RIFERIMENTO BIBLIOGRAFICO Marcellini-Sbordone Esercitazioni di Matematica vol.1 parte 2 paragrafo 2F

$$f(x) = \frac{\log(x) - 1}{\log(x) + 1}$$

C.E.

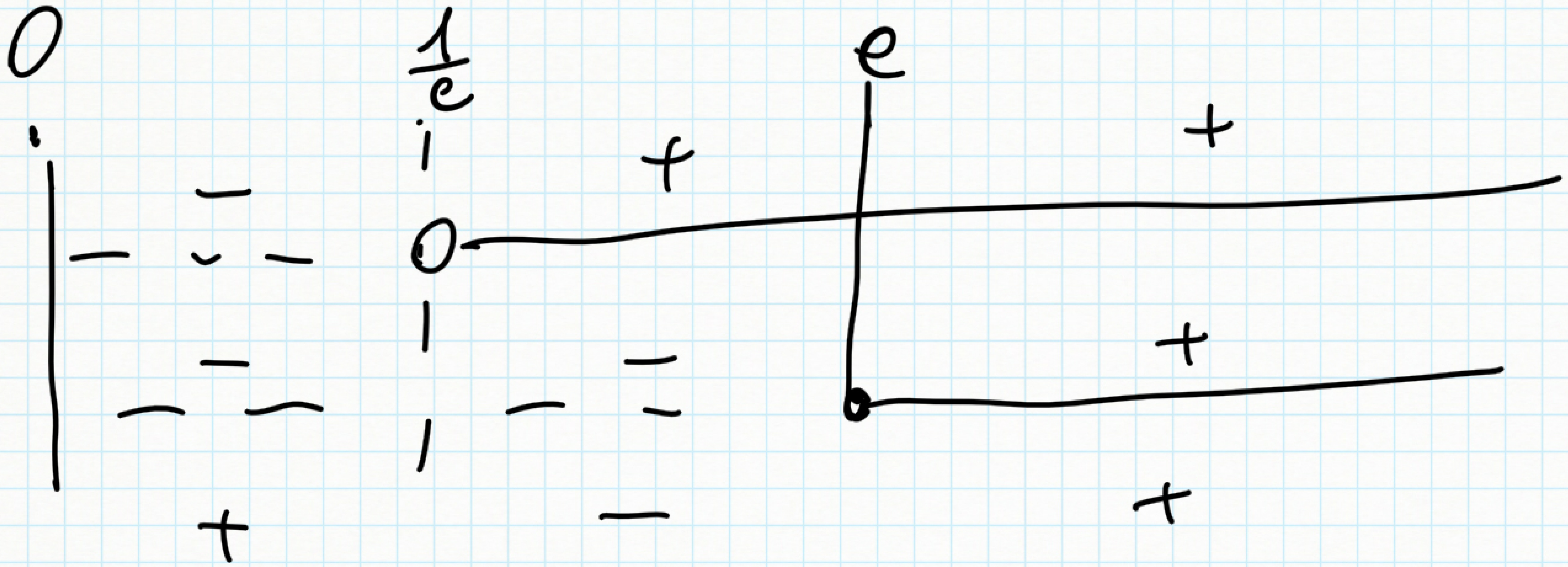
$$\begin{cases} x > 0 \\ \log(x) + 1 \neq 0 \end{cases} \qquad \begin{cases} x > 0 \\ \log x \neq -1 \end{cases} \qquad \begin{cases} x > 0 \\ e^{\log x} \neq e^{-1} \end{cases}$$

$$\begin{cases} x > 0 \\ x \neq e^{-1} \end{cases} \qquad]0; e^{-1}[\cup]e^{-1}; +\infty[$$

Positività

$$\begin{cases} \log x - 1 \geq 0 \\ \log x + 1 > 0 \end{cases} \qquad \begin{cases} \log x \geq 1 \\ \log x > -1 \end{cases} \qquad \begin{cases} e^{\log x} \geq e^{1} \\ e^{\log x} > e^{-1} \end{cases} \qquad \begin{cases} x \geq e \\ x > \frac{1}{e} \end{cases}$$

Vi ricordo $x > 0$ per il C.E.



Positiva in $]0; \frac{1}{e}[\, \cup \,]e; +\infty[$

LIMITI AGLI ESTREMI

$$\lim_{x \to 0^+} \frac{\log(x)-1}{\log(x)+1} = 1 \qquad \lim_{x \to +\infty} \frac{\log(x)-1}{\log(x)+1} = 1 \qquad Y = 1$$

ASINT. OR.

Guardando la positività

$$\lim_{x \to \left(\frac{1}{e}\right)^-} \frac{\log(x)-1}{\log(x)+1} = +\infty$$

$X = \frac{1}{e}$ ASIN. VERT.

$$\lim_{x \to \left(\frac{1}{e}\right)^+} \frac{\log(x)-1}{\log(x)+1} = -\infty$$

CRESCENZA

$$f(x) = \frac{\log(x) - 1}{\log(x) + 1}$$

$$f'(x) = \frac{\frac{1}{x}(\log(x) + 1) - (\log(x) - 1) \cdot \frac{1}{x}}{(\log(x) + 1)^2} =$$

$$f'(x) = \frac{\frac{1}{x}(\cancel{\log(x)} + 1 - \cancel{\log(x)} + 1)}{(\log(x) + 1)^2} = \frac{2}{x(\log(x) + 1)^2}$$

$$\left\| \begin{array}{l} x \geq 0 \\ (\log(x)+1)^2 \geq 0 \end{array} \right.$$

Entrambi positivi nel C.E.

Quindi funzione sempre crescente

$$f'(x) = \frac{2}{x\,(\log(x)+1)^2}$$

CONVESSITÀ

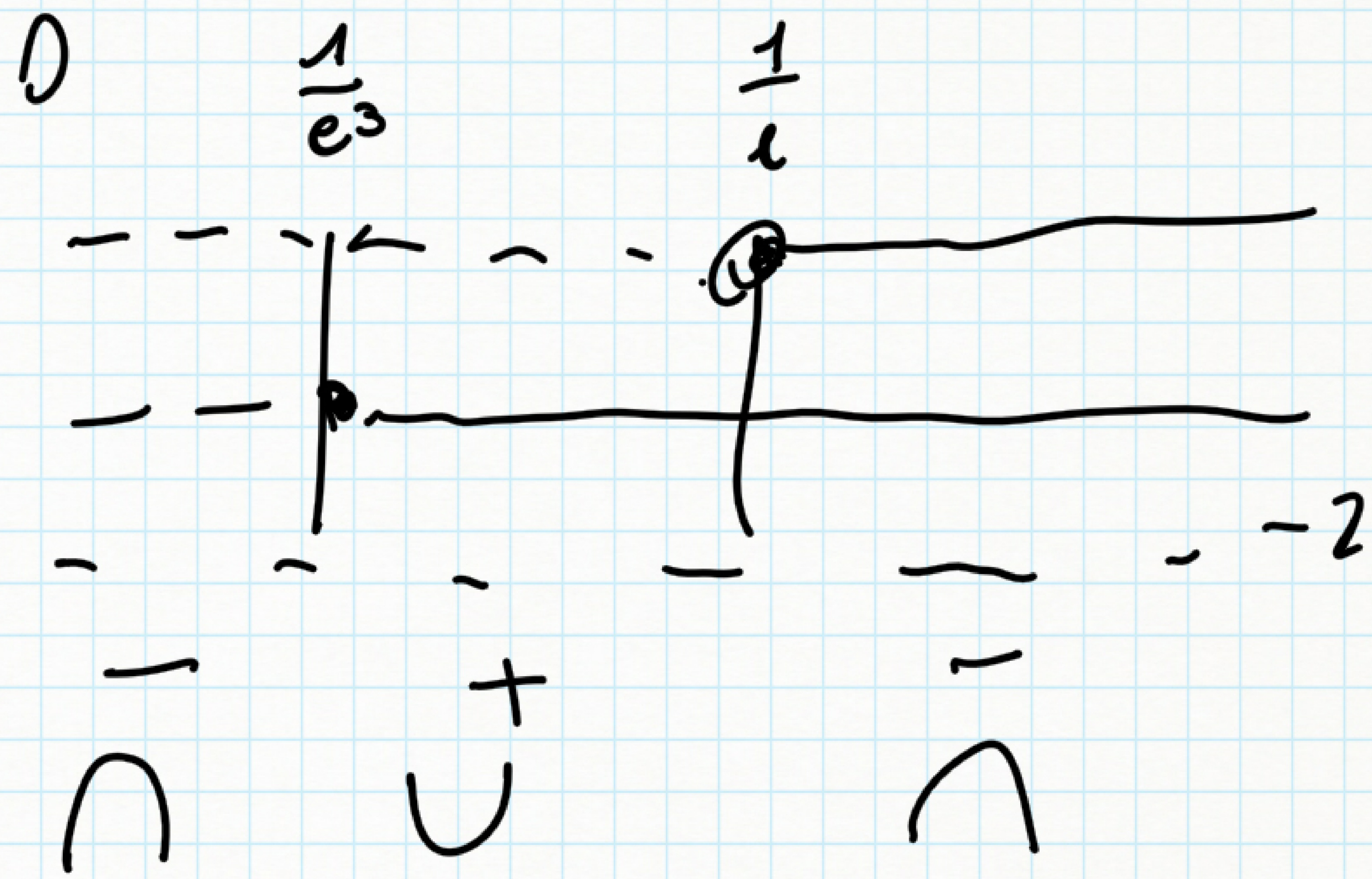
$$f''(x) = 2\, D\left(\frac{1}{x\,(\log(x)+1)^2}\right)$$

$$2D\left(x(\log(x)+1)^2\right) = (\log(x)+1)^2 + \cancel{x}\,2(\log(x)+1)\cdot\frac{1}{\cancel{x}} =$$

$$= (\log(x)+1)(\log(x)+1+2) = (\log(x)+1)(\log(x)+3)$$

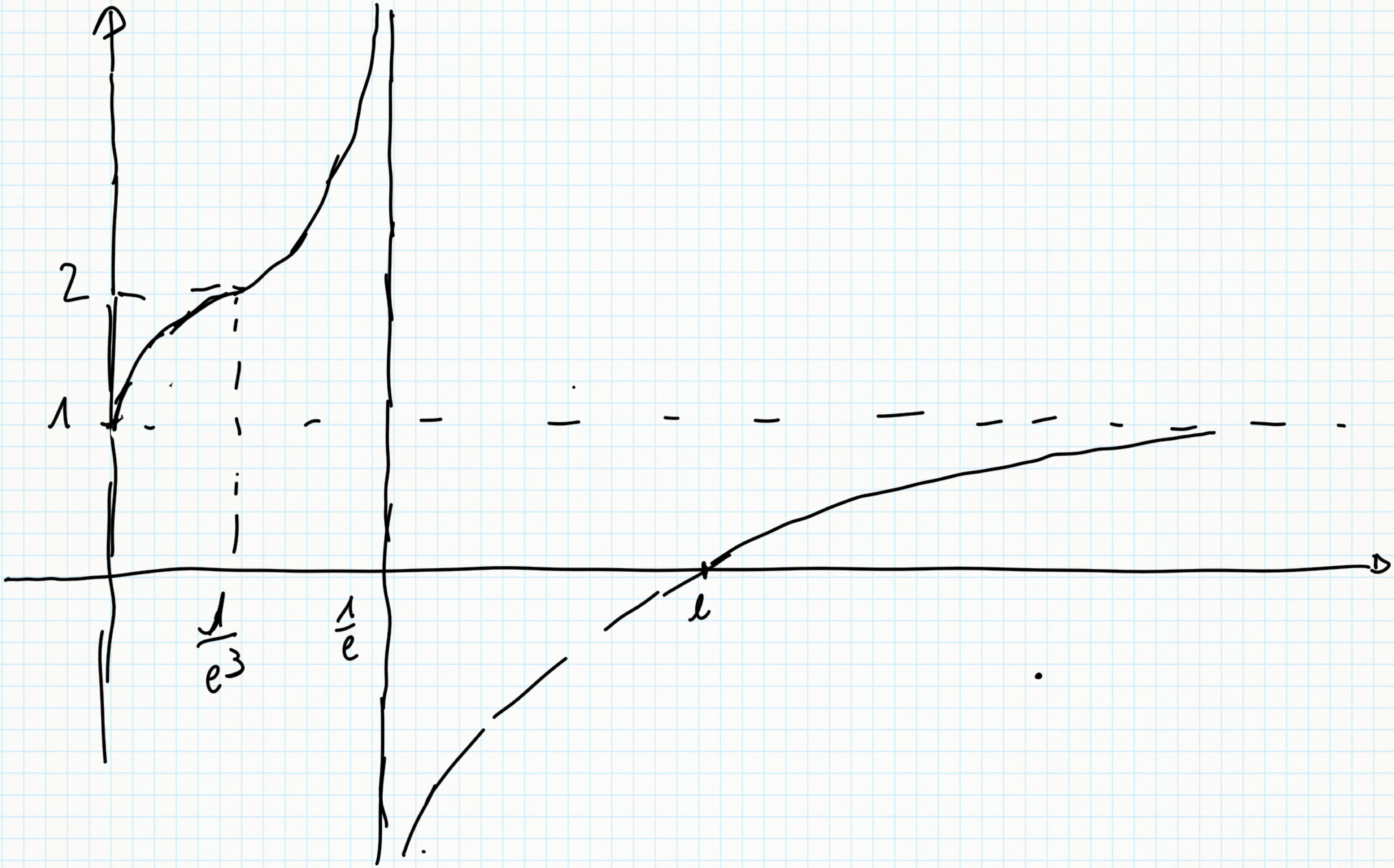
$$2D\left(\frac{1}{x(\log(x)+1)^2}\right) = 2\cdot\frac{-(\cancel{\log(x)+1})(\log(x)+3)}{x^2(\log(x)+1)^{\cancel{4}\,3}} = \frac{-2\cdot(\log(x)+3)}{x^2(\log(x)+1)^3}$$

$$\begin{cases}\log(x)+3 \geq 0\\ \log(x)+1 > 0\end{cases} \quad \begin{cases}\log(x) \geq -3\\ \log(x) > -1\end{cases} \quad \begin{cases}x \geq e^{-3}\\ x > e^{-1}\end{cases}$$



e^{-3} punto di flesso

$$f(e^{-3}) = \frac{-3-1}{-3+1} = \frac{-4}{-2} = 2$$



2
1
$\frac{1}{e^3}$
$\frac{1}{e}$
e