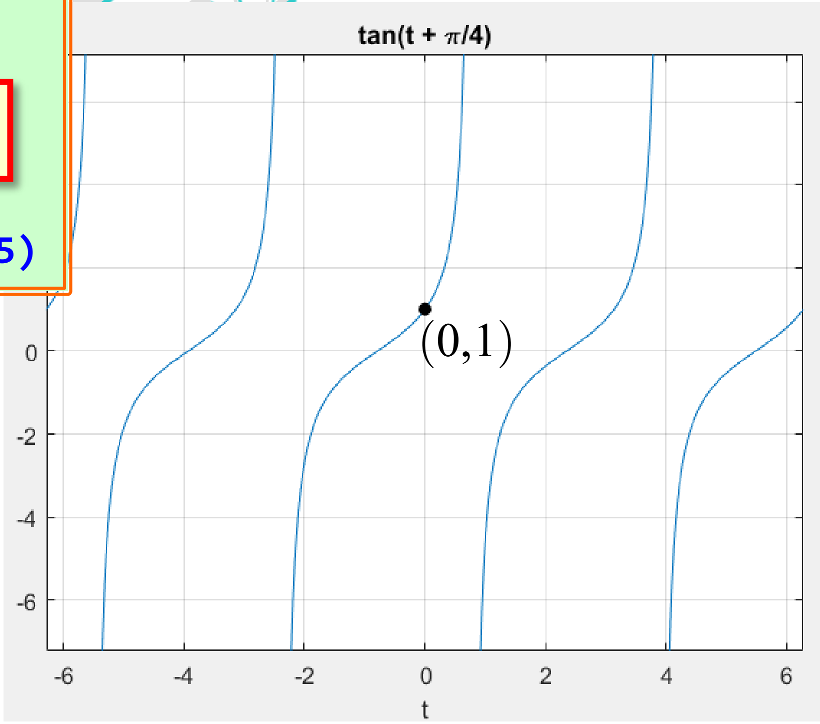
particular solution

initial condition

$$y' = 1 + y^2$$

$$\begin{cases} y' = 1 + y^2 \\ y(0) = 1 \end{cases}$$

IVP: Initial Value Problem



# Symbolic Differential Equations: examples [2]

## 2nd order differential equation: IVP

$$\begin{cases} y'' = \cos(2t) - y \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

```
syms y(t)
D1y=diff(y,t);  D2y=diff(y,t,2);
Y=dsolve(D2y == cos(2*t)-y, y(0) == 1, D1y(0) == 0);
simplify(Y)
ans =
1 - (8*sin(t/2)^4)/3
simplify(diff(Y,t,2) - cos(2*t) + Y)
ans =
0
```

$y'$ and $y''$

check the solution

**2 initial conditions**

## system of 2 differential equations del 1° ordine

$$\begin{cases} f' = +3f + 4g \\ g' = -4f + 3g \end{cases}$$

```
syms f(t) g(t); eqns=[diff(f,t) == 3*f+4*g, diff(g,t) == -4*f+3*g];
[F,G]=dsolve(eqns); F=simplify(F), G=simplify(G)
F =
exp(3*t)*(C2*cos(4*t) + C1*sin(4*t))
G =
exp(3*t)*(C1*cos(4*t) - C2*sin(4*t))
```

general solution

```
S=dsolve(eqns)
S = struct with fields:
    g: C1*cos(4*t)*exp(3*t) - C2*sin(4*t)*exp(3*t)
    f: C2*cos(4*t)*exp(3*t) + C1*sin(4*t)*exp(3*t)
```

```
eqns=…; conds=[f(0) == 0, g(0) == 1];
[F,G]=dsolve(eqns, conds)
F =
sin(4*t)*exp(3*t)
G =
cos(4*t)*exp(3*t)
```

particular solution

```
[F,G]=dsolve(eqns, f(0) == 0, g(0) == 1)
F =
sin(4*t)*exp(3*t)
G =
cos(4*t)*exp(3*t)
```

particular solutions

# Symbolic Differential Equations: examples [3]



Let us consider the following PDE (Partial Differential Equation) problem for the 1D wave equation equipped by Initial Conditions (IC) and Boundary Conditions (BC):

$$
\begin{cases}
\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}, & 0 < x < L, \quad t > 0 \\
u(x,0^+) = \dfrac{x \sin(3x)}{6} & \text{IC} \\
\dfrac{\partial u}{\partial t}(x,0^+) = \dfrac{\sin(3x)}{6} + \dfrac{x\cos(3x)}{2} & \\
u(0,t) = \dfrac{t \sin(3t)}{6} & \text{BC} \\
u(L,t) = \dfrac{(L+t)\sin[3(L+t)]}{6} &
\end{cases}
$$

1D wave equation

a single spatial variable ($x$) and a time variable ($t$)

analytical solution

$$u(t,x) = \frac{(x+t)\sin[3(x+t)]}{6}$$

$$F(s) = \mathscr{L}[f(t)] = \int_0^\infty f(t) e^{-st}\,\mathrm{d}t, \quad \mathrm{Re}(s) > \sigma_0$$

$\sigma_0$ *abscissa of convergence*

In order to solve the PDE, we can apply the **Laplace Transform (LT) method** w.r.t. t, by computing $U(x,s) = \mathscr{L}_t[u(t,x)]$; it transforms the PDE problem into an ODE problem producing the following Boundary Value Problem (BVP):

$$
\begin{cases}
U'' = s^2 U - (sx+1)\dfrac{\sin(3x)}{6} - x\dfrac{\cos(3x)}{2}, & 0 < x < L, \quad s \in \mathbb{C} \\
U(0,s) = \dfrac{s}{(s^2+9)^2} & \\
U(L,s) = \dfrac{s\cos(3L) - 3\sin(3L)}{(s^2+9)^2} + \dfrac{(sL+1)\sin(3L) + 3L\cos(3L)}{6(s^2+9)} &
\end{cases}
$$

analytical solution

$$U(x,s) = \frac{(sx+1)\sin(3x) + 3x\cos(3x)}{6(s^2+9)} + \frac{s\cos(3x) - 3\sin(3x)}{(s^2+9)^2}$$

# Symbolic Differential Equations: examples [3a]

The complex variable *s*, introduced by Laplace Transform, is considered as a parameter, and it will be ignored in solving the BVP simbolically by means of the **dsolve()** function.

```
syms x L real;   syms s;   syms U(x) % s is considered as a parameter
ODE = diff(U,x,2) == s^2*U - (s*x+1)*sin(3*x)/6 - x*cos(3*x)/2;

cond1 = U(0) == s/(s^2+9)^2;
cond2 = U(L) == (s*cos(3*L)-3*sin(3*L))/(s^2+9)^2 + ...
                          ((s*L+1)*sin(3*L)+3*L*cos(3*L))/(6*(s^2+9));
conds = [cond1, cond2]; % boundary conditions

Usol = dsolve(ODE, conds); % solve BVP simbolically

% true analytical solution (for a comparison)
Utrue = ((s*x+1)*sin(3*x)+3*x*cos(3*x))/(6*(s^2+9)) + ...
                                    (s*cos(3*x)-3*sin(3*x))/(s^2+9)^2;
% compare symbolic and analytical solutions
fprintf('\nCheck if the solution is correct: Usol - Utrue = ');
disp(simplify(Usol - Utrue))
Check if the solution is correct: Usol - Utrue =
0
```

Download live script: **wave_BVP.mlx**

# Symbolic Differential Equations: examples [3b]

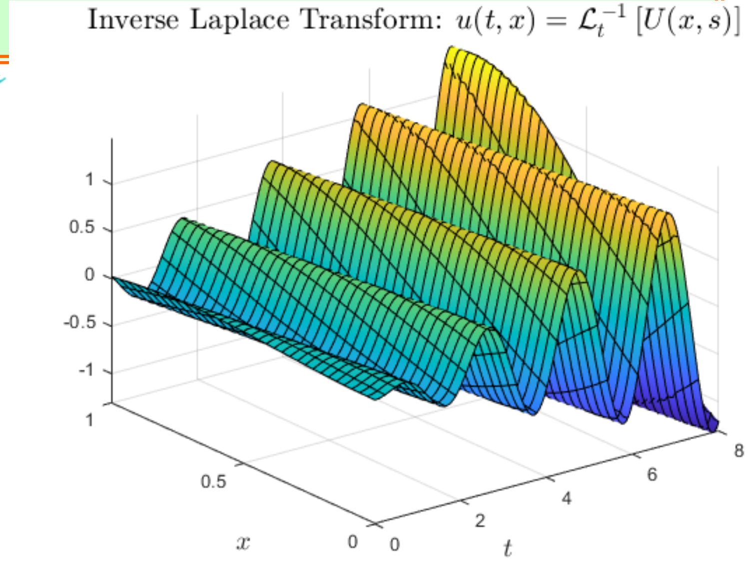
Once the Laplace Transform $U(x,s)$ has been computed by solving the BVP, to find the symbolic solution $u(x,t)$ of the PDE problem, we have to compute the Inverse Laplace Transform of $U(x,s)$ by means of the **ilaplace()** symbolic function:

```
syms t real;  u = simplify(ilaplace(Usol,s,t))
   u =
```

$$\frac{\sin(3\,t+3\,x)\,(t+x)}{6}$$

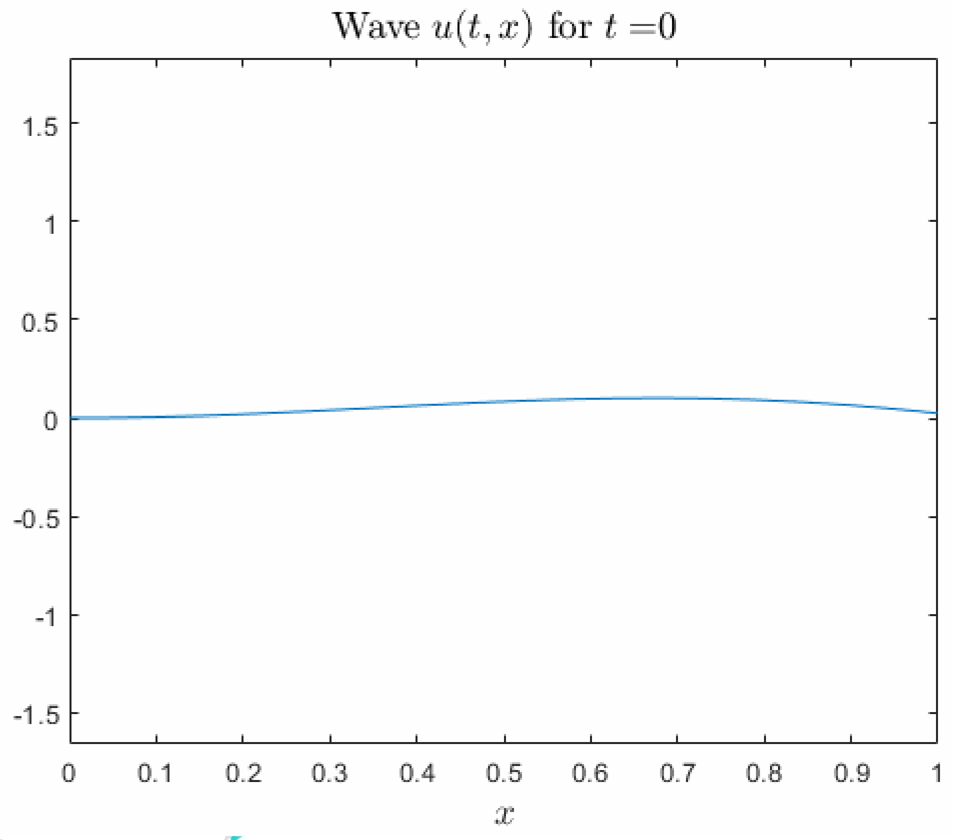
```
uTRUE = (x+t)*sin(3*(x+t))/6; % known analytical solution
% comparison
fprintf('\nCheck if the symbolic Inverse LT is correct: u - uTRUE =');
disp(simplify(u - uTRUE))
Check if the symbolic Inverse LT is correct: u – uTRUE =
0
```

Download live script: **wave_BVP.mlx**



# Symbolic Differential Equations: examples [3c]

At last, the animated wave solution is created

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