$f(x) = x^2 e^{-\frac{1}{x}}$ C.E. $x \neq 0$ $]-\infty; 0[\cup]0; +\infty[$

Positività

$\left\| \begin{array}{l} x^2 \geq 0 \\ e^{-\frac{1}{x}} > 0 \end{array} \right\| \begin{array}{l} \forall x \in \mathbb{R} \\ \forall x \in]-\infty; 0[\cup]0; +\infty[\end{array}$

quindi $f(x) > 0$ $\forall x \in]-\infty; 0[\cup]0; +\infty[$

LIMITI AGLI ESTREMI

Ricordando che $\lim_{x \to -\infty} -\frac{1}{x} = 0$ $\quad \lim_{x \to +\infty} -\frac{1}{x} = 0$

$\lim_{x \to -\infty} x^2 e^{-\frac{1}{x}} = +\infty$ $\quad \lim_{x \to +\infty} x^2 e^{-\frac{1}{x}} = +\infty$

$\lim_{x \to -\infty} \frac{\cancel{x^2} e^{-\frac{1}{x}}}{\cancel{x}} = -\infty$ $\quad \lim_{x \to +\infty} \frac{\cancel{x^2} e^{-\frac{1}{x}}}{\cancel{x}} = +\infty$

NO ASINT. OBL.

Ricordando che

$$\lim_{x \to 0^-} -\frac{1}{x} = +\infty \qquad \lim_{x \to 0^+} -\frac{1}{x} = -\infty$$

$$\lim_{x \to 0^+} x^2 e^{-\frac{1}{x}} = 0 \cdot 0 = 0$$

$$\lim_{x \to 0^-} x^2 e^{-\frac{1}{x}} = \lim_{x \to 0^-} \frac{e^{-\frac{1}{x}}}{\frac{1}{x^2}} = \left(\text{sostituzione } t = -\frac{1}{x}\right)$$

$$\lim_{t \to +\infty} \frac{e^t}{t^2} = +\infty \qquad x = 0 \text{ AS. VERT.}$$

MONOTONIA

$$f(x) = x^2 \cdot e^{-\frac{1}{x}}$$

$$f'(x) = 2x\, e^{-\frac{1}{x}} + \cancel{x^2}\left(e^{-\frac{1}{x}}\right) \cdot \left(\frac{1}{\cancel{x^2}}\right) =$$

$$= e^{-\frac{1}{x}}(2x+1)$$

$$e^{-\frac{1}{x}} \geq 0 \qquad \forall x \in \,]-\infty;0[\, \cup \,]0;+\infty[$$

$$2x+1 \geq 0 \qquad x \geq -\frac{1}{2}$$

$$\begin{array}{ccccc} & -\frac{1}{2} & & & \\ - & & + & & f' \\ \searrow & & \nearrow & & \end{array}$$

$$f\left(-\frac{1}{2}\right) = \frac{1}{4} \cdot e^2$$

$-\frac{1}{2}$ punto di minimo relativo $\quad \frac{1}{4}e^{2}$ minimo relativo

CONVESSITÀ

$$f'(x) = e^{-\frac{1}{x}}(2x+1)$$

$$f''(x) = e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)(2x+1) + 2e^{-\frac{1}{x}} = e^{-\frac{1}{x}}\left[\frac{2x+1}{x^2} + 2\right] =$$

$$= e^{-\frac{1}{x}}\left[\frac{2x^2+2x+1}{x^2}\right]$$

$e^{-\frac{1}{x}} \geqslant 0$	$\forall x \in \left]-\infty;0\right[\cup \left]0;+\infty\right[$	$\forall x \in \left]-\infty;0\right[\cup \left]0;+\infty\right[$
$x^2 \geqslant 0$	$\forall x \in \mathbb{R}$	$\forall x \in \mathbb{R}$
$2x^2+2x+1 \geqslant 0$	$\Delta = 4-8=-4<0$	$\forall x \in \mathbb{R}$

e^2/4
-1/2