Riferimento bibliografico: Marcellini-Sbordone Esercitazioni di Matematica volume 1 parte 2 paragrafo 2G

$$f(x) = e^{\frac{x+1}{2x-2}}$$

C.E. $2x-2 \neq 0 \quad 2x \neq 2$

$]-\infty; 1[\cup]1; +\infty[$

Positività

$e^{\frac{x+1}{2x-2}} > 0 \quad \forall x \in C.E.$

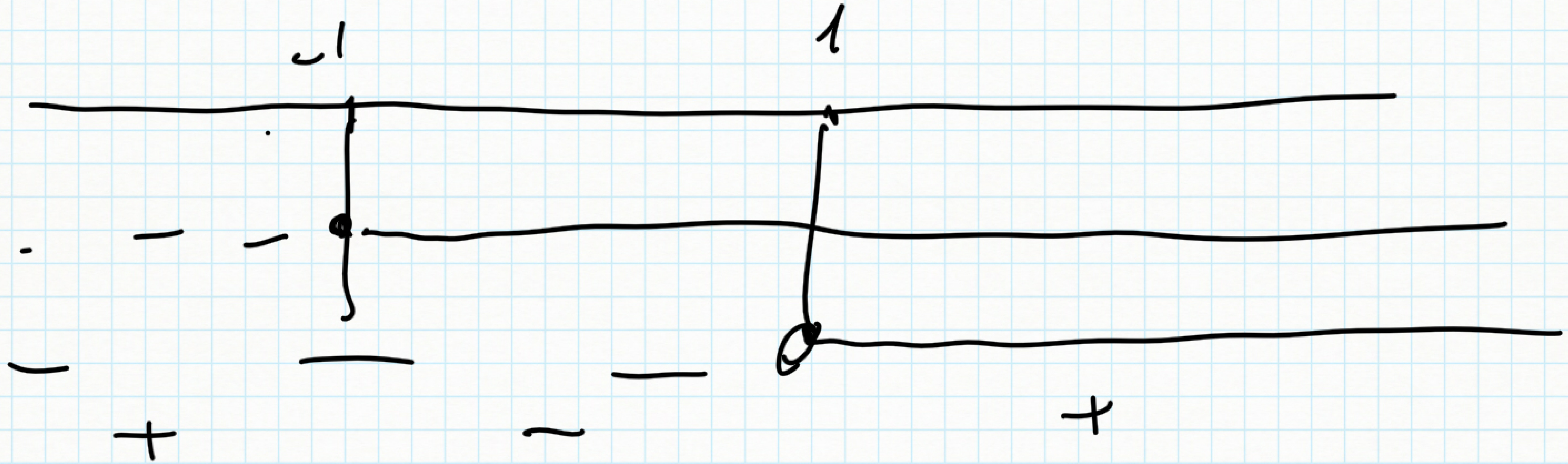
Poiché

$$\lim_{x \to c} e^{\frac{x+1}{2x-2}} = e^{\lim_{x \to c} \frac{x+1}{2x-2}}$$

prima di fare si limiti agli estremi studiamo l'esponente

$$g(x) = \frac{x+1}{2x-2}$$

$$\begin{cases} x+1 \geq 0 \\ 2x-2 > 0 \end{cases} \qquad \begin{cases} x \geq -1 \\ 2x > 2 \end{cases} \qquad \begin{cases} x \geq -1 \\ x > 1 \end{cases}$$



$$\lim_{x \to -\infty} e^{\frac{x+1}{2x-2}} = e^{\frac{1}{2}} = \sqrt{e}$$ $y = \sqrt{e}$ AS. OR. ~~S~~x

$$\lim_{x \to +\infty} e^{\frac{x+1}{2x-2}} = e^{\frac{1}{2}} = \sqrt{e}$$ $y = \sqrt{e}$ AS. OR. Dx

$$\lim_{x \to 1^-} e^{\frac{x+1}{2x-2}} = e^{\lim_{x \to 1^-} \frac{x+1}{2x-2}} = 0$$

$$\lim_{x \to 1^+} e^{\frac{x+1}{2x-2}} = e^{\lim_{x \to 1^+} \frac{x+1}{2x-2}} = +\infty$$ $x = 1$ AS. VER.

MONOTONIA

$$f(x) = e^{\frac{x+1}{2x-2}}$$

$$f'(x) = e^{\frac{x+1}{2x-2}} \left(\frac{2x-2-(x+1)(2)}{(2x-2)^2} \right) =$$

$$= e^{\frac{x+1}{2x-2}} \left(\frac{\cancel{2x}-2-\cancel{2x}-2}{(2x-2)^2} \right) = \frac{-4\,e^{\frac{x+1}{2x-2}}}{(2x-2)^2}$$

$$f'(x) < 0 \qquad \forall\, x \in \left]-\infty; 1\right[\cup \left]1; +\infty\right[$$

CONVESSITÀ

$$f'(x) = -4 \cdot \frac{e^{\frac{x+1}{2x-2}}}{(2x-2)^2} = \frac{-\cancel{4}}{\cancel{4}} \cdot \frac{e^{\frac{x+1}{2x-2}}}{(x-1)^2} = -\frac{e^{\frac{x+1}{2x-2}}}{(x-1)^2}$$

$$f'' = -\frac{\cancel{-\frac{e^{\frac{x+1}{2x-2}}}{(x-1)^2} \cdot (x-1)^2} - e^{\frac{x+1}{2x-2}} \cdot 2(x-1)}{(x-1)^4} =$$

$$= \frac{e^{\frac{x+1}{2x-2}} + 2e^{\frac{x+1}{2x-2}} \cdot (x-1)}{(x-1)^4} = \frac{e^{\frac{x+1}{2x-2}}}{(x-1)^4}\left[1+2x-2\right] =$$

$$f''(x) = \frac{e^{\frac{x+1}{2x-2}}}{(x-1)^4}(2x-1)$$

$2x-1 \geq 0 \qquad x \geq \frac{1}{2}$

$$\begin{array}{cccc} - & \frac{1}{2} & + & f'' \\ \frown & & \smile & \end{array}$$

$$f\left(\frac{1}{2}\right) = e^{\frac{\frac{3}{2}}{-1}} = e^{-\frac{3}{2}}$$


$\sqrt{e}$
$\frac{1}{\sqrt{e}}$
$\frac{1}{\sqrt{e^3}}$
$x=\frac{1}{2}$
$x=1$