

The Macroeconomics of Innovation: *Models of Economic Growth -* Exogenous technology growth

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Economics of innovation

Introduction

- Economic growth is defined as a situation where GDP per capita increases over time.
- Innovation is central to economic growth →
Microeconomists define an innovation as something that increases “value” to an enterprise, perhaps by raising sales or lowering costs. At the economy level, GDP measures the aggregate value created by all enterprises. Hence, innovation at the firm level will be an important driver of GDP growth.

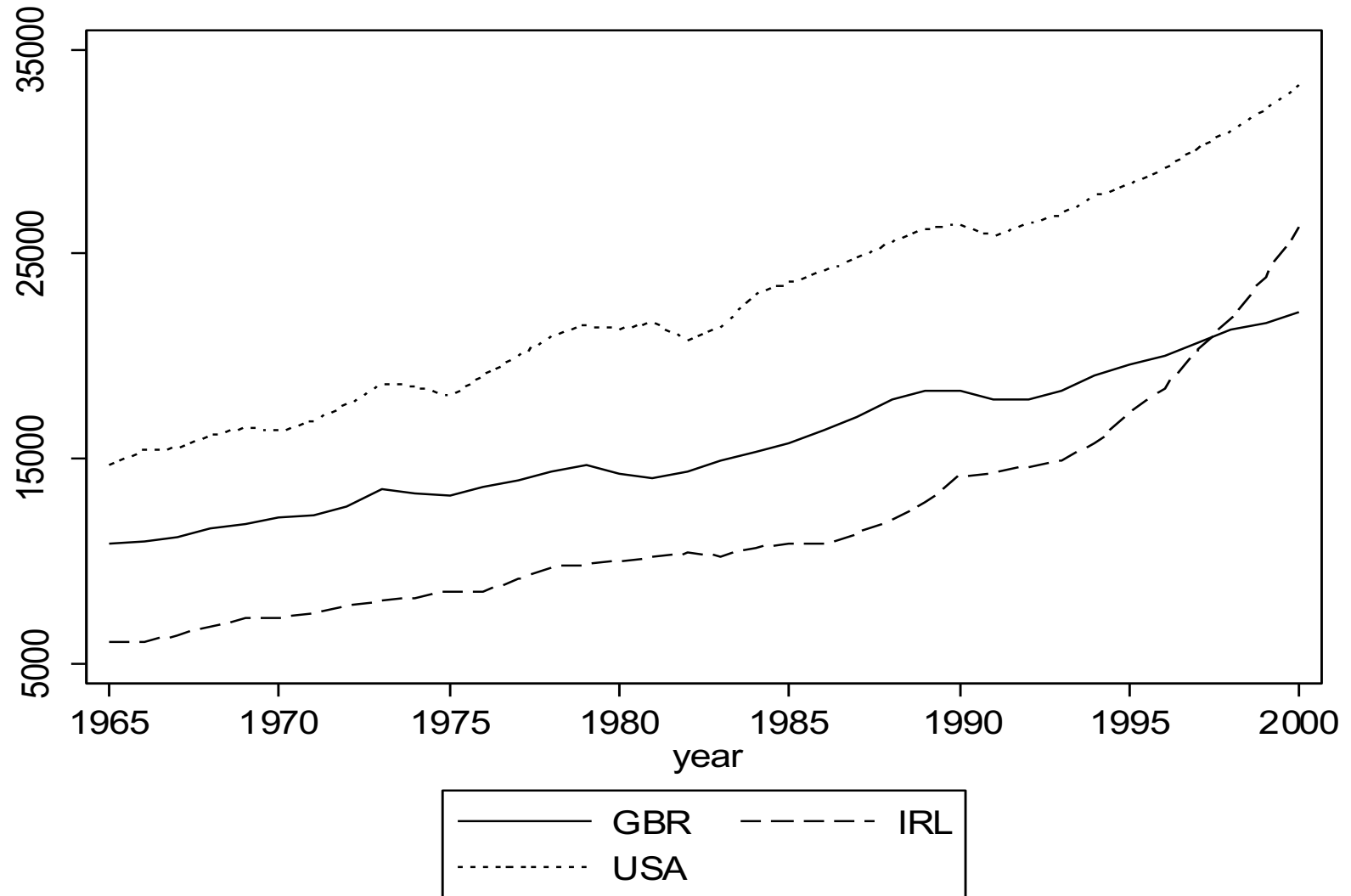
The very long run

Growth of GDP per capita (average annual percentage changes)

	1500-1820	1820-1900	1900-2000
OECD		1.2	2.0
Non-OECD		0.4	0.6
World	0.04	0.8	1.9

Source: Boltho and Toniolo (1999, Table 1) OECD refers to North America, Western Europe, Japan, Australia and New Zealand.

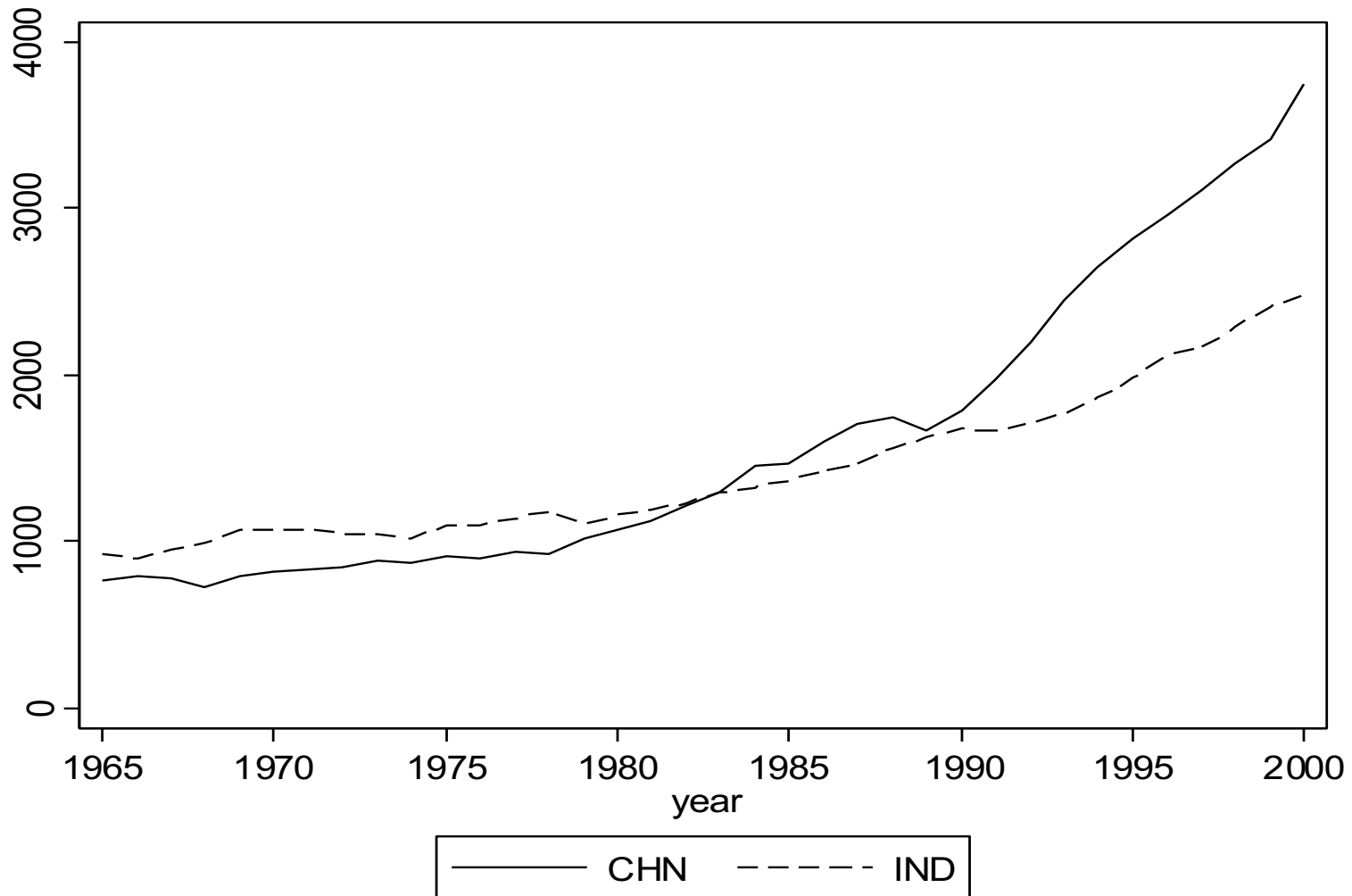
USA, UK and IRL



Growth of GDP p.c: USA=2.2%, GBR=2.0%, Ireland=3.7% (but post-93, 8.5%)

GDP per capita is US\$ 1996 constant prices. Source: Penn World Table 6.1

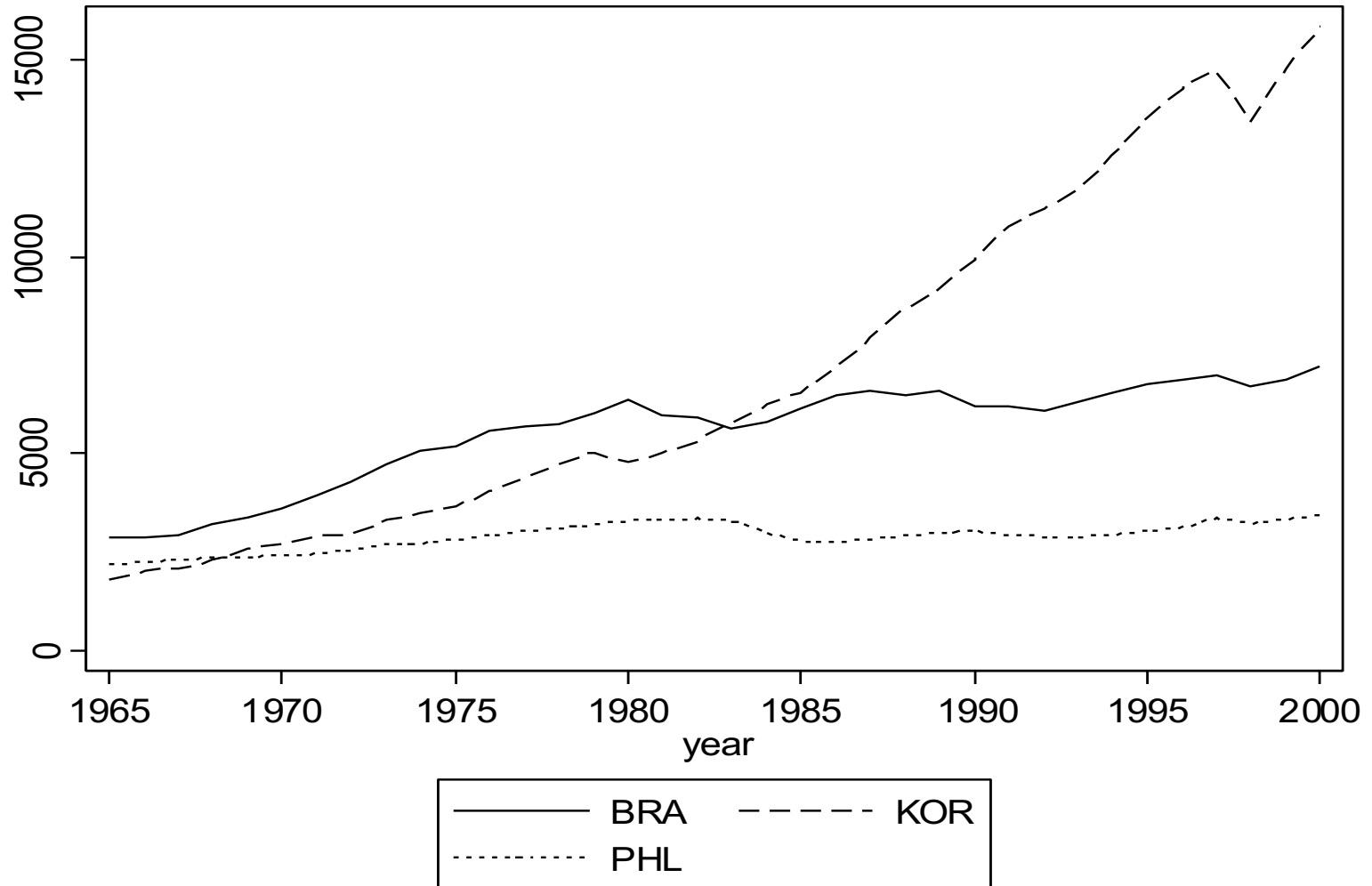
China and India



Growth: pre-90 China 3.7%, India 4.4%. 1990-2000: China 7.0%, India 4.4%

Source: Penn World Table 6.1

Brazil, S. Korea, Philippines



Source: Penn World Table 6.1 (<http://pwt.econ.upenn.edu/aboutpwt.html>)

Neoclassical model

This is one of the key models that modern economists use to think about the process of economic growth.

This model most often attributed to Robert Solow (1956) – US Nobel prize winner but Trevor Swan (1956) (a less well known Australian economist) published (independently) a very similar paper in the same year – hence refer to Solow-Swan model

Model Background

The Solow growth model is the starting point to analyze why growth differs among similar countries

- The model has been developed in the **mid-1950s** by Robert Solow of the MIT and was the basis for the Nobel Prize he received in 1987
- It is inspired to the **neo-classical** theory
- It can be considered an **extension** to the 1946 Harrod–Domar model by including new concepts as productivity, technological progress. Today, economists use Solow's sources-of-growth accounting to estimate the separate effects on economic growth coming from technological change, capital, and labor
- Important contributions to the model came from the work done by **Robert Solow and Trevor Swan** in 1956
- The Solow model is **the basis for the modern theory** of economic growth.
- Solow's model estimated the US economic growth with some success but not the same for other country's application

Model Background: more details

- The model is an exogenous model of economic growth that analyzes **changes in the level of output** in an economy over time as a result of changes in the population growth rate, the savings rate and the technological progress
- It is based on the **hypothesis** that the accumulation of capital is the engine of long-run economic growth
- It has been built on the basis of the **Cobb-Douglas** production function $Y = f(K, L)$ by adding a theory of capital accumulation
- The Cobb-Douglas production function is based on the assumption of **constant return to scale**
- Although constant return to scale has become the canonical way celebrated in many economic textbooks to analyze growth, recent re-appraisal of Harrod's work has **contested** the explicit use a fixed proportions production function

Model equations

$$Y = Af(K, L) \quad (\text{production function})$$

$Y = \text{GDP}$, $A = \text{technology}$,

$K = \text{capital}$, $L = \text{labour}$

$$\frac{dK}{dt} = sY - \delta K \quad (\text{capital accumulation equation})$$

$s = \text{proportion of GDP saved}$ ($0 < s < 1$)

$\delta = \text{depreciation rate (as proportion)}$ ($0 < \delta < 1$)

Solow-Swan analyse how these two equations interact.

Y and K are endogenous variables; s , δ and growth rate of L and/or A are exogenous (parameters).

Outcome depends on the **exact** functional form of production function and parameter values.

Long-run implications: convergence

- The standard Solow model predicts that in the long run, **different economies converge** to their steady state equilibrium and that permanent growth is achievable only through the introduction of technological progress as **growth in saving** and in **populational** cause only marginal effects in the long-run
- An interesting implication of Solow's model is that **poor countries** should grow faster and eventually catch-up to richer countries.
- Then the so-called «convergence» hypothesis can be explained by:
 - lags in the diffusion of knowledge -> gaps in real income might shrink as poor countries receive better technology and information than the advanced ones;
 - efficient allocation of international capital flows -> since the rate of return on capital is going to be higher in poorer countries, capitals move from advanced countries to less developed countries

Simplified Representation of the model.

Assumptions

1. The **population grows** at a constant rate g . Therefore, current population (represented by N) and future population (represented by N') are linked through the population growth equation $N' = N(1+g)$. **If the current population is 100 and the growth rate of population is 2%, the future population is 102.**
2. All **consumers** in the economy save a constant proportion 's' of their incomes and consume the rest. Therefore, consumption (represented by C) and output (represented by Y) are linked through the consumption equation $C = (1-s)Y$. **If a consumer earns 100 units of output as income and the savings rate is 40%, then the consumer consumes 60 units and saves 40 units.**
3. All **firms** in the economy produce output using the same production technology that takes in capital and labor as inputs. Therefore, the level of output (represented by Y), the level of capital (represented by K), and the level of labor (represented by L) are all linked through the production function equation $Y = a f(K,L)$.

Simplified Representation of the model.

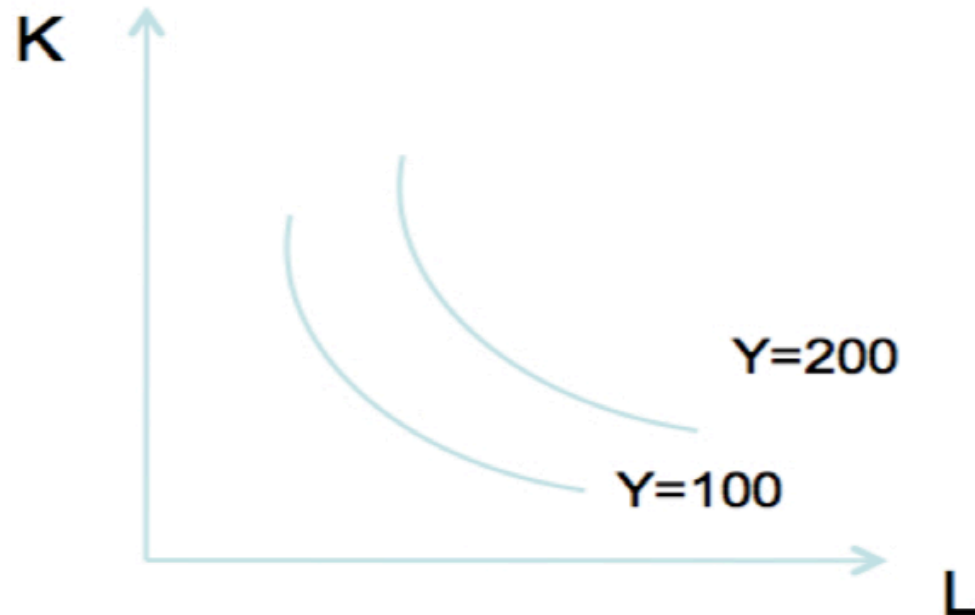
Assumptions

4. The Solow Growth Model assumes that the production function exhibits **constant-returns-to-scale**. Under such an assumption, by doubling both the level of capital stock and the level of labor, the level of output will exactly doubled. As a result, much of the mathematical analysis of the Solow model focuses on output per worker and capital per worker instead of aggregate output and aggregate capital stock.
5. Actual capital stock (represented by K), future capital stock (represented by K'), the rate of capital depreciation (represented by d), and level of capital investment (represented by I) are linked through the **capital accumulation equation**:

$$K' = K(1-d) + I.$$

Representation of constant returns to scale through isoquants

The isoquants of the CRS production function provide a geometric representation of the production function where, by doubling K and L , also output Y double



The model

- The **production function** is assumed to take the following form:

$$Y = aK^bL^{1-b} \text{ where } 0 < b < 1$$

- The production function is known as the Cobb-Douglas production function, the most widely used neoclassical production function. Together with the assumption that firms are competitive (i.e., they are price-taking firms) the coefficient b is the capital share (the share of income that capital receives) and the coefficient $1-b$ represents the labor share.
- It is possible to measure **output per worker**, given by the following equation:

$$y = ak^b \text{ where: } y = Y/L$$

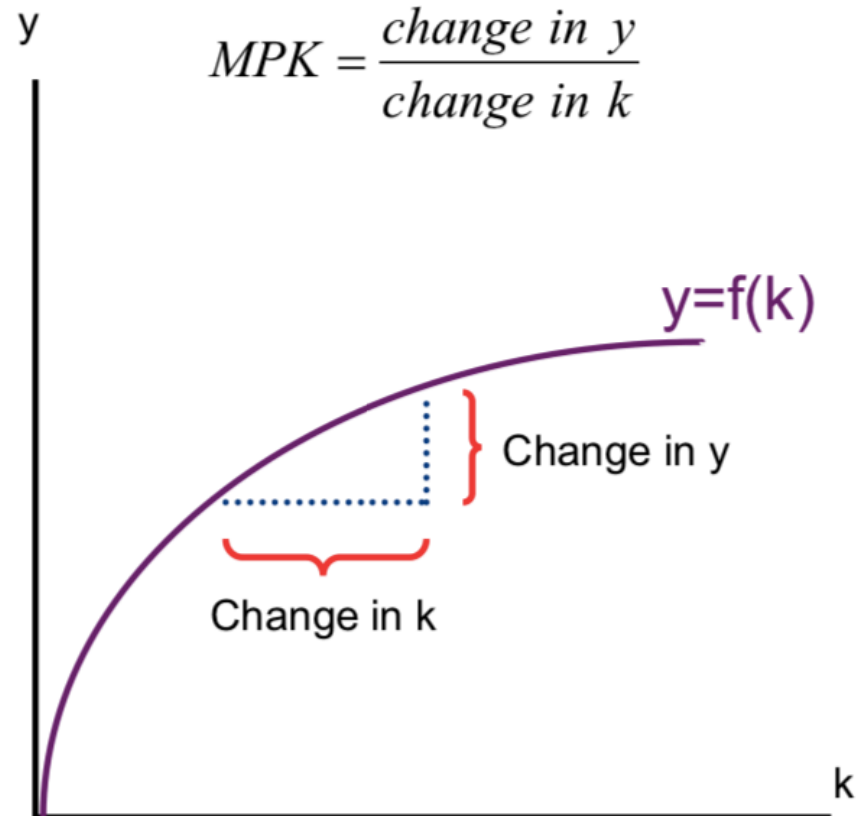
$y = Y/L$ (output per worker) and $k = K/L$ (capital stock per worker)

Model: graphic representation

- The curve is concave
- The slope of the curve is the marginal product of capital per worker.

$$MPK = f(k+1) - f(k)$$

- **MPK explains the change in output per worker following by the increase of one unit of capital**



Model development

- Under the assumption of **competitive equilibrium**, the following relations can be written:
 - The income-expenditure identity holds as an equilibrium condition:
$$Y = C + I$$
 - Consumer's budget constraint is: $C = Y - S$
 - In equilibrium: $I = S = sY$.
 - The capital accumulation equation is: $K' = (1-d)K + sY$
- The output increases up to the **steady state**.
- **The steady state is a state where the level of capital per worker does not change**

Steady State Equilibrium

- Solow model operates to **reach the steady state** equilibrium, as it follows:
- Substituting $f(k)$ for (y) , the investment per worker function ($i=s*y$) becomes a function of capital per worker ($i = s*f(k)$).
- then, adding a depreciation rate (d)
- The impact of both investment and depreciation on capital can be developed to evaluate the need of capital change:

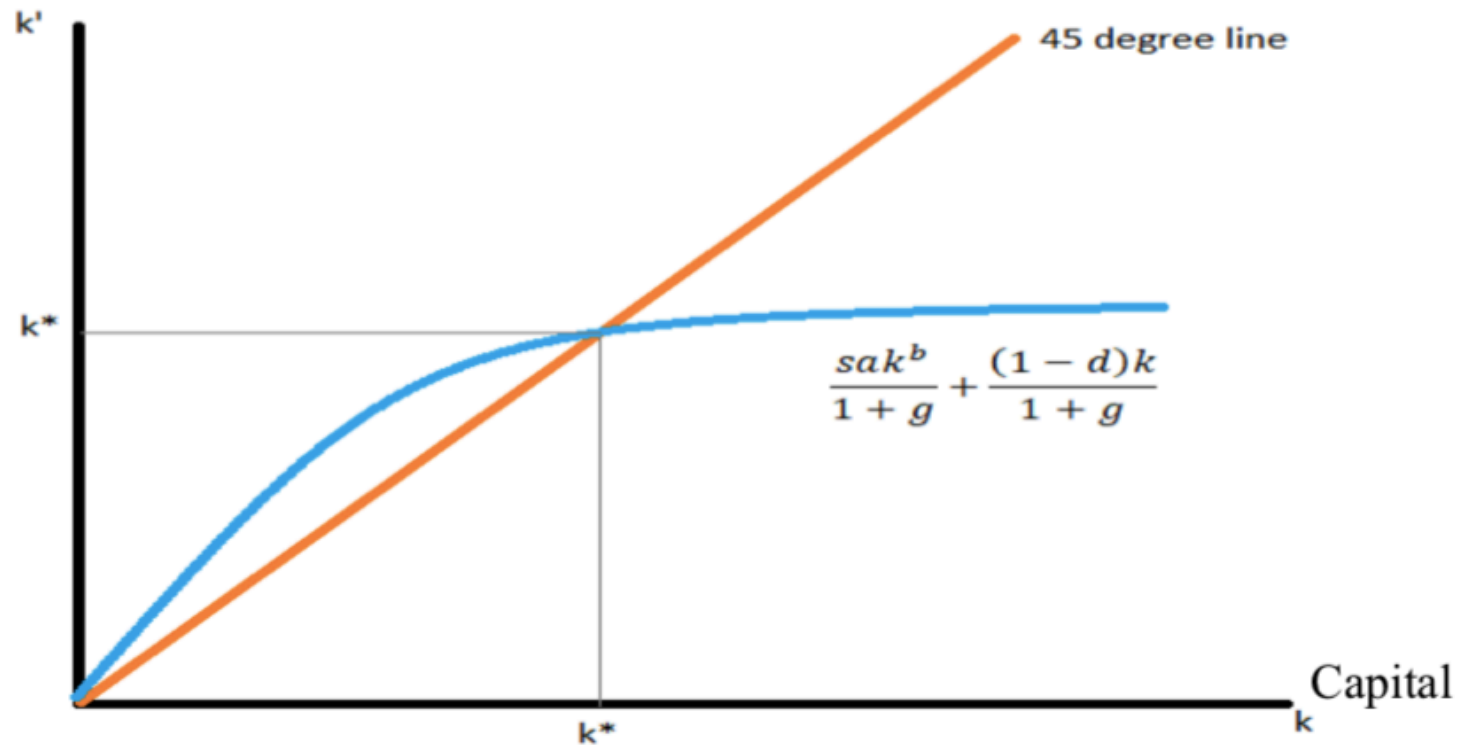
$$dk = i - dk \text{ ...substituting for } (i = s*f(k))$$

$$dk = s*f(k) - dk$$

Model development.

Steady state

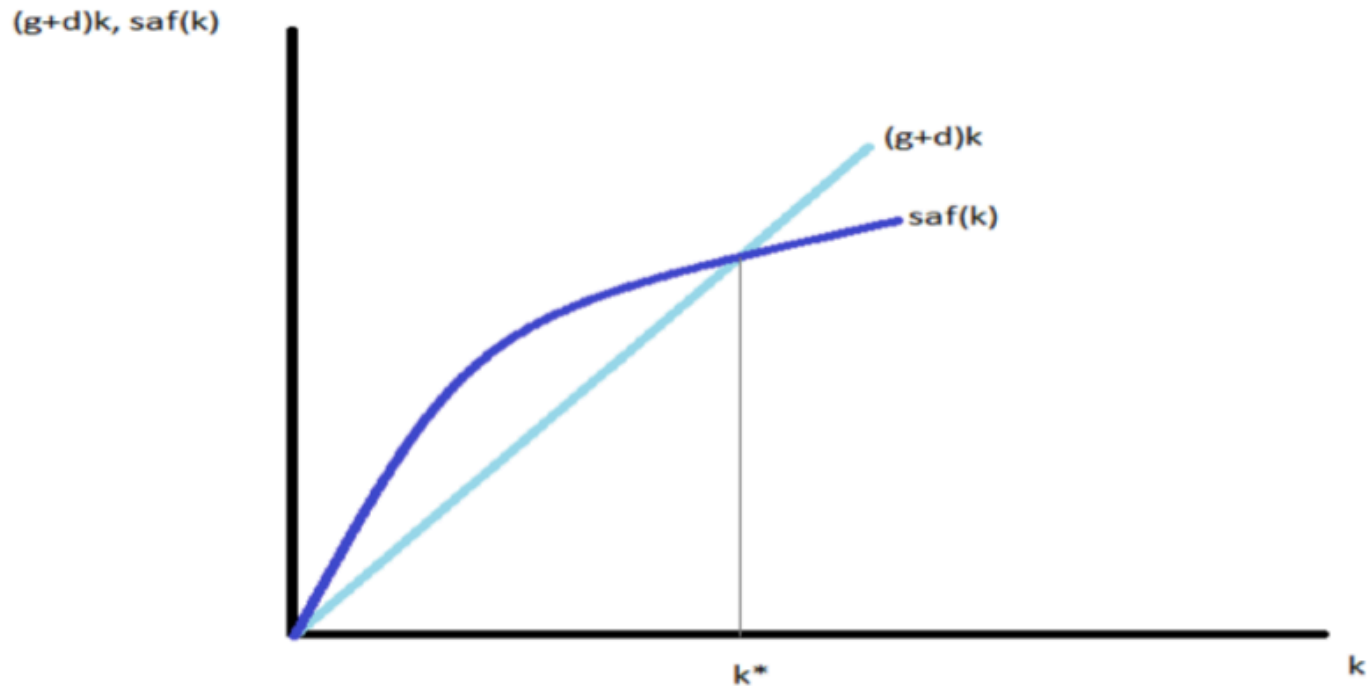
Investment, Depreciation



Model development.

Steady state per worker

It is possible to measure the **steady state value of capital per worker** and the **steady state value of output per worker**



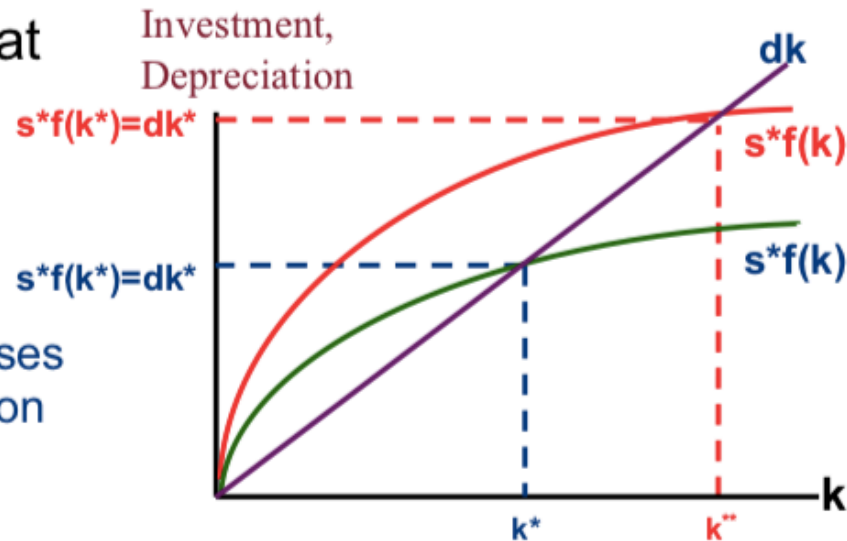
Changing the exogenous variable: savings

The steady state is reached at the point where $s^*f(k)=dk$

What happens if savings increase?:

- the slope of the investment function increases and cause the function to shift up.
- an higher steady state level of capital will be reached

Similarly a lower savings rate leads to a lower steady state level



Golden rule

- The 'golden rule' is the 'optimal' saving rate (s_G) that maximises consumption per head.
- Assume A is constant, but population growth is n .
- Can show that this occurs where the marginal product of capital equals $(\delta + n)$

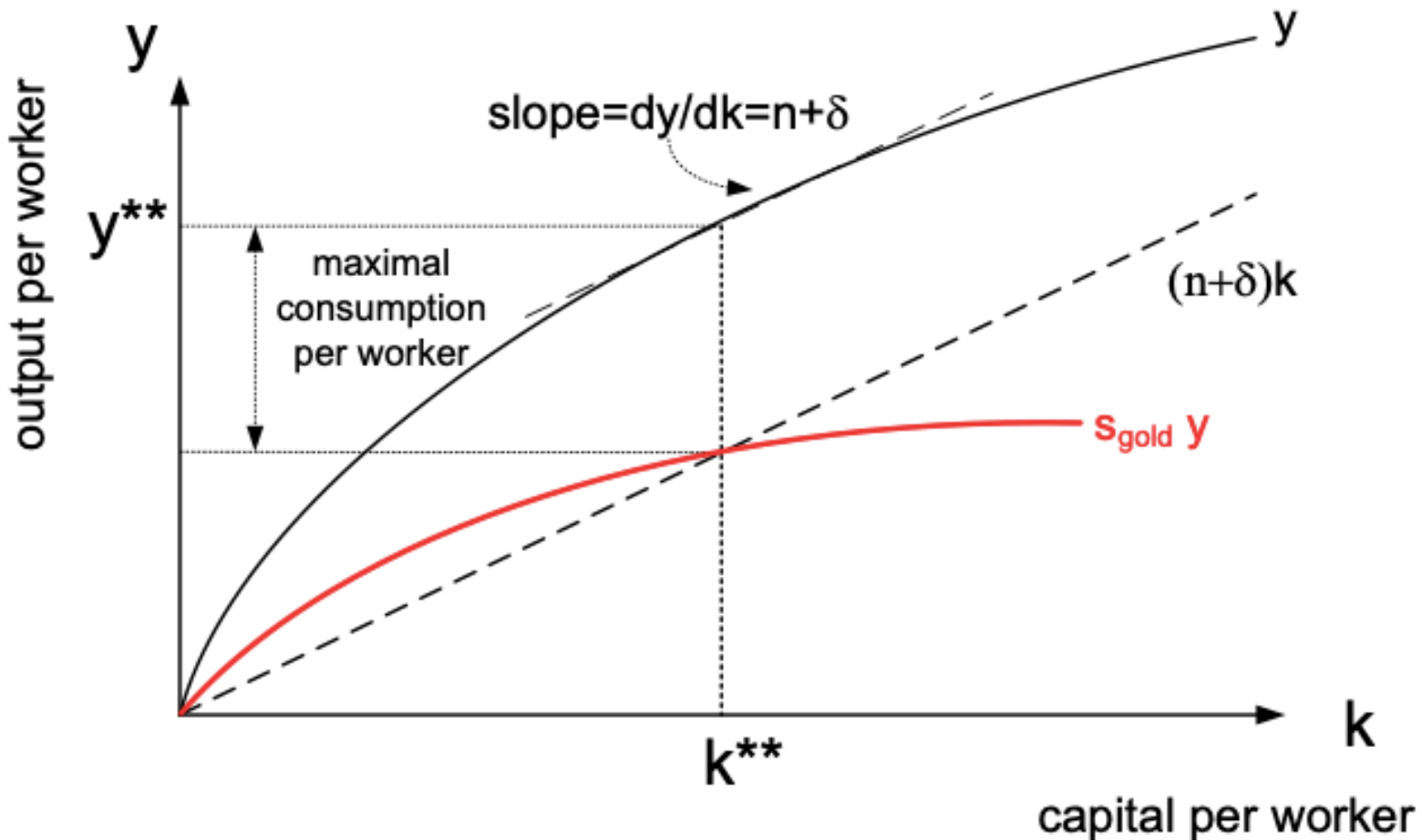
Proof: $\frac{dk}{dt} = sy - (\delta + n)k = 0$ at steady state,

hence $sy^* = (\delta + n)k^*$, where $*$ indicates steady state equilibrium value

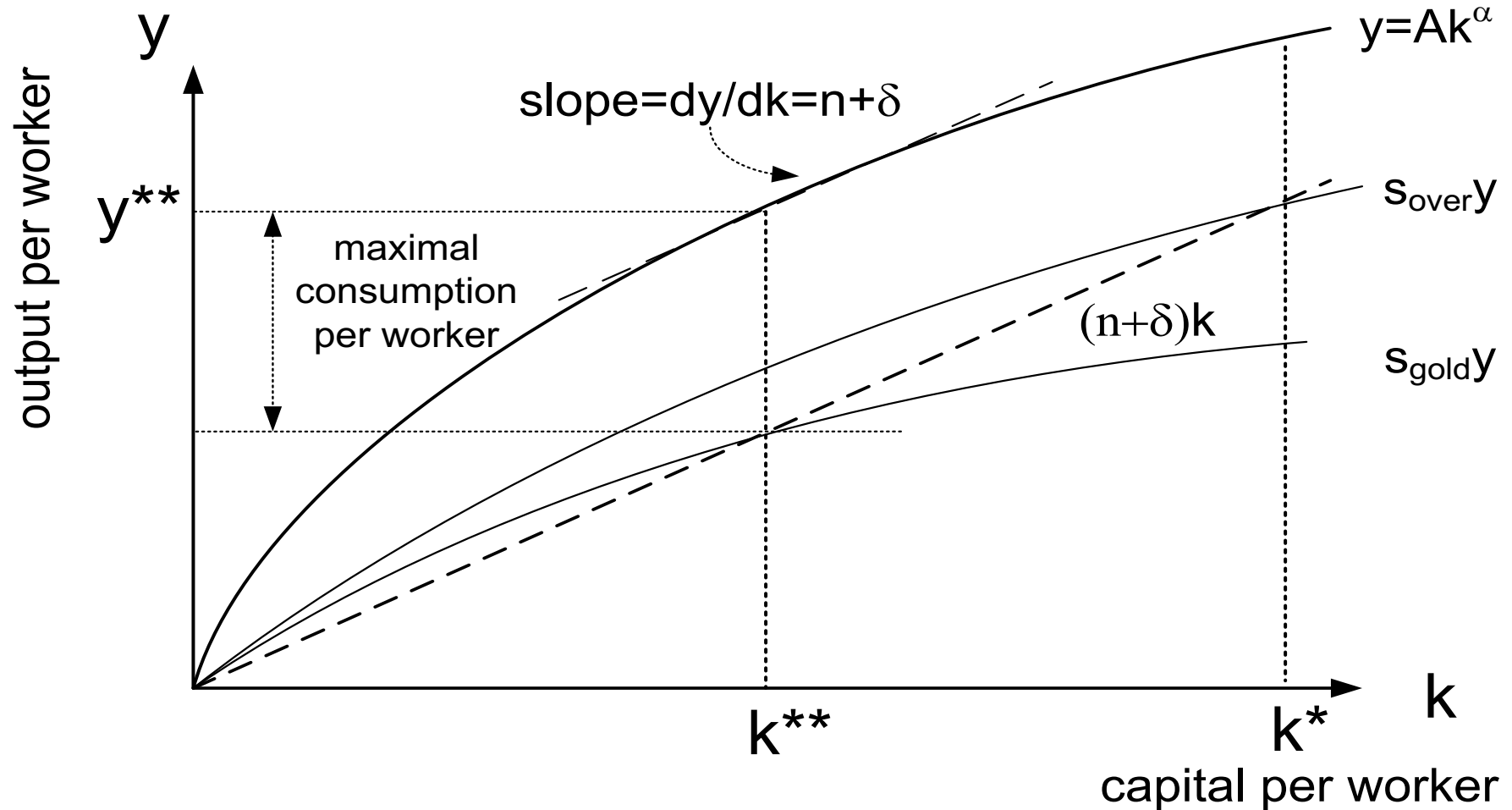
The problem is to: $\max_k c = y - sy = y^* - (\delta + n)k^*$

First order condition: $0 = \frac{dy^*}{dk^*} - (\delta + n)$ hence $MP_k = \frac{dy^*}{dk^*} = \delta + n$

Graphically find the maximal distance between two lines



... over saving



Economies can **over save**. Higher saving does increase GDP per worker, but real objective is consumption per worker.

Solow's surprise*

- Solow's model states that investment in capital cannot drive **long run** growth in GDP per worker
- Need technological change (growth in A) to avoid **diminishing returns to capital**
- Easterly (2001) argues that “capital fundamentalism” view widely held in World Bank/IMF from 60s to 90s, despite lessons of Solow model
- Policy lesson: don't advise poor countries to invest without due regard for technology and incentives

What if technology (A) grows?

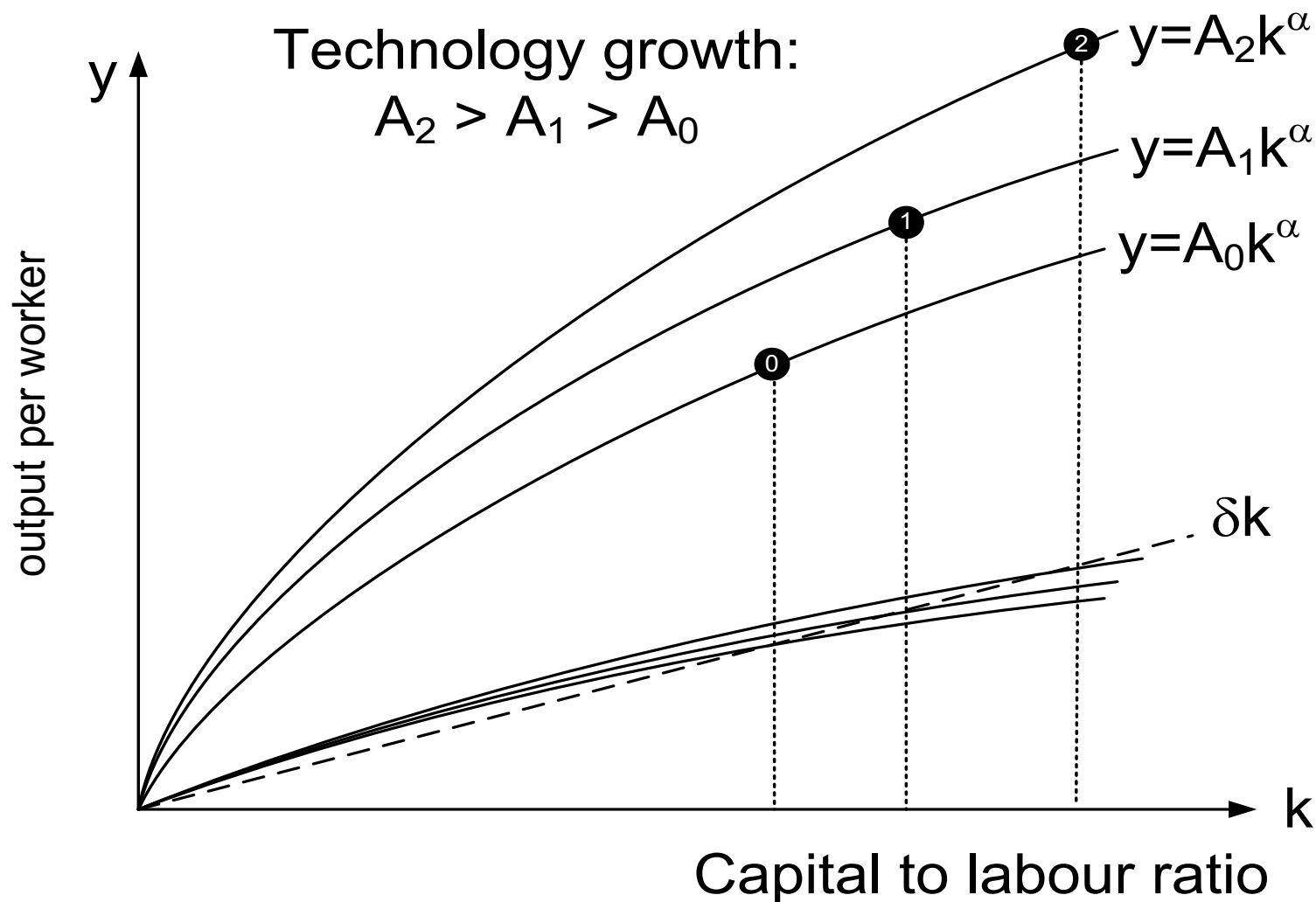
- Consider $y=Ak^\alpha$, and $sy=sAk^\alpha$, these imply that output can go on increasing.
- Consider marginal product of capital (MP_k)

$$MP_k=dy/dk =\alpha Ak^{\alpha-1},$$

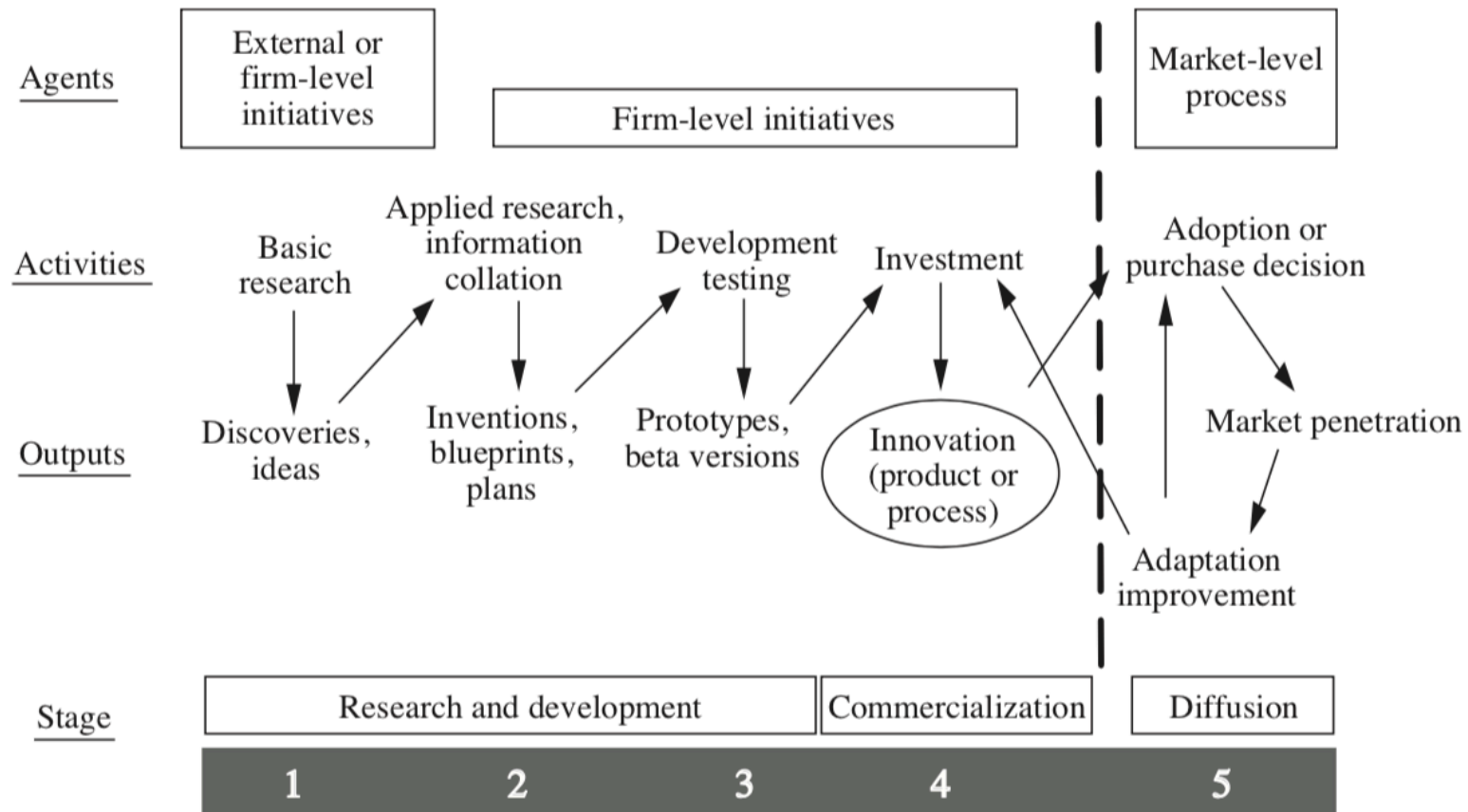
if A increases then MP_k can keep increasing (no 'diminishing returns' to capital)

- implies **positive long run growth**

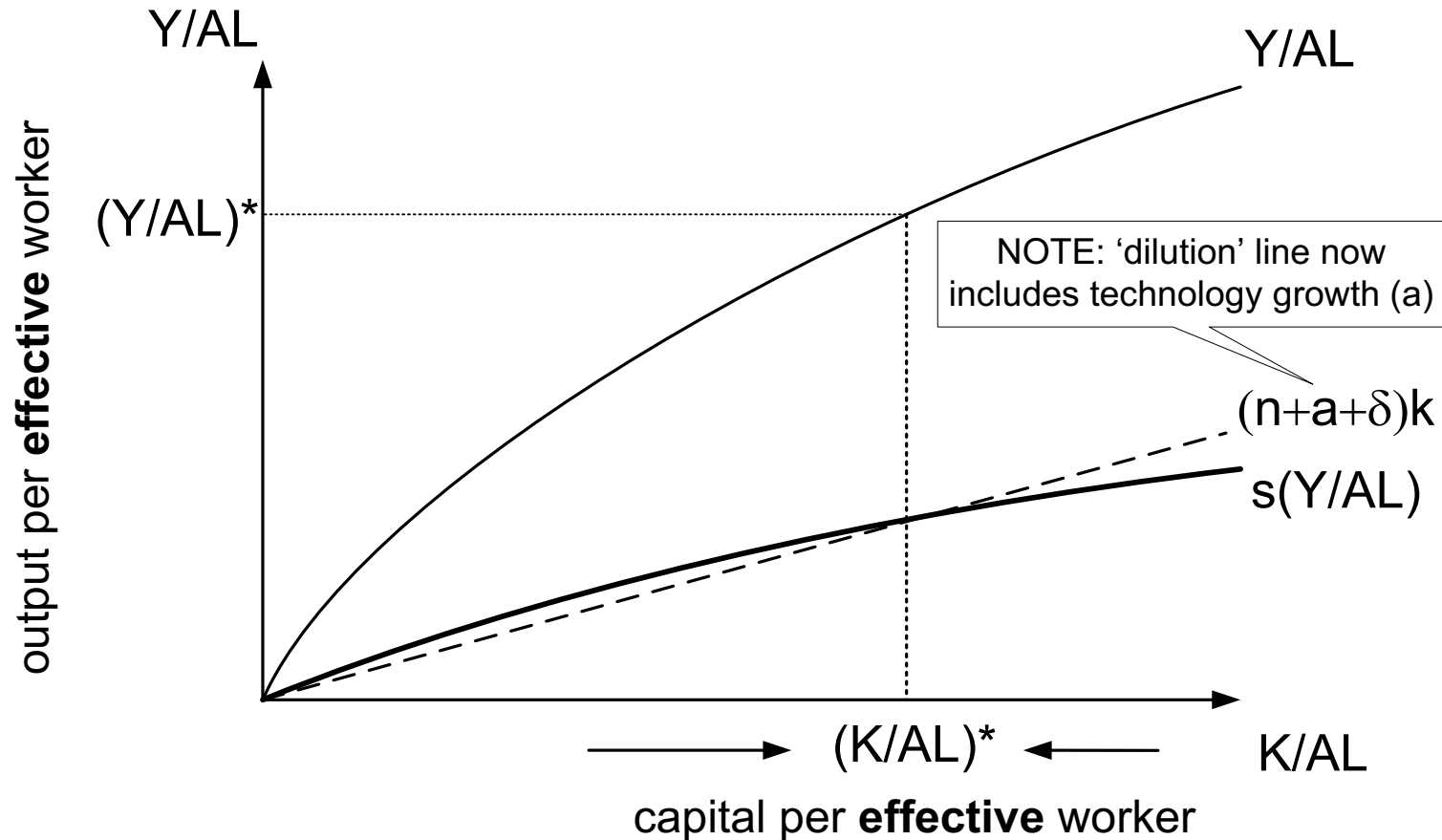
.... graphically, the production function simply shifts up



The stages of innovation



Output (capital) per effective worker diagram



If Y/AL is a constant, the growth of Y must equal the growth rate of L plus growth rate of A (i.e. $n+a$)

And, **growth in GDP per worker must equal growth in A .**

Summary of Solow-Swan

- Solow-Swan, or neoclassical, growth model, implies countries converge to steady state GDP per worker (if no growth in technology)
- if countries have same steady states, poorer countries grow faster and ‘converge’
 - call this classical convergence or ‘convergence to steady state in Solow model’
- changes in savings ratio causes “level effect”, but no **long run** growth effect
- higher labour force growth, ceteris paribus, implies lower GDP per worker
- Golden rule: economies can over- or under-save (note: can model savings as endogenous)