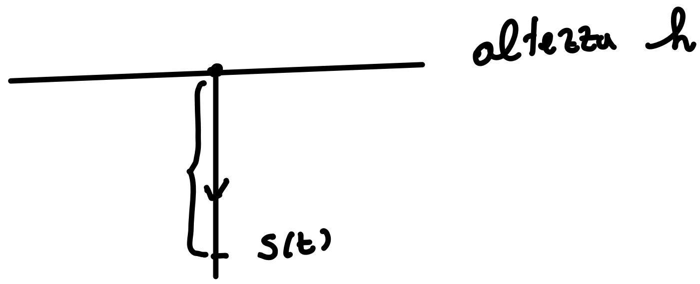
Lezione del 28/10/2022

Concetto di funzione

1) h



$$s(t) = \frac{1}{2} g t^2 \qquad t > 0 \longrightarrow s(t)$$

2) Volume di una sfera di raggio $r > 0$

$$V = \frac{4}{3} \pi r^3$$

3) i $\qquad K(i) = \left(1 + \frac{i}{12}\right)^{12} \qquad 0 \le i \le 1$

Def. A, B insiemi qualsiasi non vuoti $(A, B \neq \emptyset)$

Una funzione tra A e B è una corrispondenza (o una legge) f tra A e B che associa ad ogni elemento $x \in A$ uno ed un solo elemento $y \in B$. Se f è una funzione tra A

e B, utilizziamo la notazione

$$f : A \longrightarrow B$$
$$x \longmapsto y = f(x)$$

$x \in A$, l'unico corrispondente di x mediante f, si denota con $y = f(x)$ e si chiama "immagine di x mediante f".

$D_f = A$ = insieme di definizione di f, o dominio di f

B = " " arrivo di f

$$f(A) = \text{codominio di } f = \{ \underbrace{f(x)}_{\in B} : x \in A \} \subseteq B$$

Es.

1) $f(x) = c$, $c \in \mathbb{R}$, $\forall x \in \mathbb{R}$ funzione costante

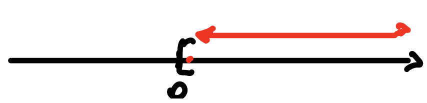
2) $f(x) = x+1$ $\forall x \in \mathbb{R}$. $D_f = \mathbb{R}$

$f(0) = 0+1 = 1$, $f(2) = 2+1 = 3$, ...

3) $f(x) = x^2$ $\forall x \in \mathbb{R}$, $D_f = \mathbb{R}$

$f(3) = 3^2 = 9$

4) $f(x) = \sqrt{x}$ $x \geq 0$, $D_f = \{x \in \mathbb{R} : x \geq 0\} = [0, +\infty[$



5) $$f(x)=\begin{cases} 0 & \text{se } x\in\mathbb{Z} \\ 1 & \text{altrimenti} \end{cases} \qquad D_f=\mathbb{R}$$

$$f(-4)=0\,,\qquad f(\pi)=1\,,\qquad f\left(\frac{m}{n}\right)=1$$

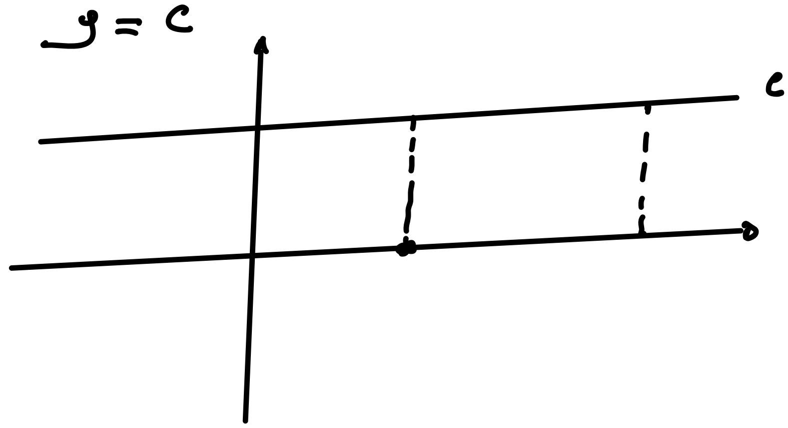
Def. $A,B\neq\emptyset$ $\underset{\text{v}}{A,B\subseteq\mathbb{R}}$ $f:A\to B$: si dice grafico di f, l'insieme

$$G_f=\{(x,f(x)) : x\in A\}\subseteq A\times B$$

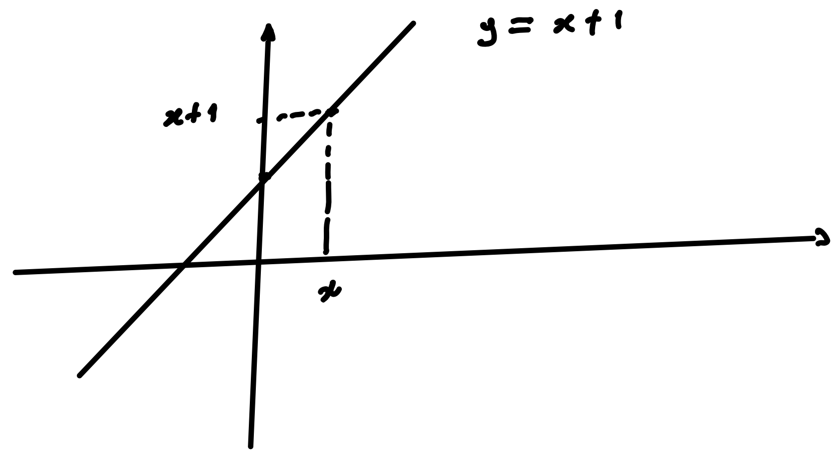
$$=\{(x,y) : x\in A,\ \underbrace{y}_{}=f(x)\}$$

$y=f(x)$ equazione del grafico di f

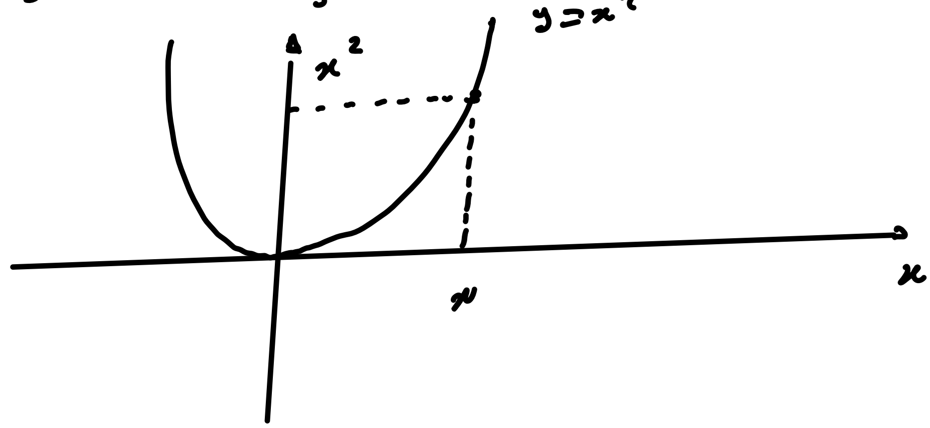
ES $f(x)=c$: $y=c$



$f(x)=x+1\,,\qquad y=x+1$

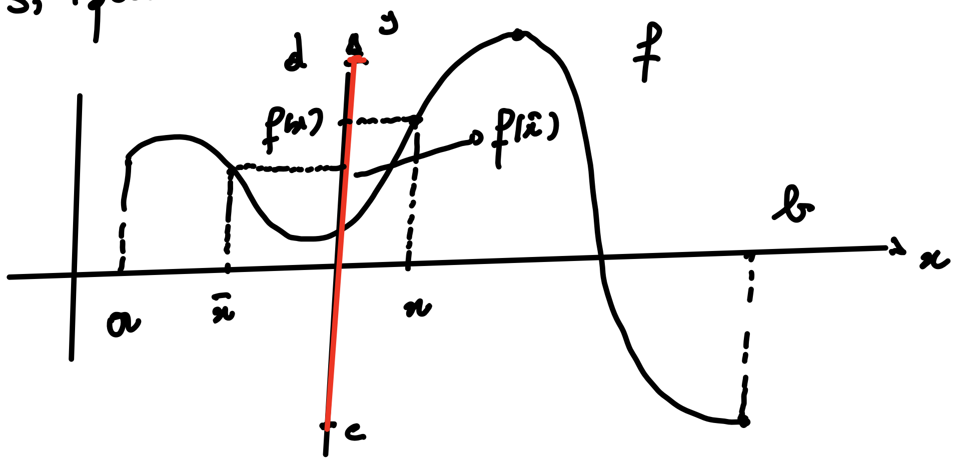


$f(x) = x^2$, $y = x^2$ parabola



$f(\mathbb{R}) =$ codominio di $f = [0, +\infty[$

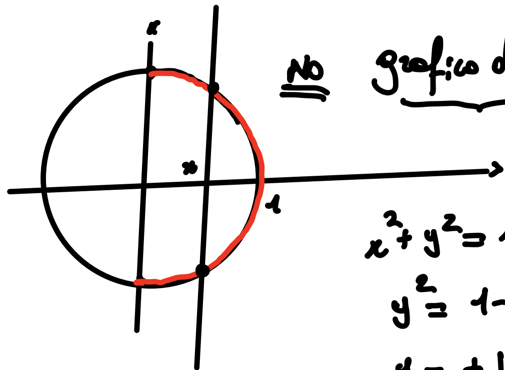
In generale, si può avere una situazione del tipo:



$[a,b] = \{ x \in \mathbb{R} : \ a \leq x \leq b \}$

$f : [a,b] \longrightarrow \mathbb{R}$

$f([a,b]) = [c,d]$

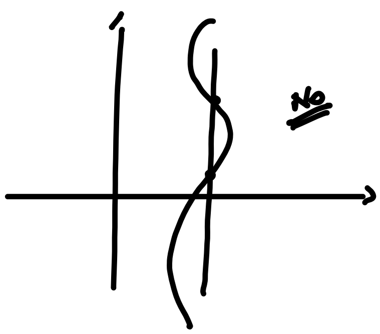


No grafico di una funzione

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm \sqrt{1-x^2}$



No

Funzioni limitate

$f : A \longrightarrow \mathbb{R} \quad , A \subseteq \mathbb{R}$

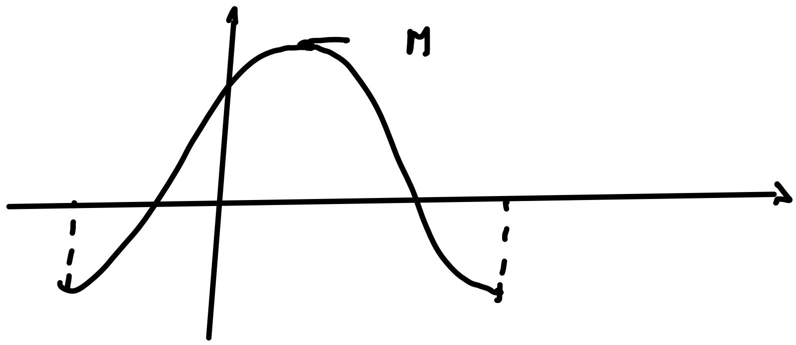
$A = D_f$ dominio di f

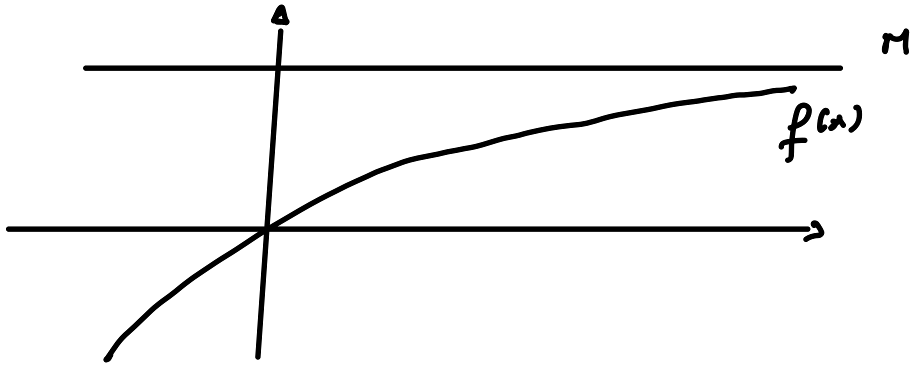
"funzioni reali di una variabile reale" $\qquad f = \underbrace{f(x)}_{\in \mathbb{R}} \quad x \in \mathbb{R}$

Def Si dice che f è limitata superiormente **se**

$\exists M \in \mathbb{R}$ $\underbrace{\text{t.c.}}$ $\quad f(x) \leq M, \ \forall x \in A$

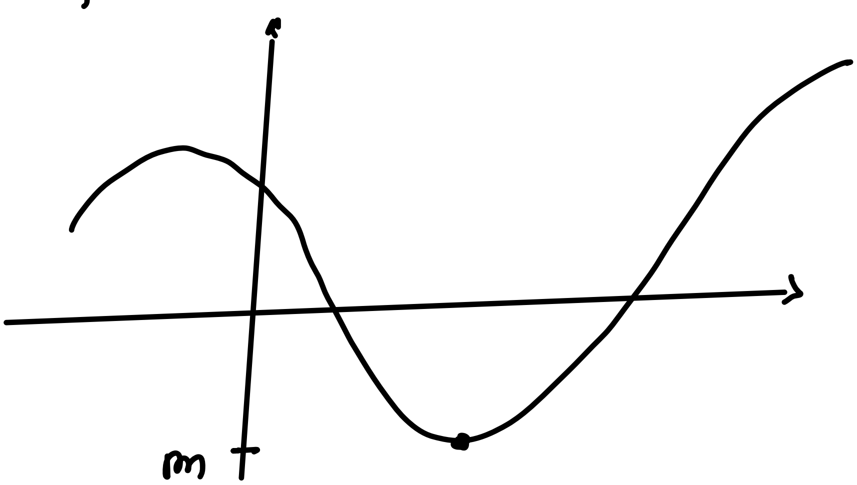
"tale che"

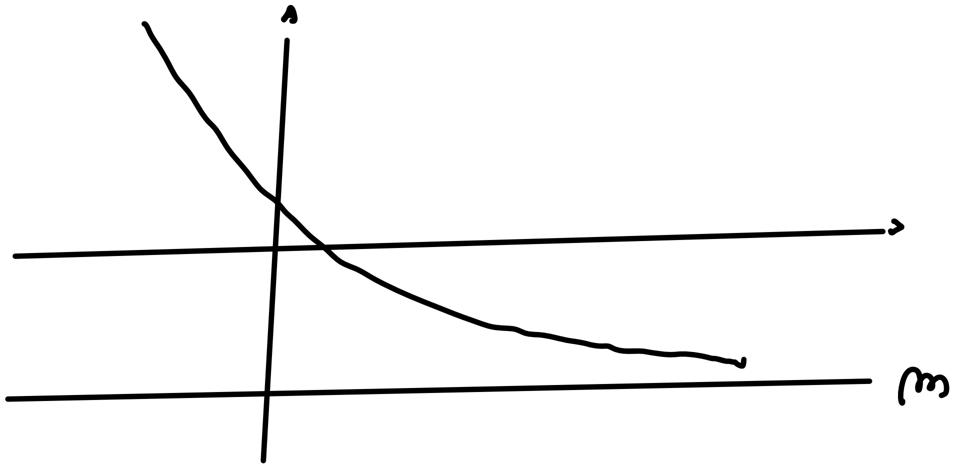




f si dice <u>limitata inferiormente</u> se $\exists\, m \in \mathbb{R}$

tale che $\quad f(x) \geq m \,, \quad \forall x \in A$





Se f è limitata superiormente $\Rightarrow$ il codominio di

f " " o superiormente $\Rightarrow$ " "

$f(A)$ è dotato di estremo

superiore

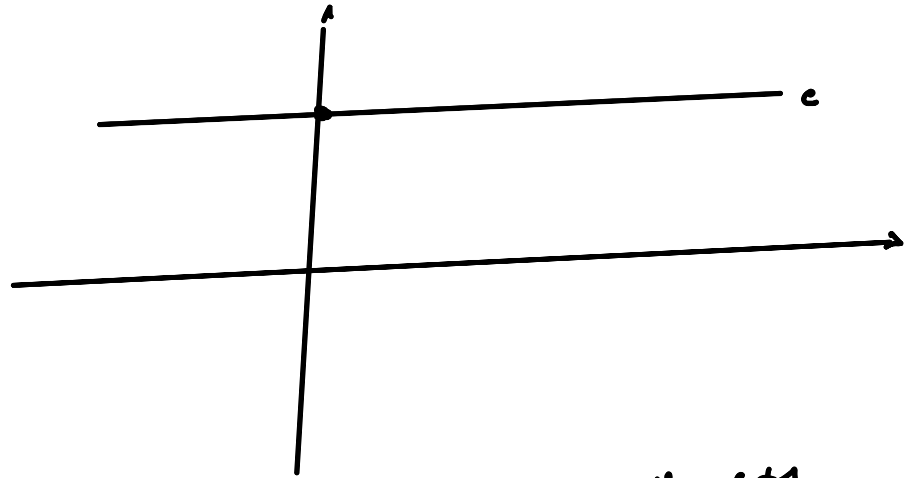
$$\sup f(A) = \text{"estremo superiore di } f\text{"}$$
$$= \sup_{x \in A} f(x)$$

Se f è limitata inferiormente $\Rightarrow$

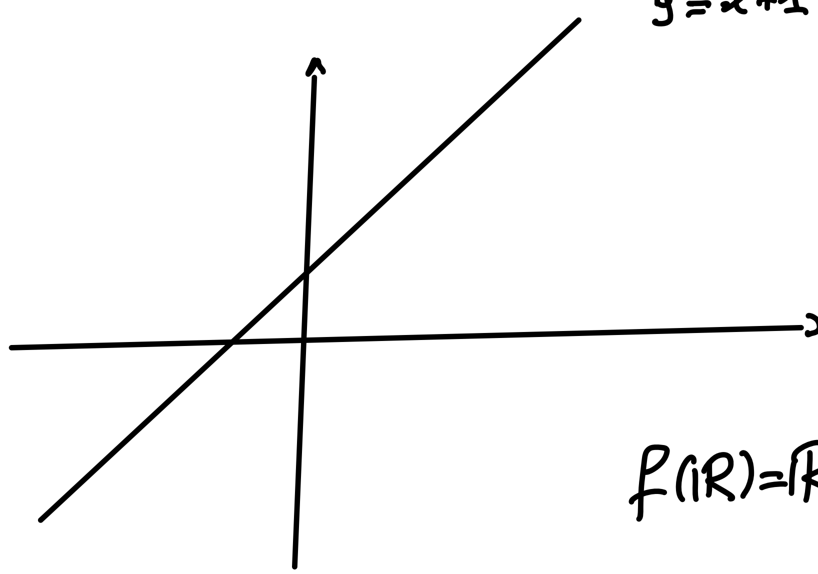
$$\exists \inf f(A) = \inf_{x \in A} f(x) = \text{"estremo}$$

inferiore di f"

Es. $f(x) = c$

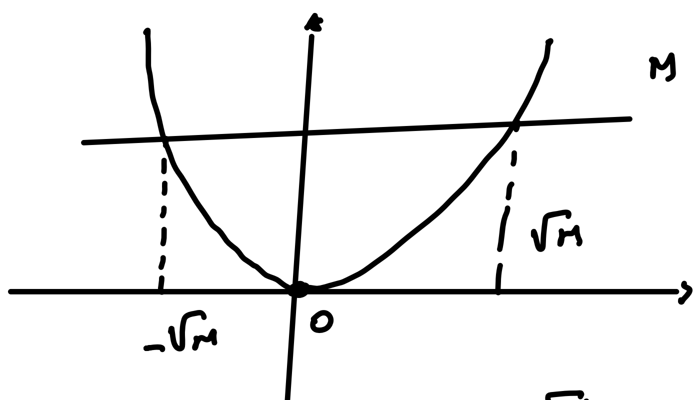


$f(x) = x + 1$



$f(\mathbb{R}) = \mathbb{R}$

$f(x) = x^2$

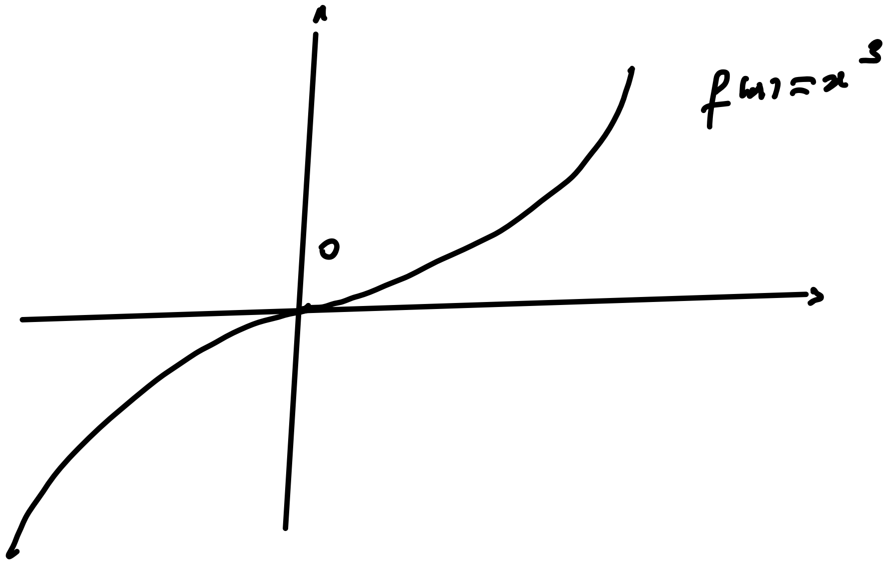


$\pm\sqrt{M}$

$M > 0 \; : \; f(x) > M \iff x^2 > M \iff x < -\sqrt{M},$
$x > \sqrt{M}$

$f(x) = x^3$

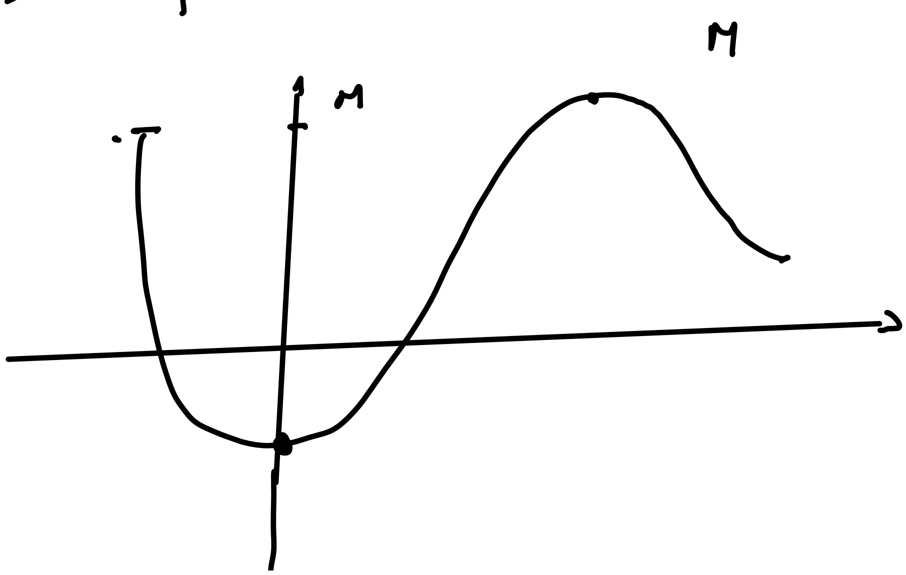
NĒ LIMITATA
SUPERIORMENTE
NĒ INFERIORMENTE

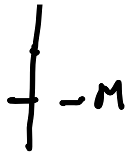


Def $f : A \to \mathbb{R}$ limitata se ē limitata inferiormente e superiormente $\Leftrightarrow$ $\exists\, M>0$ tale che

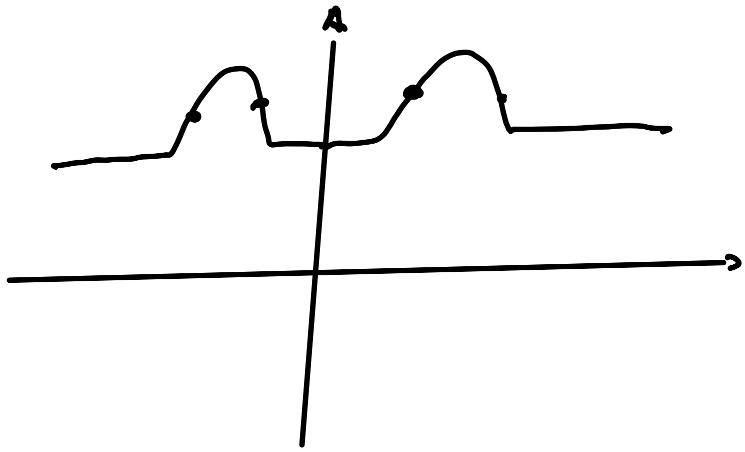
$$-M \le f(x) \le M \qquad (|a|\le b \Leftrightarrow -b \le a \le b)$$

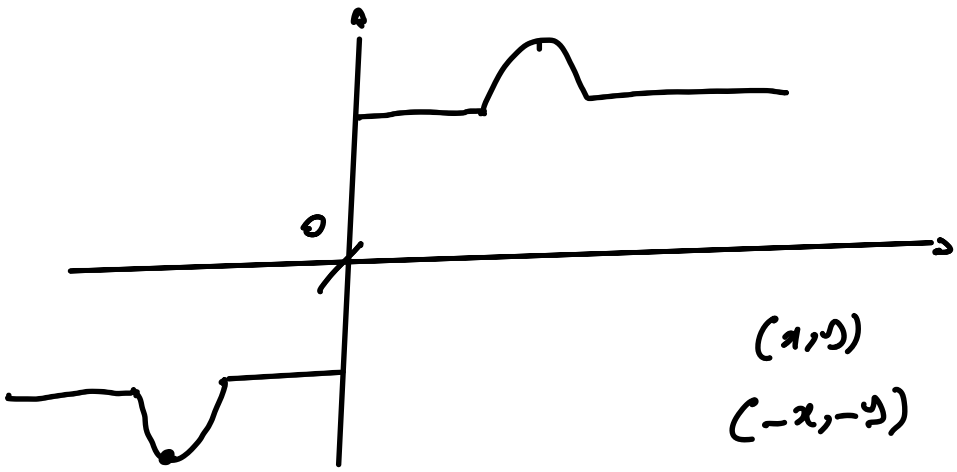
$$\Leftrightarrow \quad |f(x)| \le M \qquad f(x) \in [-M, M]$$





Funzioni simmetriche





(x,y)

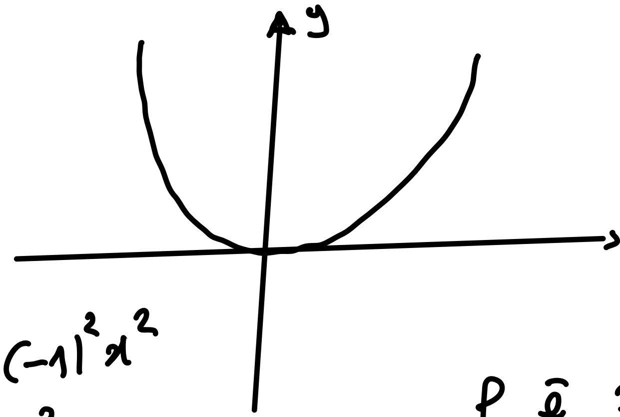
$(-x,-y)$

Def. $f=f(x)$ funzione definita in un intervallo $]-a,a[$

$$f:[-a,a]\to\mathbb{R}$$

Si dice che f è pari se: $f(-x)=f(x)$ $\forall x\in\,]-a,a[$

Es $f(x)=x^2$

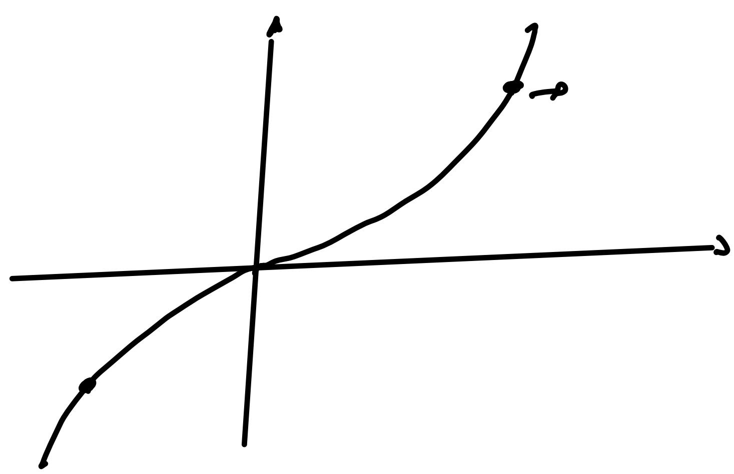


$f(-x) = (-x)^2 = (-1)^2 x^2$
$= x^2$
$= f(x)$

f è pari

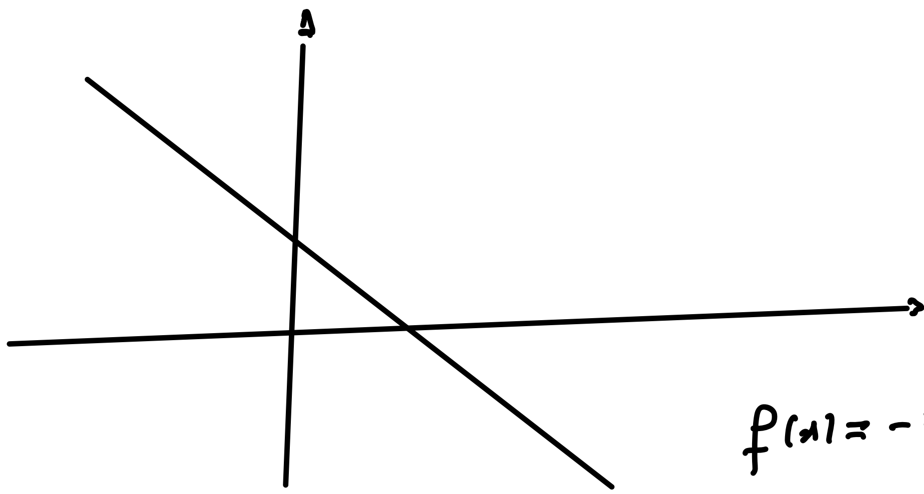
Def.
Si dice che f è dispari se $f(-x) = -f(x)$ $\forall x \in [-a, a]$

$f(x)=x^3$



$$f(-x) = (-x)^3 = (-1)^3 x^3 = -x^3 = -f(x)$$

<u>**Def.**</u> $f : A \longrightarrow B$, $A, B \neq \emptyset$: si dice che f è iniettiva se per ogni coppia $x, y \in A$ tali che $\underbrace{x \neq y}$ $\Longrightarrow$ $f(x) \underbrace{\neq} f(y)$ (*)



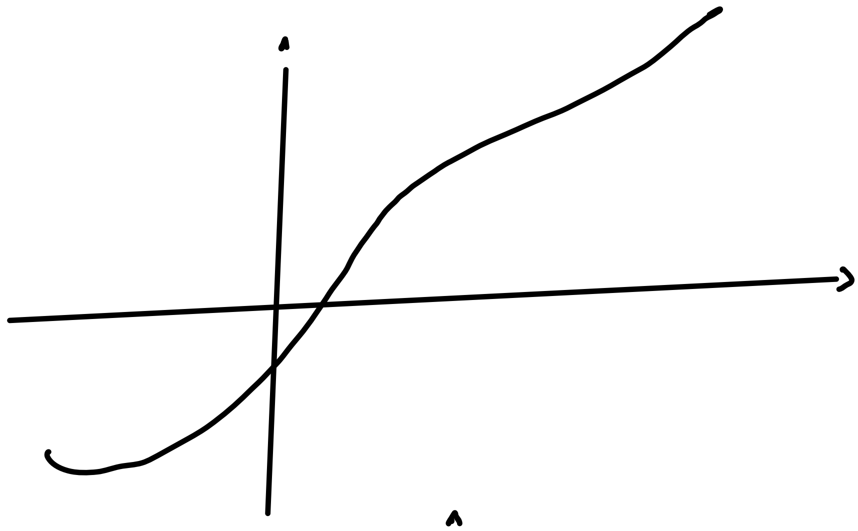
$f(x) = -x + 1$

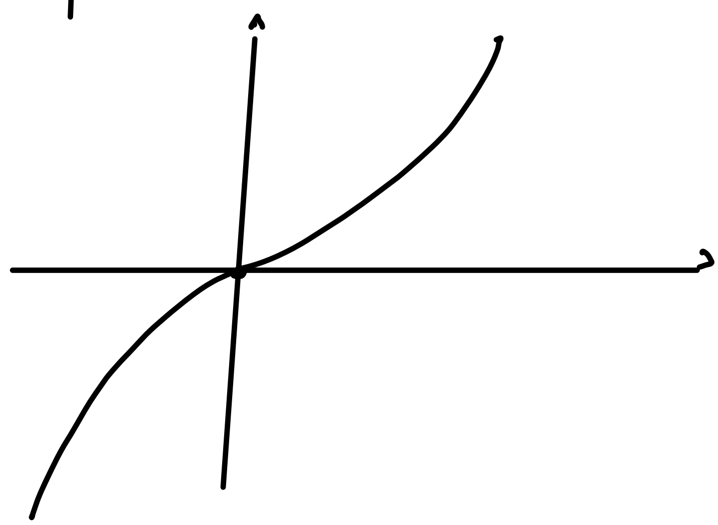
(*) <u>equivale</u> a dire che da $f(x) = f(y)$, <u>segue</u> che $x = y$

Nel caso $f(x) = -x + 1$, si ha che quando

$$f(x)=f(y) \Rightarrow -x+\cancel{1} = -y+\cancel{1}$$

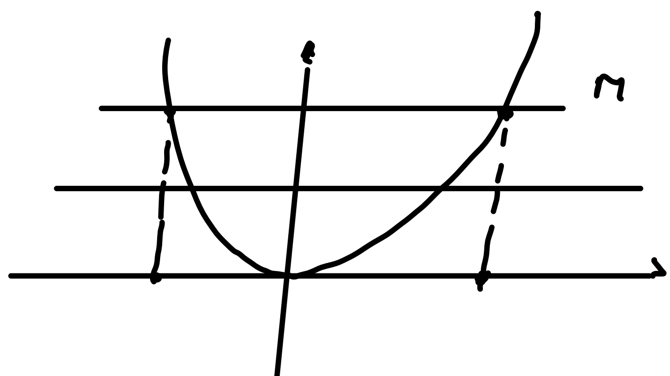
$$\Rightarrow \underline{x=y}$$





$f(x)=x^3$ è

<u>iniettiva</u>



$f(x)=x^2$ <u>No</u> iniettiva

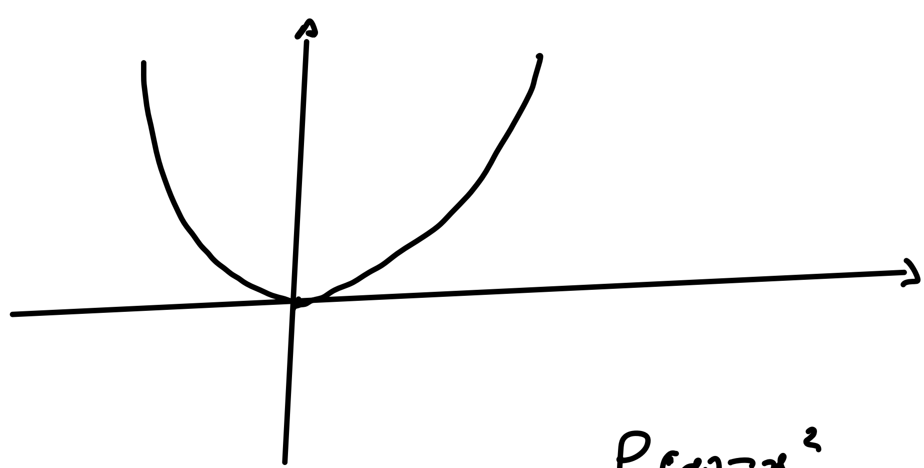
$$f(x)=M \iff x^2=M$$

$$\iff x=\underline{-\sqrt{M}}\ ,\ x=\underline{\sqrt{M}}$$

N.B. f è iniettiva se ogni retta parallela all'asse x interseca il grafico di f al più in un punto.

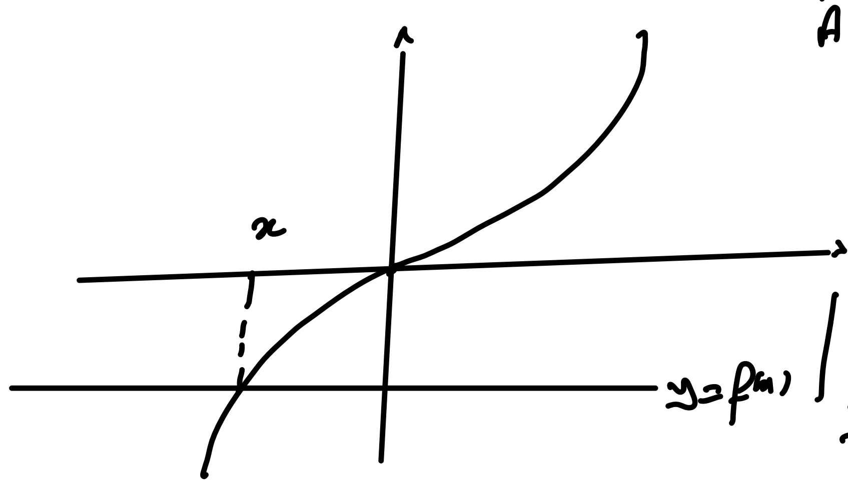
Def. $f: A \to B$ si dice suriettiva se

$$\forall y \in B \quad \exists\, x \in A \ : \ f(x) = y$$



$f(x) = x^2$

$f: \underbrace{\mathbb{R}}_{A} \to \underbrace{[0, +\infty[}_{B}$



$f(x) = x^3$

$f: \mathbb{R} \to \mathbb{R}$

$$f(-2) = -8$$

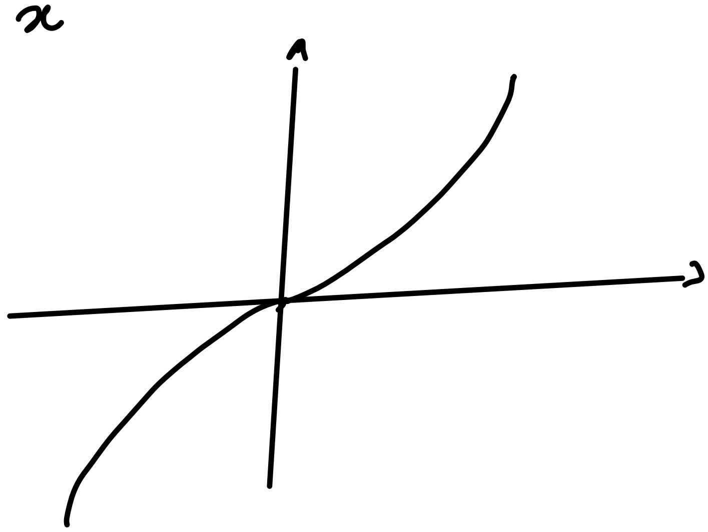
Def. $f: A \to B$ sia iniettiva che suriettiva si dice biettiva o corrispondenza biunivoca tra A e B: in particolare, f è biettiva

$$\iff \forall y \in B \ \underbrace{\exists!\, x \in A : f(x) = y}$$

Es.

$$f(x) = mx + n \qquad m \neq 0$$

$$f(x) = x^3$$



In generale,

$$f(x) = x^n$$

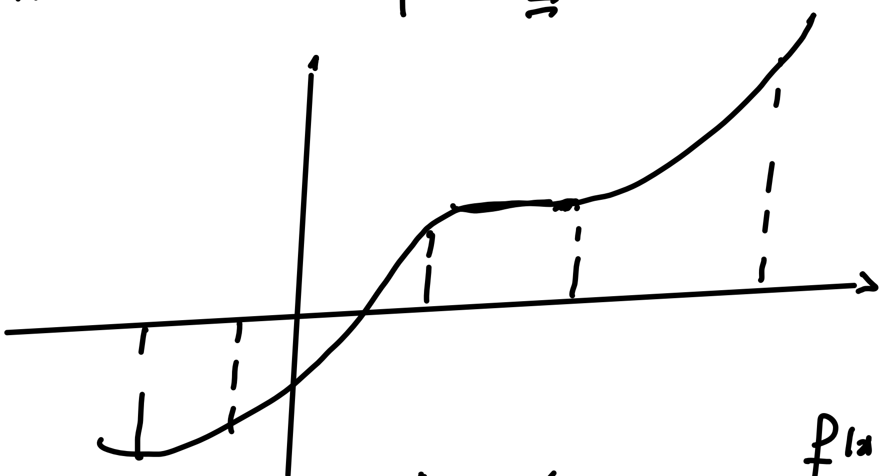
n dispari

Def. Funzioni monotone

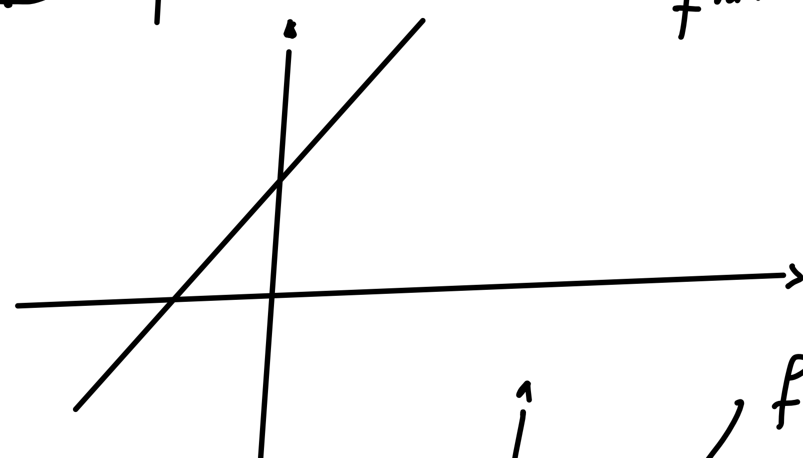
$$f : A \subseteq \mathbb{R} \longrightarrow \mathbb{R} \qquad f = f(x)$$

Si dice che f è monotona **crescente** se

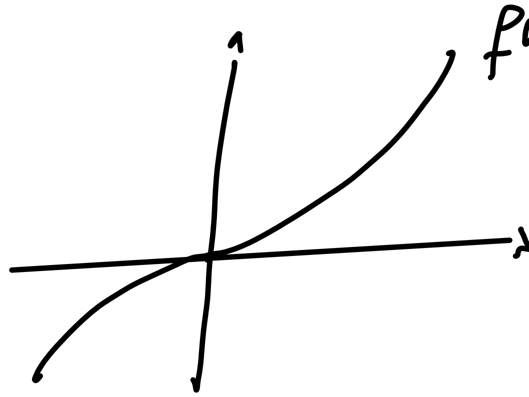
$$x < y \Longrightarrow f(x) \leq f(y) \qquad (*)$$



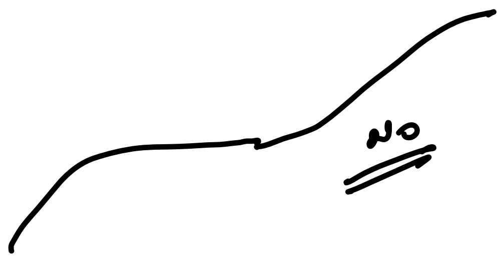
$f(x) = x+1$



$f(x) = x^3$

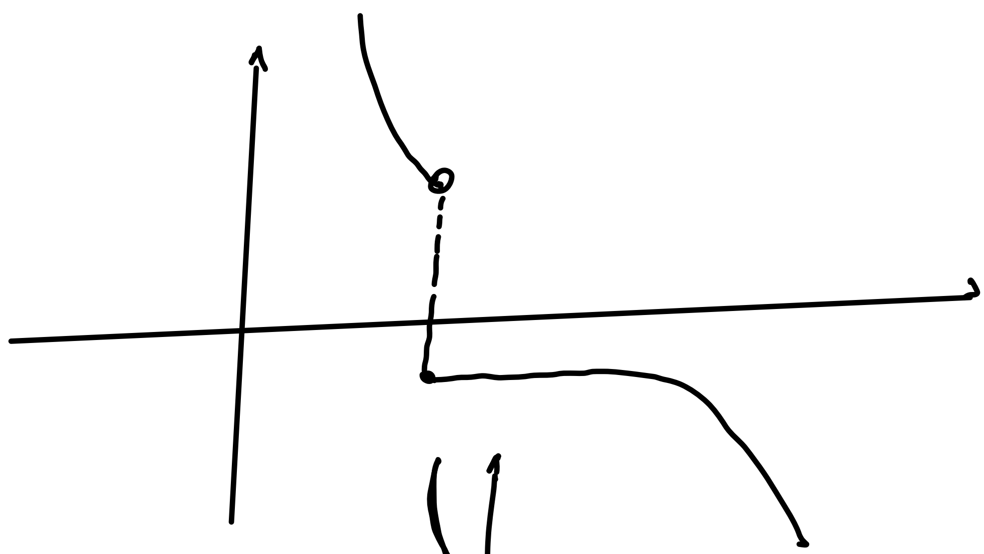


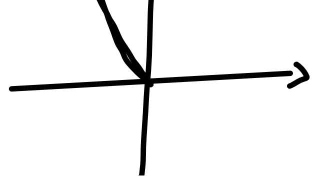
Se (*) vi è disuguaglianza stretta "$<$", si dice che f è <u>strettamente crescente</u>



f si dice monotona decrescente se

$$x<y \implies f(x) \geq f(y) \qquad (*)$$





Se in $(*)$ si sostituisce "$\geq$" a "$>$" la funzione si dice <u>strettamente</u> decrescente.