Studi di funzione; rifermento bibliografico Marcellini-Sbordone Esercitazioni di Matematica 1 volume 1 parte 2 paragrafo 2D

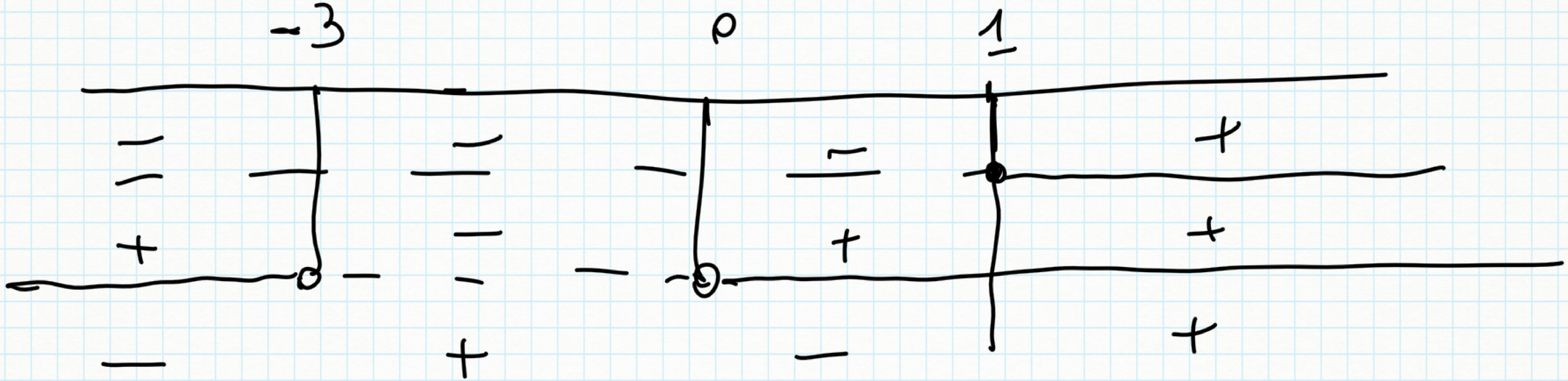
$$f(x) = \frac{x-1}{x^2+3x}$$

C.E. $x^2+3x \neq 0$ $x(x+3) \neq 0$

$]-\infty; -3[\cup]-3; 0[\cup]0; +\infty[$

Positività

$$\begin{cases} x-1 \geq 0 \\ x^2+3x > 0 \end{cases} \qquad \begin{cases} x \geq 1 \\ x < -3 \quad x > 0 \end{cases}$$



Positività $-3 < x < 0 \qquad x > 1$

Limiti agli estremi

$$\lim_{x \to -\infty} \frac{x-1}{x^2+3x} = 0$$

$$\lim_{x \to +\infty} \frac{x-1}{x^2+3x} = 0$$

Poiché grad D > grad N

quindi

$y = 0$ AS. OR. DX e SX

Utilizzando lo studio della positività

$$\lim_{x \to -3^-} \frac{x-1}{x^2+3x} = -\infty$$

$x = -3$ AS VERT.

$$\lim_{x \to -3^+} \frac{x-1}{x^2+3x} = +\infty$$

$$\lim_{x \to 0^-} \frac{x-1}{x^2+3x} = +\infty$$

$x = 0$ AS. VERT.

$$\lim_{x \to 0^+} \frac{x-1}{x^2+3x} = -\infty$$

MONOTONIA

$$f(x) = \frac{(x-1)}{x^2+3x}$$

$$f'(x) = \frac{(x^2+3x) - (x-1)(2x+3)}{(x^2+3x)^2} = \frac{x^2+3x-(2x^2+3x-2x-3)}{(x^2+3x)^2} =$$

$$= \frac{x^2+3x-2x^2-3x+2x+3}{(x^2+3x)^2} = \frac{-x^2+2x+3}{(x^2+3x)^2}$$

il denominatore è sempre positivo nel campo d'esistenza quindi

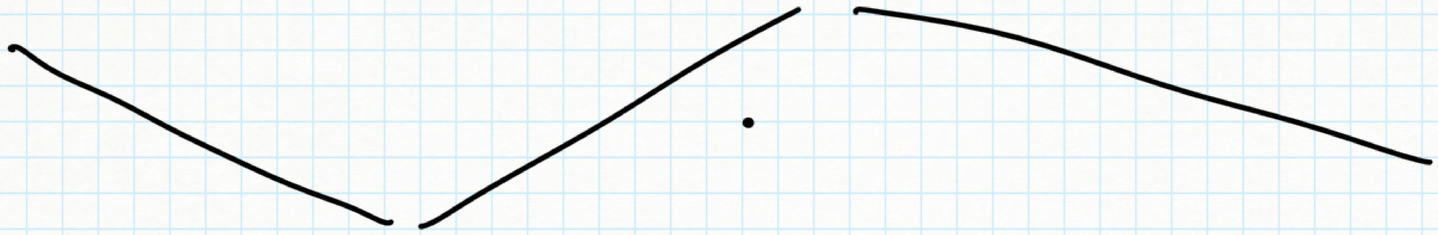
$-x^2+2x+3>0$

$\Delta = 4+12 = 16 > 0$

$x_{1,2} = \frac{-2 \pm 4}{-2}$ $\begin{cases} 3 \\ -1 \end{cases}$

$-1 \qquad\qquad 3$

$- \qquad + \qquad - \qquad 8^1$



-1 punto di minimo relativo $\quad f(-1) = \underline{1}$ minimo relativo

3 // // massimo // $\quad f(3) = \frac{1}{9}$ massimo relativo

CONCAVITA`

$$f'(x) = \frac{-x^2+2x+3}{(x^2+3x)^2}$$

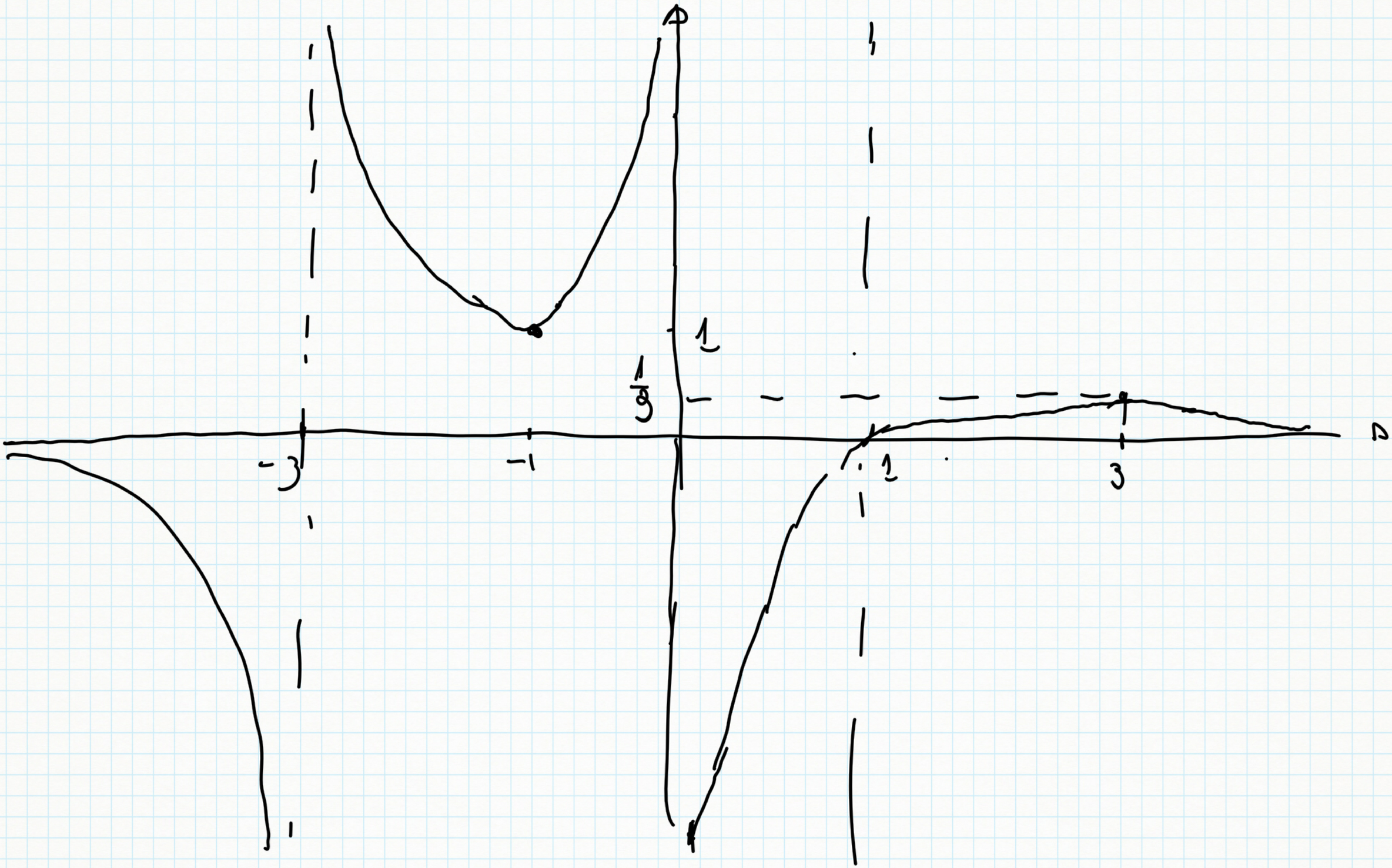
$$f''(x) = \frac{(-2x+2)(x^2+3x)^2 - (-x^2+2x+3)\,2(x^2+3x)(2x+3)}{(x^2+3x)^4} =$$

$$= \frac{2(x^2+3x)}{(x^2+3x)^{4}(x^3)}\left[(1-x)(x^2+3x) - (-x^2+2x+3)(2x+3)\right] =$$

$$= \frac{2}{(x^2+3x)^3}\left[x^2+3x-x^3-3x^2-(-2x^3-3x^2+4x^2+6x+6x+9)\right] =$$

$$= \frac{2}{(x^2+3x)^3}\left[\cancel{x^2+3x-x^3-3x^2+2x^3+3x^2-4x^2-6x-6x-9}\right] =$$

$$= \frac{2}{(x^2+3x)^3}\left[x^3-3x^2-3x-9\right] .$$


1
1/9
-3
-1
1
3