Prova A

$$f(x) = \frac{3e^{2x}}{x-4}$$

$$E[f(x)] =]-\infty, 4[\cup]4, +\infty[$$

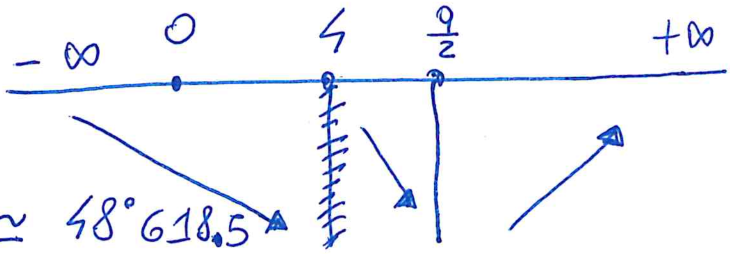
a) $$f'(x) = \frac{6e^{2x}(x-4) - 3e^{2x}}{(x-4)^2} = \frac{3e^{2x}(2x-8-1)}{(x-4)^2} =$$

$$= \frac{3e^{2x}(2x-9)}{(x-4)^2}$$

$$f'(x) \geq 0 \iff 2x-9 \geq 0 \quad \forall x \in E[f(x)]$$

$$\iff x \geq \frac{9}{2} \text{ in } E[f(x)]$$

$x = \frac{9}{2}$ p.to min rel.



$$f\left(\frac{9}{2}\right) = \frac{3e^{\cancel{2}\cdot\frac{9}{\cancel{2}}}}{\frac{9}{2}-4} = \frac{3}{\frac{1}{2}} \cdot e^9 = 6e^9 \simeq 48°618.5$$

(48 mila 618 virgola 5)

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b) $$I = \left[\frac{17}{4}; \frac{21}{4}\right] \subset E[f(x)]$$

$\frac{17}{4} = 4.25$; $\frac{21}{4} = 5.25$

Candidati: $x = \frac{17}{4}$; $x = \frac{21}{4}$; $x = \frac{9}{4} = 4.5$

$$f\left(\frac{17}{4}\right) = \frac{3e^{-\cancel{2}\cdot\frac{17}{\cancel{4}_2}}}{\frac{17}{4}-4} = \frac{3}{\frac{1}{4}}\cdot e^{\frac{17}{2}} = 3\cdot 4e^{17/2} = 12e^{17/2} \simeq 58^{\circ}977.2$$

(58 mila
977 virgola 2)

$$f\left(\frac{21}{4}\right) = \frac{3e^{\overset{1}{\cancel{2}}\cdot\frac{21}{\cancel{4}_2}}}{\frac{21}{4}-4} = \frac{3}{\frac{5}{4}}\, e^{21/2} = 3\cdot\frac{4}{5}\, e^{21/2} = \frac{12}{5}\, e^{21/2} \simeq 87^{\circ}157.2$$

(87 mila
157 virgola
2)

$$f\left(\frac{9}{2}\right) = 6e^{9} \simeq 48^{\circ}618.5$$

$\max f(x) = \frac{12}{5}\, e^{21/2}$; $x = \frac{21}{4}$ p.to max assoluto.

$\min f(x) = 6e^{9}$; $x = \frac{9}{2}$ p.to minimo assoluto.

Prova B

$$f(x) = \frac{\log x}{3x}$$

$$E[f(x)] =]0, +\infty[$$

a) $$f'(x) = \frac{\cancel{\frac{1}{x}\cdot 3x} - \log x \cdot 3}{(3x)^2} = \frac{\cancel{3}^{\,1}}{\cancel{9}_{\,3}\,x^2}(1 - \log x) =$$

$$= \frac{1}{3x^2}(1 - \log x)$$

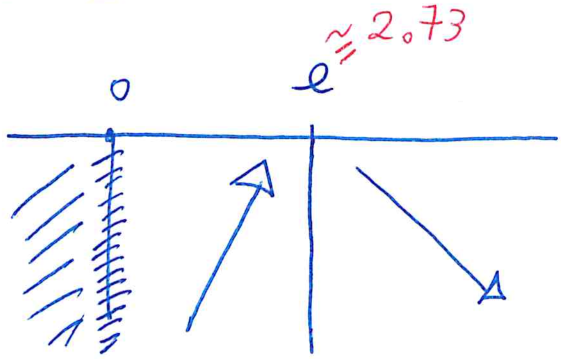
$$f'(x) \geq 0 \iff 1 - \log x \geq 0 \quad \forall x \in E[f(x)]$$

$$\iff \log x - 1 \leq 0 \iff \log x \leq 1 \iff$$

$$\iff 0 < x \leq e \quad \text{in } E[f(x)]$$

$x = e$ p.to max rel.

$$f(e) = \frac{\log e}{3e} = \frac{1}{3e} \simeq 0.123$$



b) $I = [1, 5] \subset E[f(x)]$

Candidati: $x = 1$; $x = 5$; $x = e$

$$f(1) = \frac{\log 1}{3} = 0$$

$$f(5) = \frac{\log 5}{15} \simeq 0.107$$

$$f(e) = \frac{1}{3e} \simeq 0.123$$

$\max f(x) = \frac{1}{3e}$; $x = e$ p.to max assoluto

$\min f(x) = 0$; $x = 1$ p.to min assoluto

Prova C

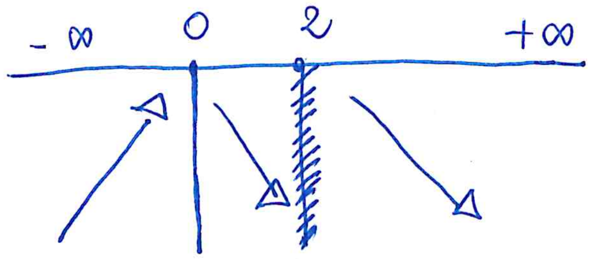
$$f(x) = \frac{e^{-\frac{1}{2}x}}{x-2}$$

$$E[f(x)] = \{ x \in \mathbb{R} - \{2\}\} =]-\infty, 2[\cup]2, +\infty[$$

a) $$f'(x) = \frac{-\frac{1}{2} e^{-\frac{1}{2}x}(x-2) - e^{-\frac{1}{2}x}}{(x-2)^2} = \frac{-e^{-\frac{1}{2}x}\left(\frac{1}{2}x - 1 + 1\right)}{(x-2)^2} =$$

$$= \frac{-\frac{1}{2}x\, e^{-\frac{1}{2}x}}{(x-2)^2}$$

$f'(x) \geq 0 \iff -x \geq 0 \iff x \leq 0 \quad \forall x \in E[f(x)]$



$x = 0$ p.to max rel.

$f(0) = -\frac{1}{2} = -0.5$

b) $I = [-1, 1] \subset E[f(x)]$

Candidati: $x = 0$; $x = -1$; $x = 1$

$f(0) = -\frac{1}{2} = -0.5$

$$f(-1) = \frac{e^{-\frac{1}{2}(-1)}}{-1-2} = -\frac{1}{3} e^{1/2} \simeq -0.55$$

$$f(1) = \frac{e^{-\frac{1}{2}(1)}}{1-2} = -e^{-1/2} \simeq -0.6$$

$\max f(x) = -\frac{1}{2}$; $x = 0$ p.to max assoluto

$\min f(x) = -e^{-1/2}$; $x = 1$ p.to min assoluto

Prova D

$$f(x) = \frac{x}{\log x}$$

$$E[f(x)] = \{ x \in \mathbb{R} : x > 0 \text{ e } \log x \neq 0 \} =]0, 1[\cup]1, +\infty[$$

C.E.

$$\begin{cases} x > 0 \\ \log x \neq 0 \end{cases} \iff \begin{cases} x > 0 \\ \log x \neq \log 1 \end{cases} \iff \begin{cases} x > 0 \\ x \neq 1 \end{cases} \iff$$

$$\iff 0 < x < 1 \quad \vee \quad x > 1 .$$

2)

$$f'(x) = \frac{\log x - \cancel{x} \cdot \frac{1}{\cancel{x}}}{\log^2 x} = \frac{\log x - 1}{\log^2 x}$$

$$f'(x) \geq 0 \iff \log x - 1 \geq 0 \quad \forall x \in E[f(x)]$$

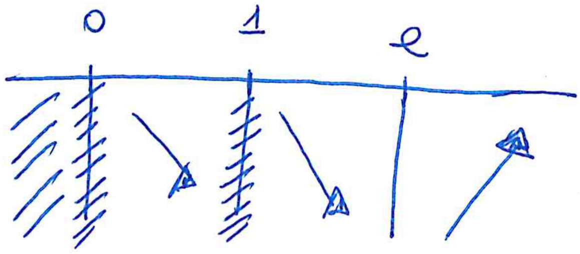
$$\iff \log x \geq 1 \iff \log x \geq \log e$$

$$\iff x \geq e \simeq 2.73 \quad \in E[f(x)]$$

(il punto e appartiene al dominio della funzione)

$x = e$ p.to min relativo

$$f(e) = \frac{e}{\log e} = \frac{e}{1} = e \simeq 2.73$$



b) $I = [2,3] \subset E[f(x)]$

Candidati : $x = e$; $x = 2$; $x = 3$

$f(e) = e \simeq 2.7183$ (più piccolo) → [specificare almeno 4 cifre decimali, altrimenti il confronto risulta difficile]

$f(2) = \frac{2}{\log 2} \simeq 2.8854$ (più grande)

$f(3) = \frac{3}{\log 3} \simeq 2.7307$

$\max f(x) = \frac{2}{\log 2}$; $x = 2$ p.to max assoluto.

$\min f(x) = e$; $x = e$ p.to min assoluto