Riferimenti bibliografici

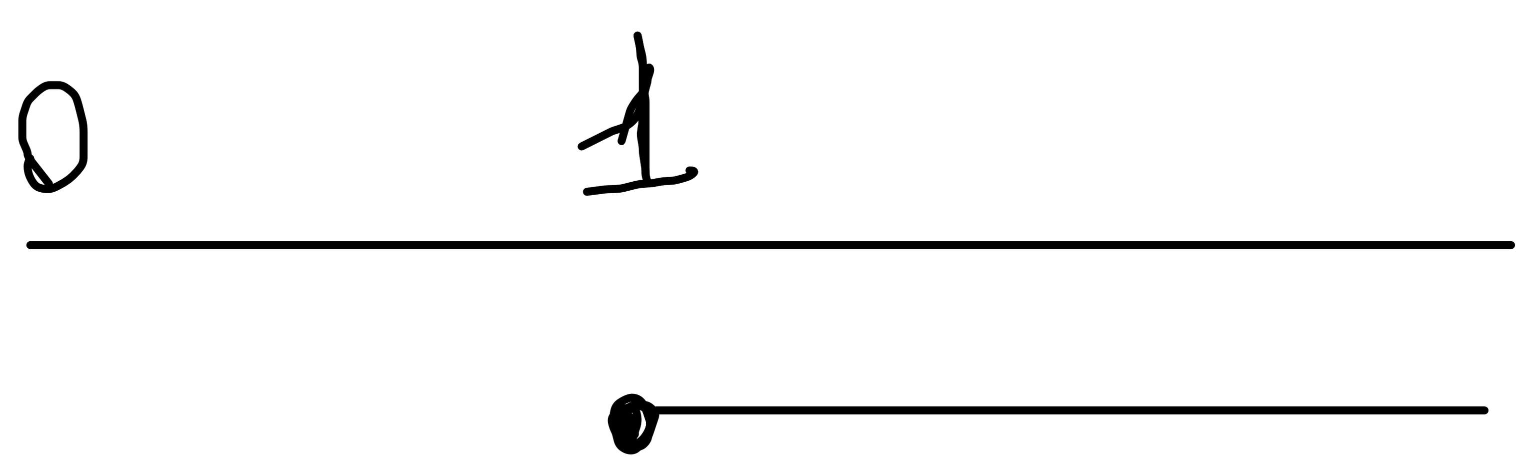
Marcellini - Sbordone Esercitazioni di Matematica

Vol I part. 2 paragrafo 2F

$$f(x) = x^2 \log x \qquad C.E. \quad x>0 \qquad]0;+\infty[$$

Positività $\qquad x^2 \log x \geq 0$

$$\begin{cases} x^2 \geq 0 & \forall x \in]0;+\infty[\\ \log x \geq 0 \end{cases} \qquad \begin{cases} \forall x \in]0;+\infty[\\ e^{\log x} \geq e^0 \end{cases} \qquad \begin{cases} \forall x \in]0;+\infty[\\ x \geq 1 \end{cases}$$



$$f(x) > 0 \iff x > 1$$

LIMITI AGLI ESTREMI

$$\lim_{x \to +\infty} x^2 \log x = +\infty$$

$$\lim_{x \to 0^+} x^2 \log x = \lim_{x \to 0^+} \frac{\log x}{\frac{1}{x^2}} \overset{H}{=} \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{2}{x^3}} =$$

$$= \lim_{x \to 0^+} \frac{1}{-2\cancel{x}} \cdot \cancel{x^3}^{\,2} = 0$$

$$\lim_{x \to +\infty} \frac{x^2 \log x}{x} = \lim_{x \to +\infty} x \log x = +\infty$$ no asint. obl.

CRESCENZA

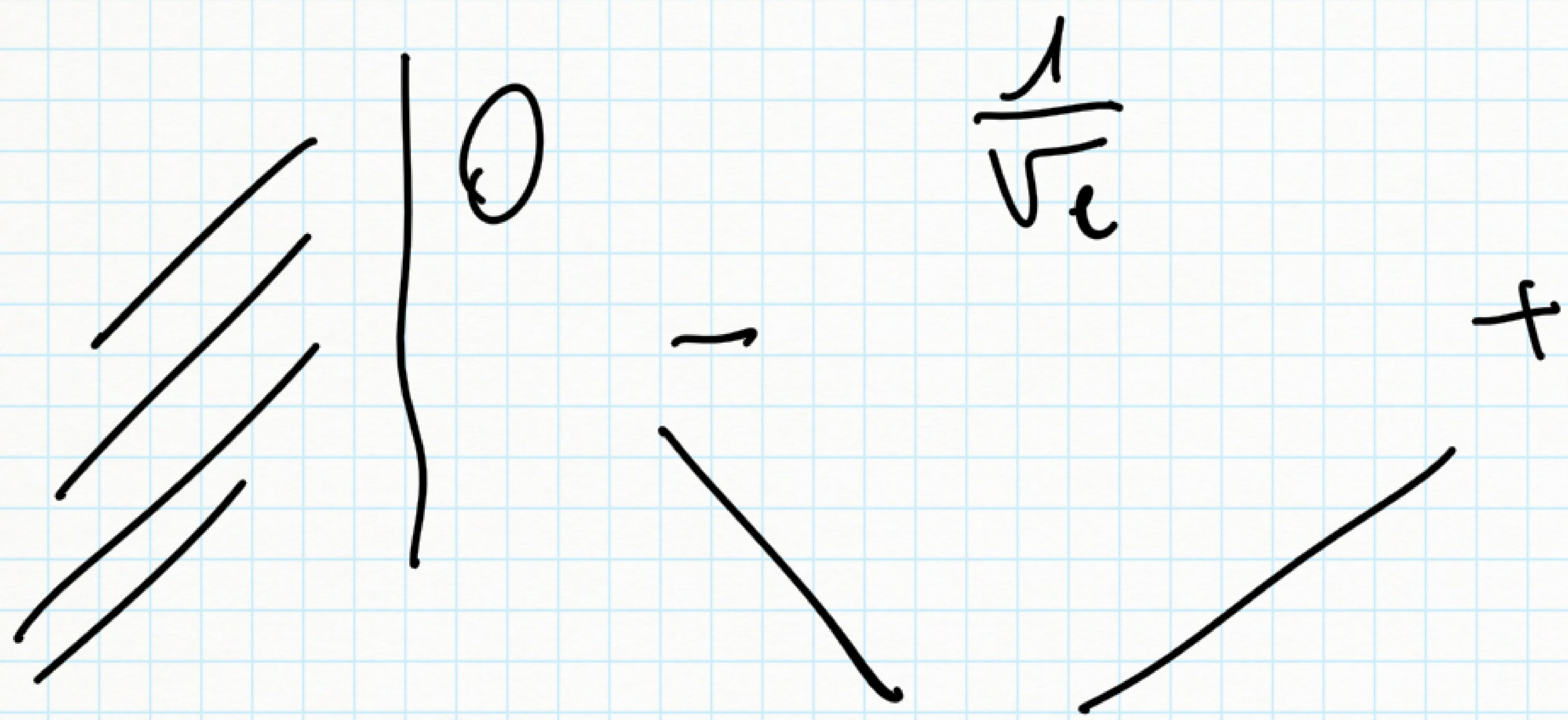
$$f(x) = x^2 \log x$$

$$f'(x) = 2x\log x + x^{\cancel{2}} \cdot \frac{1}{\cancel{x}} = 2x \cdot \log x + x =$$

$$= x(2\log x + 1)$$

$$\left\| \begin{array}{l} 2\log x + 1 \geq 0 \\ x \geq 0 \end{array} \right. \quad \left\| \begin{array}{l} 2\log x \geq -1 \\ x \geq 0 \quad \forall x \in]0;+\infty[\end{array} \right. \quad \left\| \begin{array}{l} \log x \geq -\frac{1}{2} \\ \forall x \in]0;+\infty[\end{array} \right.$$

$$\left\| \begin{array}{l} e^{\log x} \geq e^{-\frac{1}{2}} \\ \forall x \in]0;+\infty[\end{array} \right. \quad \left\| \begin{array}{l} x \geq \frac{1}{\sqrt{e}} \\ \forall x \in]0;+\infty[\end{array} \right.$$



f'

f

$\frac{1}{\sqrt{e}}$ punto di minimo assoluto.

$$f\left(e^{-\frac{1}{2}}\right) = \left(e^{-\frac{1}{2}}\right)^2 \cdot \log\left(e^{-\frac{1}{2}}\right) =$$

$$= -\frac{1}{2} \cdot e^{-1} = -\frac{1}{2e}$$

minimo assoluto

CONCAVITÀ

$$f'(x) = 2x\log x + x$$

$$f''(x) = 2\left(\log x + x \cdot \frac{1}{x}\right) + 1 = 2\log x + 2 + 1 = 2\log x + 3$$

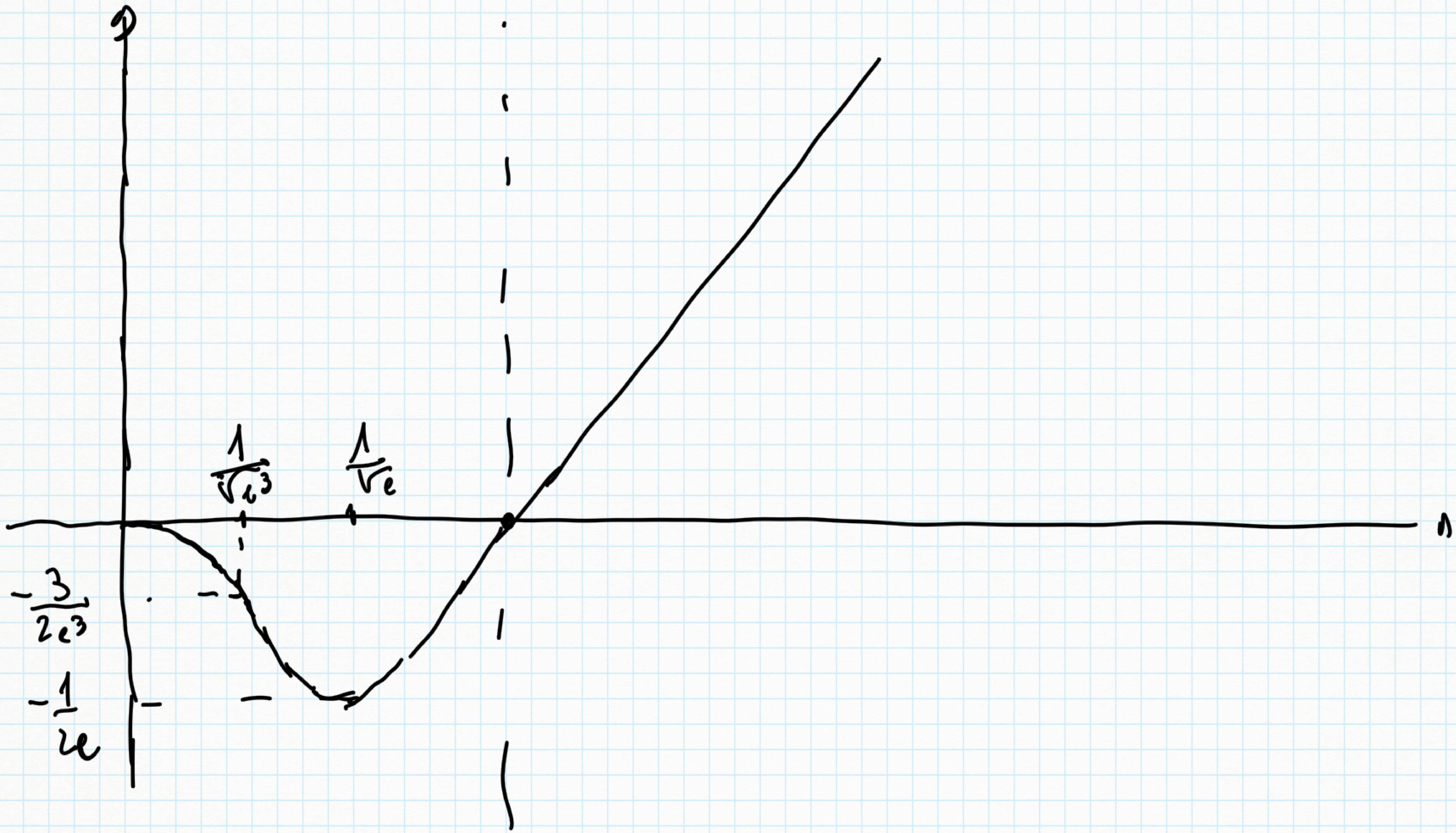
$$2\log x + 3 \geq 0 \qquad 2\log x \geq -3 \qquad \log x \geq -\frac{3}{2}$$

$$e^{\log x} \geq e^{-\frac{3}{2}} \qquad x \geq e^{-\frac{3}{2}} = \frac{1}{\sqrt{e^3}}$$

	0		$\frac{1}{\sqrt{e^3}}$	
f''		$-$		$+$
		$\cap$		$\cup$

Sottolineo che $\frac{1}{\sqrt{e^3}} \leq \frac{1}{\sqrt{e}}$

$$f\left(\frac{1}{\sqrt{e^3}}\right) = e^{-3} \cdot \left(-\frac{3}{2}\right) = -\frac{3}{2e^3}$$



$\frac{1}{\sqrt{e^3}}$
$\frac{1}{\sqrt{e}}$
$-\frac{3}{2e^3}$
$-\frac{1}{2e}$