Lezione del 04/10/2022

MATEMATICA I

(ED. LIGUORI)

P. MARCELLINI, C. SBORDONE : "ANALISI MATEMATICA I"

M. BRAMANTI, C. PAGANI, S. SALSA " " " "

P. MARCELLINI, C. SBORDONE : "ESERCITAZIONI DI ANALISI MATEMATICA"

ED. LIGUORI

ZANICHELLI

1) ALGEBRA LINEARE : VETTORI (APPLICATI, LIBERI)

OPERAZIONI TRA VETTORI, SPAZIO VETTORIALE; MATRICI :

DETERMINANTE, RANGO ; SISTEMI LINEARI DI EQUAZIONI

2) CONCETTO DI FUNZIONE $f(x)$, $f: I \subseteq \mathbb{R} \to \mathbb{R}$

" DI LIMITE $\lim_{x \to x_0} f(x) = \ell$

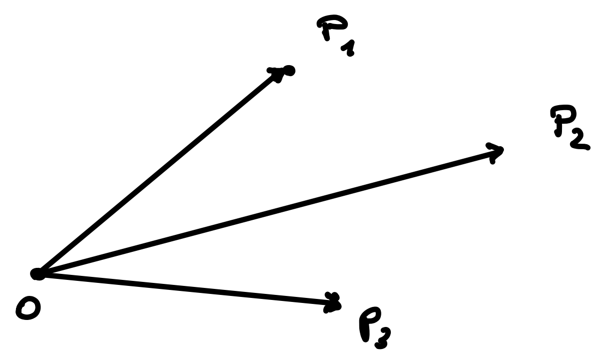
CALCOLO INFINITESIMALE

CONCETTO DI DERIVATA $f'(x)$

CALCOLO DIFFERENZIALE

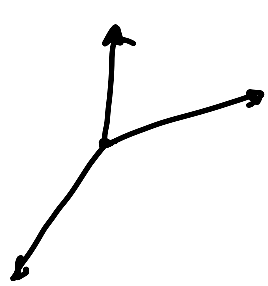
CONCETTO DI INTEGRALE $\int_a^b f(x)dx$

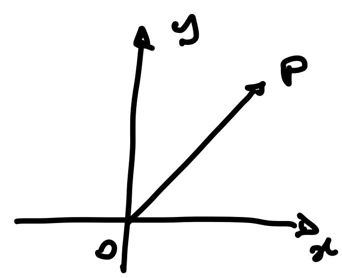
VETTORI NEL PIANO E NELLO SPAZIO

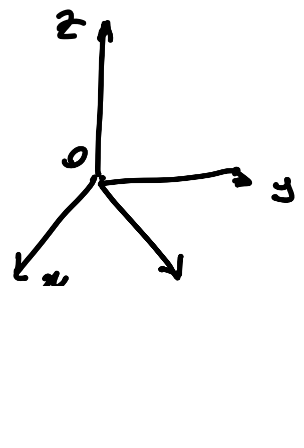


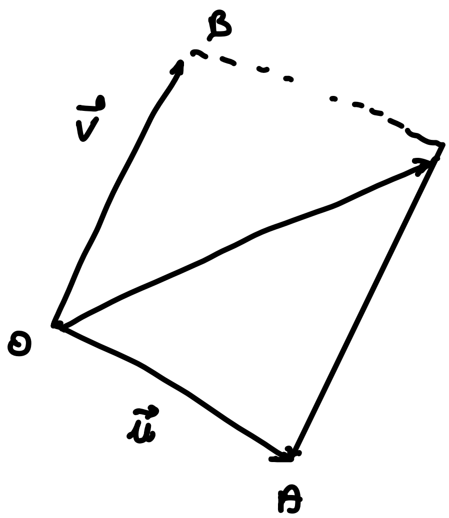
Def. Un vettore applicato del piano o dello spazio è un segmento orientato per cui vengono assegnati:

1) Il punto di applicazione O
2) La direzione, ossia la retta su cui giace il segmento
3) La lunghezza del segmento
4) Il verso









$\overrightarrow{OA} = A - O$

$\overrightarrow{OB} = B - O$

$\vec{u} = \overrightarrow{OA}$, $\vec{v} = \overrightarrow{OB}$ $\qquad \vec{u} + \vec{v}$

$O \equiv A$, $\vec{u} = \overrightarrow{OA} = \underset{\to}{0}$ $\qquad \vec{u} \quad \underset{\to}{u}$

O coincide con A : $O \equiv A$

$\vec{u}$

<u>Numeri reali</u> $\qquad \mathbb{R}$ $\qquad$ campo dei numeri reali

$\mathbb{N}_0 = \{0, 1, 2, 3, 4, \ldots, n, \ldots\}$ insieme numeri naturali

$\mathbb{N} = \{1, 2, 3, 4, \ldots, n, \ldots\}$

$\mathbb{Z}$ = insieme dei numeri interi relativi $= \{0, \pm 1, \pm 2, \pm 3, \ldots \pm n, \ldots\}$

$\mathbb{Q}$ = insieme dei numeri razionali = $\left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}$

$a \in \mathbb{Z}$: $a = \frac{a}{1}$

$a, b \neq 0$

$\frac{a}{b} \cdot \frac{b}{a} = 1$

$\mathbb{R}$ campo dei numeri reali $= \overline{\mathbb{Q}} \cup (\mathbb{R} \setminus \mathbb{Q})$

unione

numeri irrazionali

$\sqrt{2}, \sqrt{3}, \ldots$

$\cup$ = unione

$\cap$ = intersezione

$+ \quad \cdot$

$0 \quad 1$

e = numero di Nepero

$\mathbb{R}$ $\quad a \in \mathbb{R} \quad a + (-a) = 0$

$a \in \mathbb{R}, a \neq 0 \quad a \cdot \frac{1}{a} = 1$

$\leq$ (minore o uguale)

$<$ (minore stretto) $\quad a \quad b$

$\setminus$ differenza insiemistica $\quad S \setminus T$

$\mathbb{R} \setminus \{0\}$

Scalari : numeri reali $\mathbb{R}$

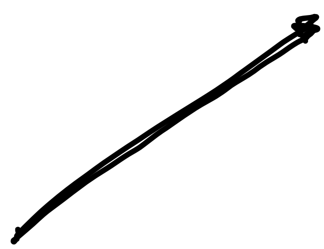
$\lambda \vec{u}$ $\quad \lambda \in \mathbb{R}$

λ = lambda

$\in$ = simbolo di appartenenza

$a \in S$

$a \notin S$



$\lambda \vec{u}$ un altro vettore, che ha la stessa direzione di $\vec{u}$, lunghezza uguale a $\underbrace{|\lambda|}_{\text{valore assoluto}} \|\vec{u}\|$

$\|\vec{u}\|$ = lunghezza di $\vec{u}$

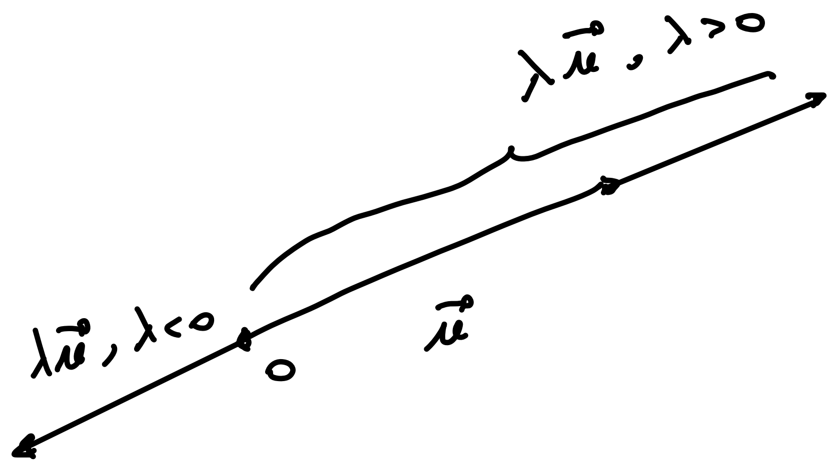
$$\|\lambda \vec{u}\| = |\lambda| \, \|\vec{u}\|$$

$$|a| = \text{valore assoluto di } a = \begin{cases} a & \text{se } a \geq 0 \\ -a & \text{se } a \leq 0 \end{cases}$$

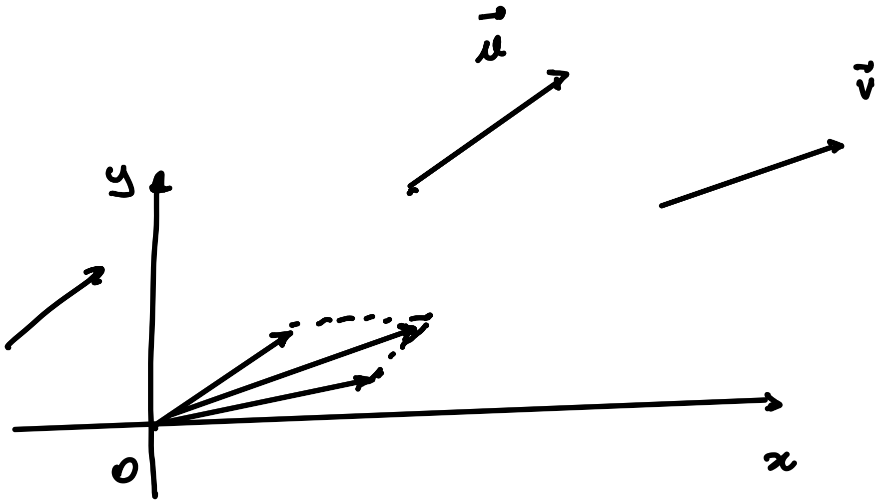
$|4| = 4, \quad |-e| = e$

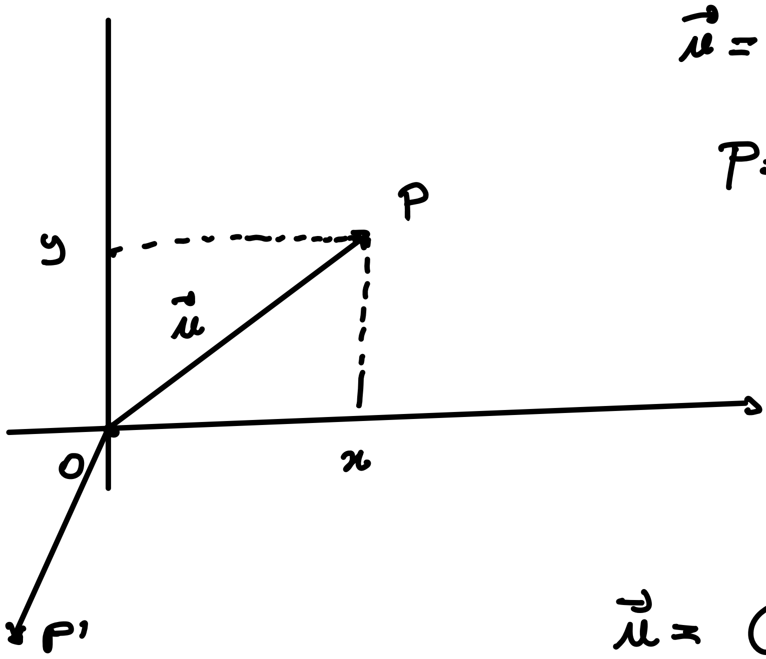
se $\lambda > 0$ (λ positivo), $\lambda \vec{u}$ ha lo stesso verso di $\vec{u}$

se $\lambda < 0$, $\lambda \vec{u}$ ha verso contrario a $\vec{u}$



<u>**Def**</u> Due vettori $\vec{u}$ e $\vec{v}$ si dicono equipollenti se hanno stessa direzione, stesso verso e stessa lunghezza
(se giacciono sulla stessa retta o sono paralleli)

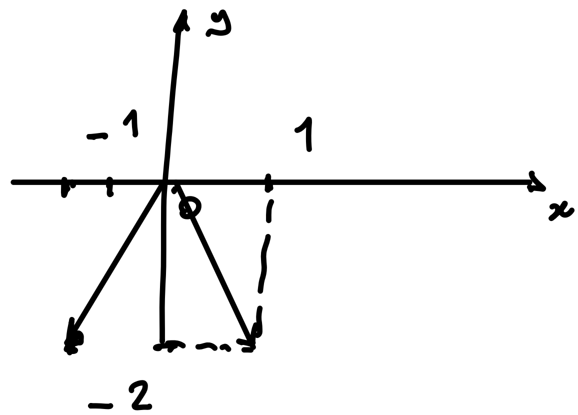




$\vec{u} = \vec{OP}$

$P = (x, y)$

$\vec{u} = (1, -2)$

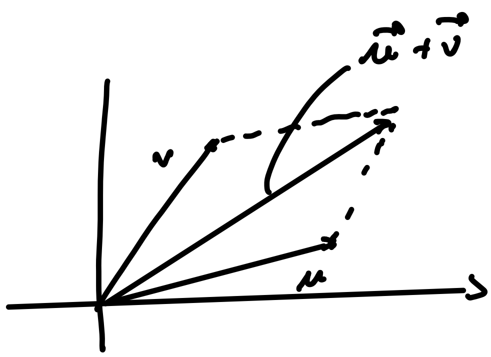


$u_1 = (-1,-2)$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{ (x,y) : x \in \mathbb{R},\ y \in \mathbb{R} \}$$

= Piano cartesiano

$\vec{u} = (x_u, y_u)$, $\vec{v} = (x_v, y_v)$



$$\vec{u} + \vec{v} = (x_u + x_v,\ y_u + y_v)$$

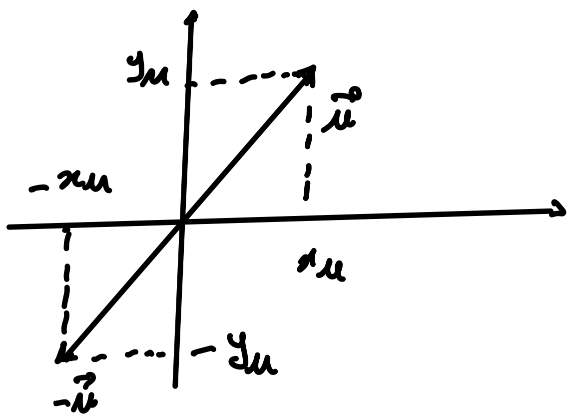
$\vec{u} = (-1, -4)$, $\vec{v} = (5, -3)$

$$\vec{u} + \vec{v} = (-1+5, -4-3) = (4, -7)$$

$\vec{u} = \left(\frac{1}{2}, -\frac{1}{3}\right) \qquad \vec{v} = (-1, 2)$

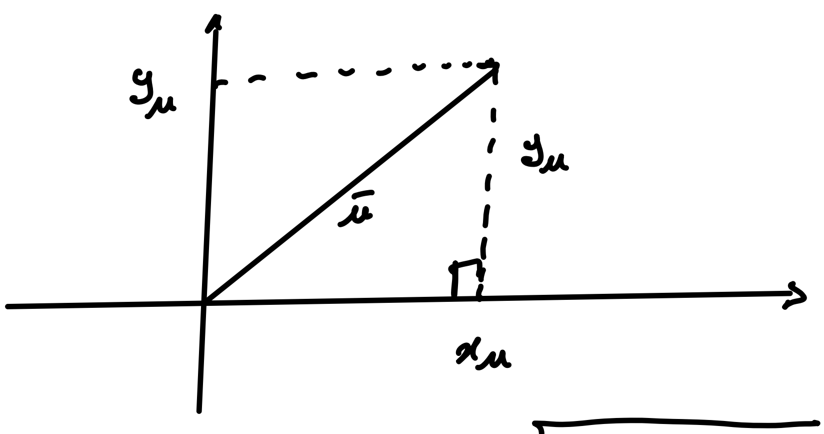
$\vec{u} + \vec{v} = \left(\frac{1}{2} - 1, -\frac{1}{3} + 2\right) = \left(-\frac{1}{2}, \frac{5}{3}\right)$

$\vec{u} = (x_u, y_u) \qquad -\vec{u} = (-x_u, -y_u)$

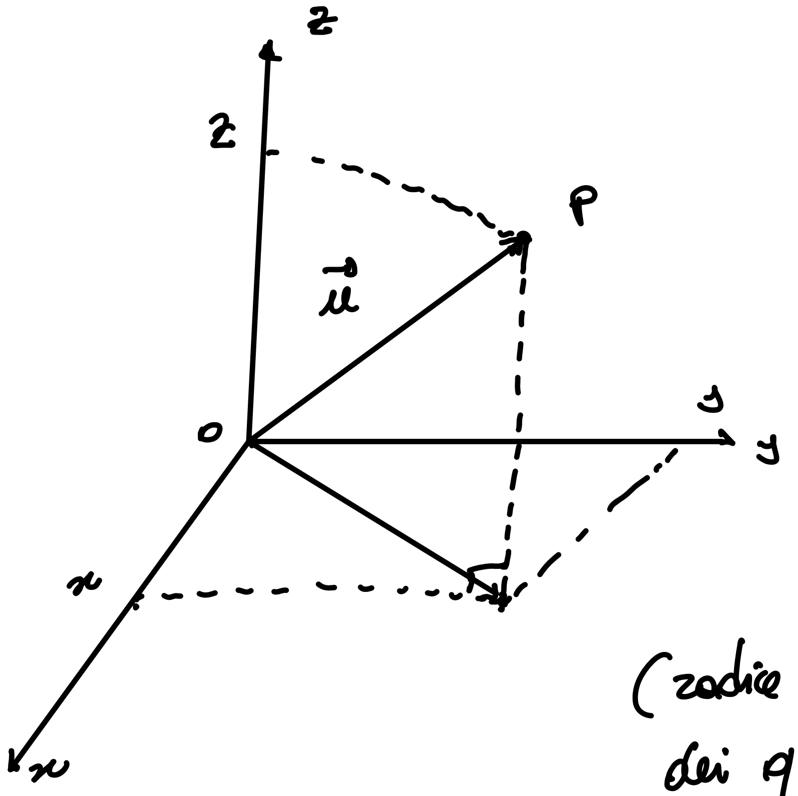


norma

$\|\vec{u}\|$ = modulo, intensità di $\vec{u}$ $= \sqrt{x_u^2 + y_u^2}$



$\vec{u} = (-3, 4) \qquad \|\vec{u}\| = \sqrt{9 + 16} = \sqrt{25} = 5$

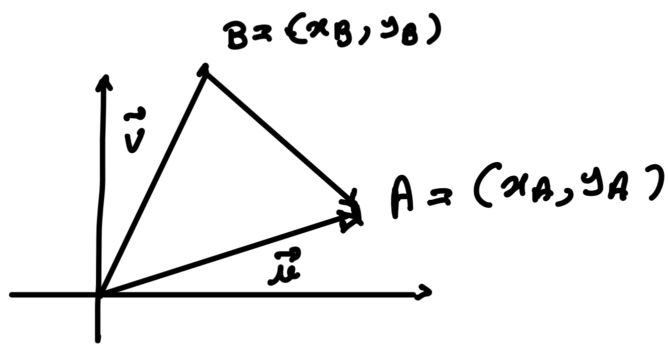


(radice della somma dei quadrati delle coordinate)

$$\|\vec{u}\| = \sqrt{x^2 + y^2 + z^2}$$

$\vec{u} = (-1, 0, 1)$ $\quad \|\vec{u}\| = \sqrt{1+1} = \sqrt{2}$

$\vec{u} = (x_A, y_A)$, $\vec{v} = (x_B, y_B)$



$$\vec{u}-\vec{v} = \vec{BA} = A-B = (x_A, y_A) + (-x_B, -y_B)$$
$$= (x_A - x_B, y_A - y_B)$$

<u>ES.</u> $A=(0,5)$, $B=(-1,3)$, $C=(-1,-2)$

$D=(0,2)$

Scrivere le componenti di $\vec{AB} = B-A$

" " $\vec{AC}$

$\vec{CD}$

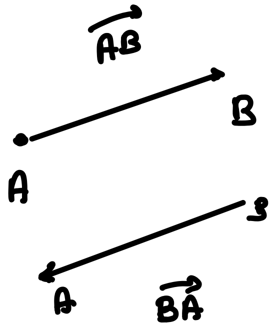
$\vec{AD}$

$\vec{BC}$

$\vec{BD}$

$$\vec{CD} = D-C = (0,2)-(-1,-2) = (1, 2+2)$$
$$= (1, 4)$$

$$\vec{BD} = D-B = (0,2)-(-1,3) = (1, 2-3) = (1,-1)$$
$$= (0,2)+(1,-3) = (1, 2-3) = (1,-1)$$



$\vec{AB} = B-A = (x_B - x_A, y_B - y_A)$

$\vec{BA} = A-B =$

$B=(x_B, y_B)$

$A=(x_A, y_A)$

$\vec{AC} = C-A$

$\overrightarrow{CA}$

$\overrightarrow{AC} = C - A = (-1,-2) - (0,5) = (-1, -2-5) = (-1, -7)$

$\lambda \vec{u} = (\lambda x_u, \lambda y_u) \qquad \lambda \in \mathbb{R}$

$\vec{u} = (x_u, y_u)$

$\vec{u} = \left(-2, \frac{1}{2}\right)$

$\lambda = \frac{1}{4}$

$\lambda \vec{u} = \frac{1}{4}\left(-2, \frac{1}{2}\right) = \left(-\frac{1}{2}, \frac{1}{8}\right)$

$\lambda = -5, \quad \vec{u} = \left(2, \frac{4}{3}\right)$

$\lambda \vec{u} = -5\left(2, \frac{4}{3}\right) = \left(-10, -\frac{20}{3}\right)$

$\|\lambda \vec{u}\| = \sqrt{(\lambda x_u)^2 + (\lambda y_u)^2} = \sqrt{\lambda^2 x_u^2 + \lambda^2 y_u^2}$

$= \sqrt{\lambda^2 (x_u^2 + y_u^2)}$

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$

$= \sqrt{\lambda^2} \cdot \sqrt{x_u^2 + y_u^2}$

$\sqrt{\lambda^2} = |\lambda| \quad \forall \lambda \in \mathbb{R}$

$= |\lambda| \, \|\vec{u}\|$

$\sqrt{(-3)^2} = 3 = |-3|$

$\forall$ = "per ogni"

$\vec{0} = (0,0)$

quantificatore universale

$\forall x \in S$

Nello spazio : $\underline{u} = (x_u, y_u, z_u)$

$\underline{v} = (x_v, y_v, z_v)$

$$\underline{u} + \underline{v} = (x_u + x_v, y_u + y_v, z_u + z_v)$$

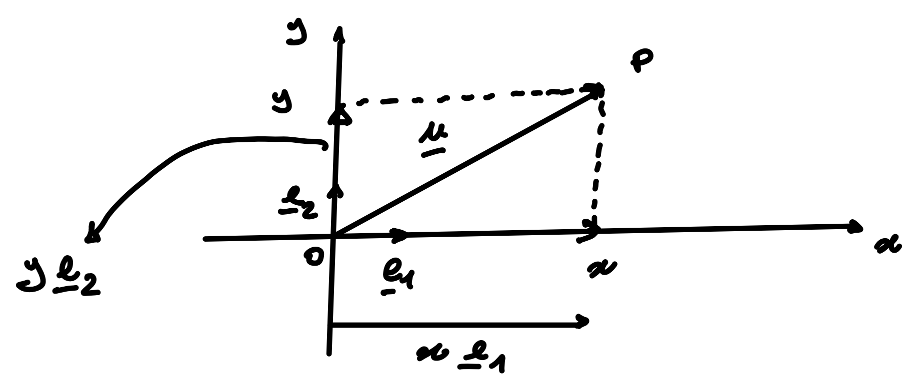
$$\underline{u} - \underline{v} = (x_u - x_v, y_u - y_v, z_u - z_v)$$

$$\lambda \in \mathbb{R} \quad \lambda \underline{u} = (\lambda x_u, \lambda y_u, \lambda z_u)$$

$\underline{u} = (1, 2, -\frac{1}{4})$, $\lambda = \frac{\sqrt{2}}{2}$

$$\lambda \underline{u} = \frac{\sqrt{2}}{2} (1, 2, -\frac{1}{4}) = (\frac{\sqrt{2}}{2}, \sqrt{2}, -\frac{\sqrt{2}}{8})$$

$\frac{\sqrt{2}}{\cancel{2}} \cdot \cancel{2}$



$\underline{e}_1 = (1,0)$

$\underline{e}_2 = (0,1)$

"versori" degli assi x e y

VERSORE : vettore $\underline{u}$ tale che $\|\underline{u}\| = 1$

$\underline{u} = x\,\underline{e}_1 + y\,\underline{e}_2 \qquad x, y \in \mathbb{R}$

$$\underline{u} = (x,y) = (x,0) + (0,y) = x\overbrace{(1,0)}^{\underline{e}_1} + y\overbrace{(0,1)}^{\underline{e}_2}$$
$$= x\,\underline{e}_1 + y\,\underline{e}_2$$

= combinazione lineare di $\underline{e}_1$ ed $\underline{e}_2$

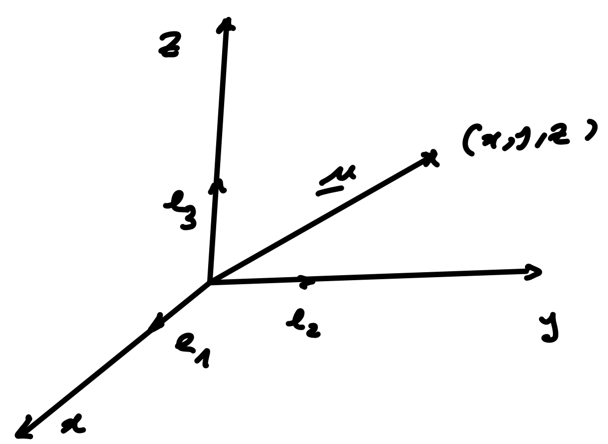
coefficienti " " sono x ed y e si chiamano **componenti** di $\underline{u}$ rispetto ad $\underline{e}_1, \underline{e}_2$.

Nello spazio $\quad \mathbb{R}^3 = \{(x,y,z) : x,y,z \in \mathbb{R}\}$

$\underline{e}_1 = (1,0,0)$ vers. asse x

$\underline{e}_2 = (0,1,0) \quad$ " " y

$\underline{e}_3 = (0,0,1) \quad$ " " z



$$\underline{u} = x\,\underline{e}_1 + y\,\underline{e}_2 + z\,\underline{e}_3$$