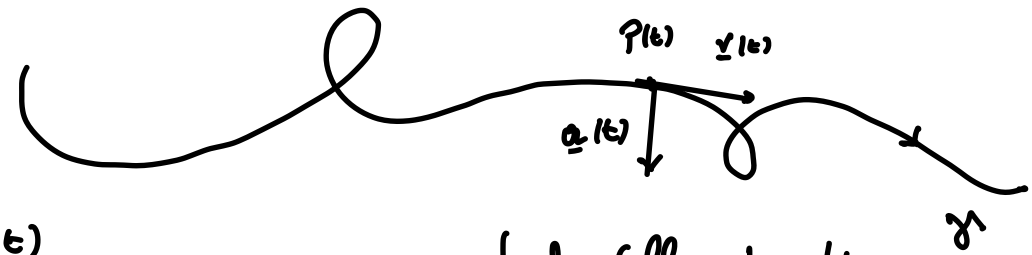
Lezione del 30/11/2022

$t \in [t_0, t_1]$

$$P(t) = (x(t), y(t), z(t))$$

$$\underline{v}(t) = (x'(t), y'(t), z'(t))$$



$$\gamma \equiv \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$

equazioni parametriche della traiettoria

$$t \longrightarrow P(t) \in \mathbb{R}^3$$

$\underline{a}(t) = (x''(t), y''(t), z''(t))$ accelerazione istantanea

Se il moto è uniforme: $\|\underline{v}(t)\| = c$

$$\sqrt{(x')^2 + (y')^2 + (z')^2} = c$$

$$(x'(t))^2 + (y'(t))^2 + (z'(t))^2 = \text{costante}$$

$$2x'x'' + 2y'y'' + 2z'z'' = 0$$

$$x'x'' + y'y'' + z'z'' = 0$$

$$\underline{v}(t) \cdot \underline{a}(t) = 0$$

Def. Una curva in $\mathbb{R}^n$ è un'applicazione continua

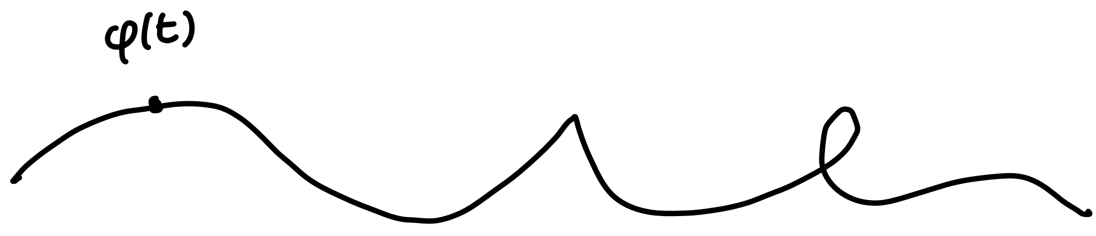
$$\varphi : I \subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$$
$$t \in I \longrightarrow \varphi(t) \quad \in \mathbb{R}^n$$

I = intervallo

$$\varphi(t) = \Big(\underset{\mathbb{R}}{\underset{\cap}{\varphi_1(t)}}, \underset{\mathbb{R}}{\underset{\cap}{\varphi_2(t)}}, \dots, \underset{\mathbb{R}}{\underset{\cap}{\varphi_n(t)}}\Big)$$

Eq. parametriche di φ :

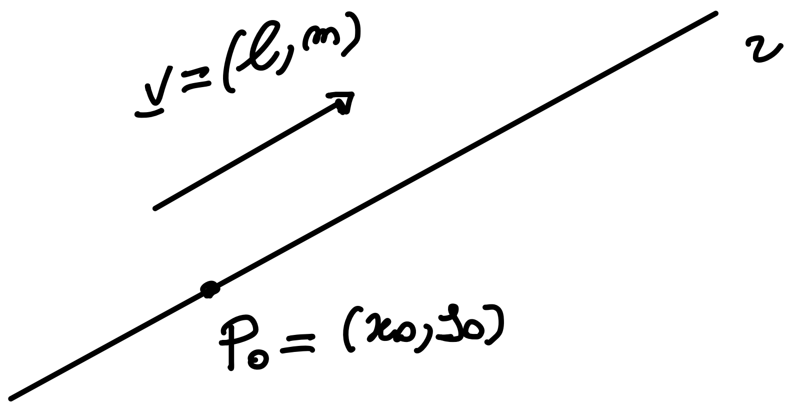
$$\begin{cases} x_1 = \varphi_1(t) \\ x_2 = \varphi_2(t) \\ \vdots \\ x_n = \varphi_n(t) \end{cases} \qquad \forall t \in I$$



Codominio di φ :

$$\varphi(I) = \{\varphi(t) : t \in I\}$$

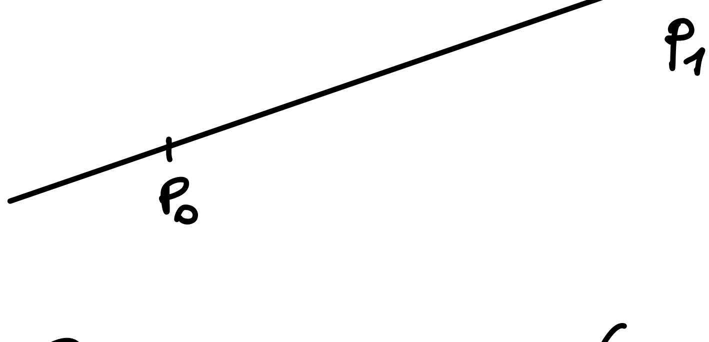
supporto di φ



$$r) \begin{cases} x = x_0 + l t \\ y = y_0 + m t \end{cases} \qquad t \in \mathbb{R}$$

$P_1 = (x_1, y_1)$

$$r) \begin{cases} x = x_0 + (x_1 - x_0) t \\ y = y_0 + (y_1 - y_0) t \end{cases}$$



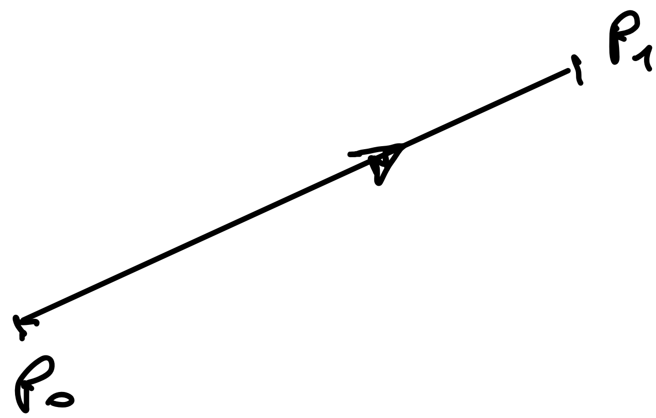
$$\varphi : t \in \mathbb{R} \longrightarrow \varphi(t) = (x_0 + l t, y_0 + m t)$$

curve $\mathbb{R}^2$

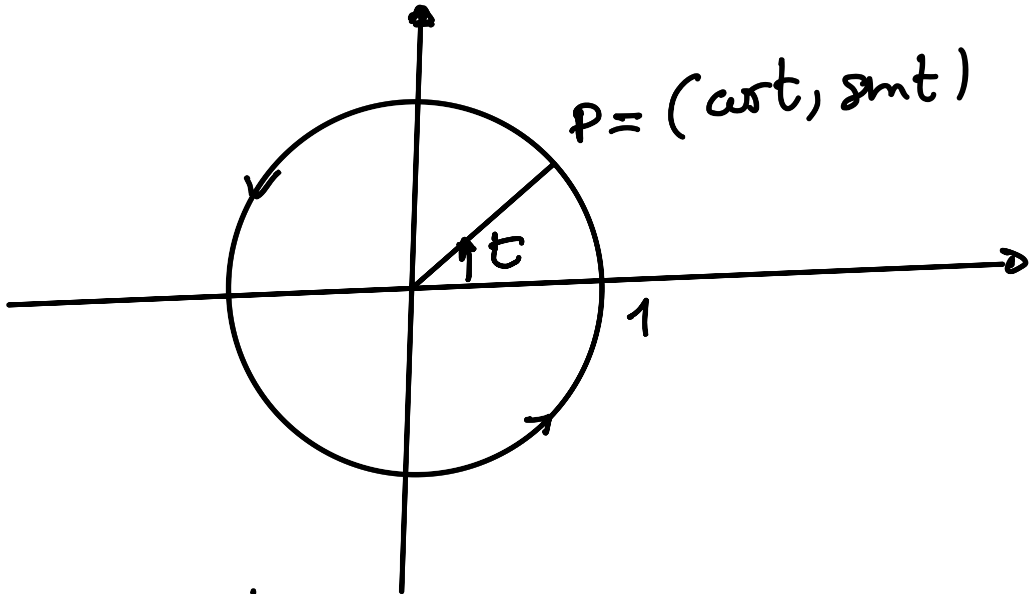
Equazioni segmento di estremi P_0, P_1

$$\begin{cases} x = x_0 + (x_1 - x_0)t \\ y = y_0 + (y_1 - y_0)t \end{cases}$$

$t \in [0, 1]$



ES. 2 Circonferenza $x^2 + y^2 = 1$



$$\varphi \equiv \begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad t \in [0, 2\pi] \qquad \text{è } \underline{\text{semplice}}$$

$$\psi \equiv \begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad t \in [0, 4\pi] \qquad \text{non è semplice}$$

φ, ψ hanno lo stesso sostegno, ma sono diverse!

ES

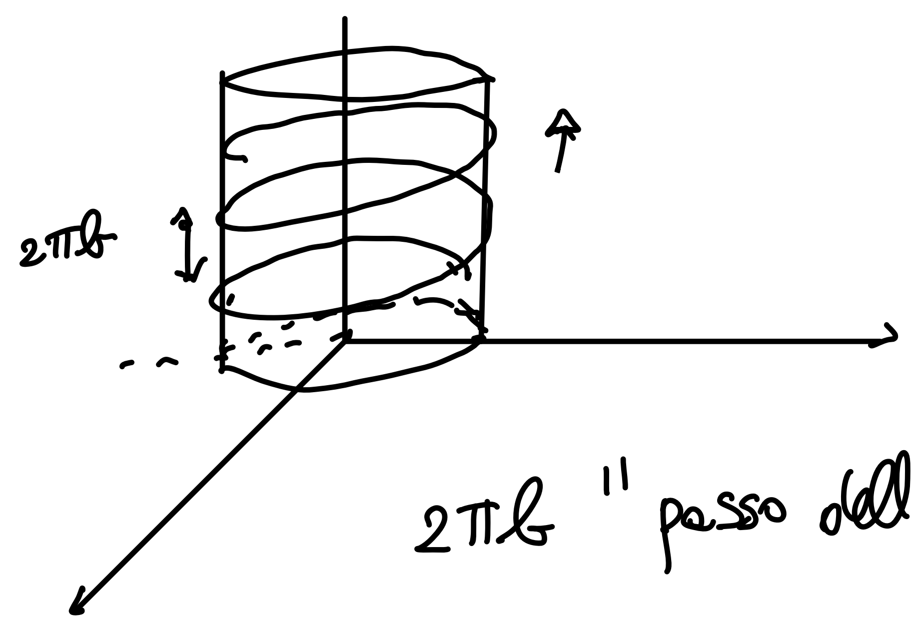
$$\varphi \equiv \begin{cases} x = a\cos t \\ y = a\sin t \\ z = bt \end{cases} \qquad \begin{matrix} a>0 \\ b>0 \end{matrix}$$

Elica cilindrica (semplice)

$t \in \mathbb{R}$

$x^2 + y^2 = a^2$ in $\mathbb{R}^3$?

Cilindro di curva direttrice la circonferenza, generatrici parallele all'asse z

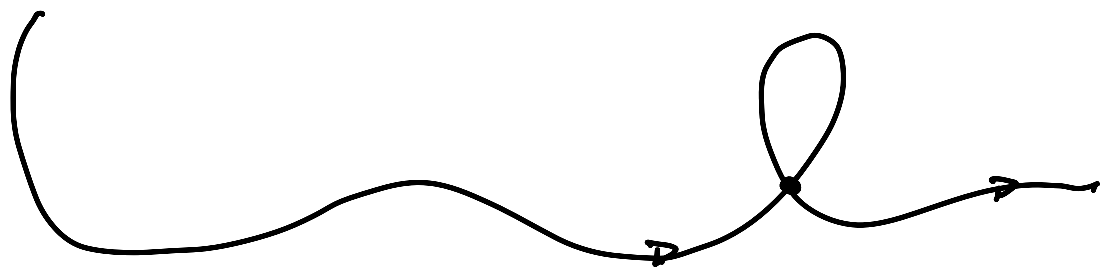


$2\pi b$ "passo dell'elica"

Def $\varphi : I \longrightarrow \mathbb{R}^n$ si dice <u>semplice</u>

se per ogni coppia di valori t_1, t_2, $t_1 \neq t_2$
e almeno uno dei due <u>interno</u> ad I, si ha

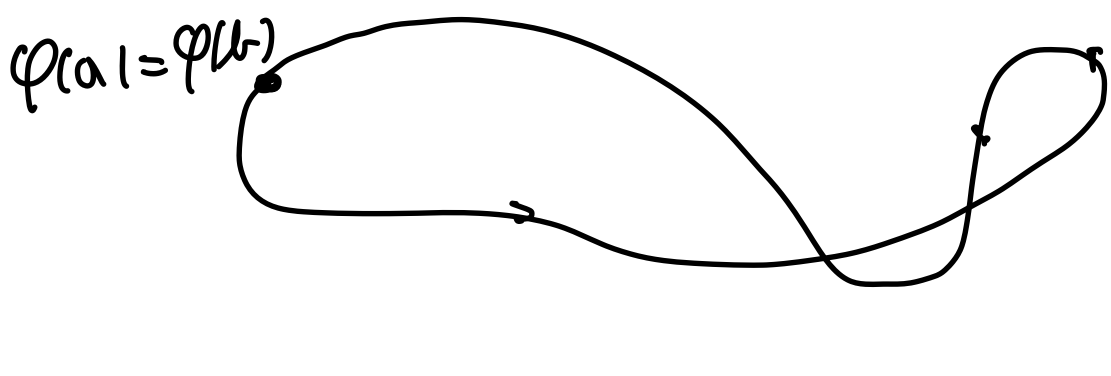
$$\varphi(t_1) \neq \varphi(t_2)$$



NON SEMPLICE

Def $\varphi : [a,b] \longrightarrow \mathbb{R}^n$ <u>chiusa</u>

se $\varphi(a) = \varphi(b)$



$$\begin{cases} x = R\cos t \\ y = R\sin t \end{cases} \qquad t \in [0, 2\pi]$$

curva chiusa e semplice

$t = 0 \longrightarrow (R, 0)$

$t = 2\pi \longrightarrow$ "

$t \in [0, 4\pi)$

chiusa e non semplice

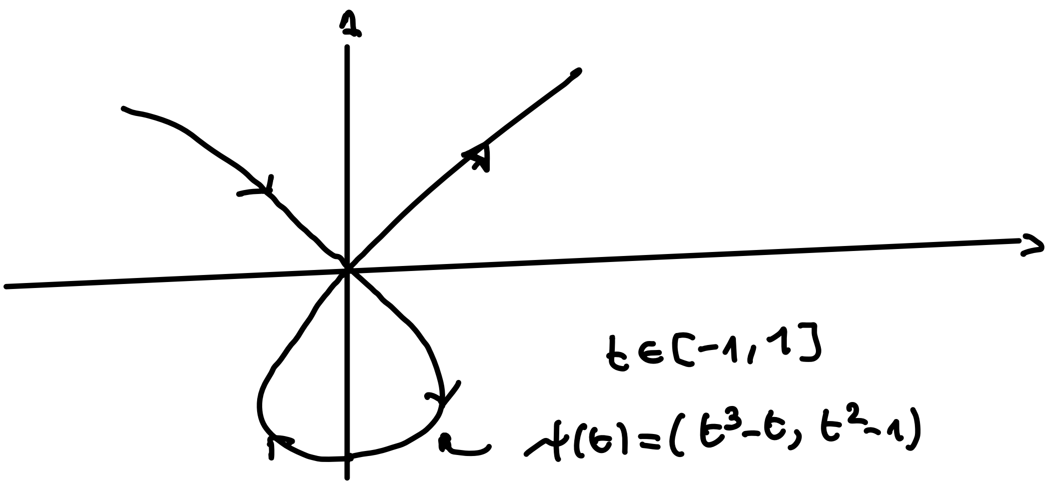
ES. $\varphi(t) = (t^3 - t,\ t^2 - 1) \quad \forall t \in \mathbb{R}$

STROFOIDE

non è chiusa

$t = -1 \Longrightarrow \varphi(-1) = (-1 + 1,\ 1 - 1) = (0, 0)$

$t = 1 \Longrightarrow \varphi(1) = (0, 0)$



CAPPIO DI STROFOIDE

curva semplice e chiusa

$\varphi(t) = (\varphi_1(t), \varphi_2(t), .., \varphi_n(t))$

<u>Def.</u> (curve regolari)

$\varphi : [a,b] \longrightarrow \mathbb{R}^n$ si dice regolare se

1) $\varphi \in C^1([a,b])$; $\varphi_1, .., \varphi_n \in C^1([a,b])$

$$\varphi'(t) = (\varphi_1'(t), \varphi_2'(t), .., \varphi_n'(t)) \in \mathbb{R}^n$$

2) $\varphi'(t) \neq 0 \quad \forall t \in]a,b[\iff$

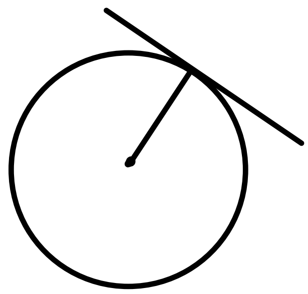
$\iff (\varphi_1')^2 + (\varphi_2')^2 + \dots + (\varphi_n')^2 > 0 \quad \forall t \in]a,b[$

$\iff \varphi_1'(t), .., \varphi_n'(t)$ <u>non</u> si annullano simultaneamente.

<u>ES.</u>

$$\begin{cases} x = R\cos t = \varphi_1(t) \\ y = R\sin t = \varphi_2(t) \end{cases} \qquad t \in [0, 2\pi]$$

$$\begin{cases} \varphi_1'(t) = -R\sin t \\ \varphi_2'(t) = R\cos t \end{cases} : \quad (\varphi_1')^2 + (\varphi_2')^2 = R^2\sin^2 t + R^2\cos^2 t = R^2 > 0$$

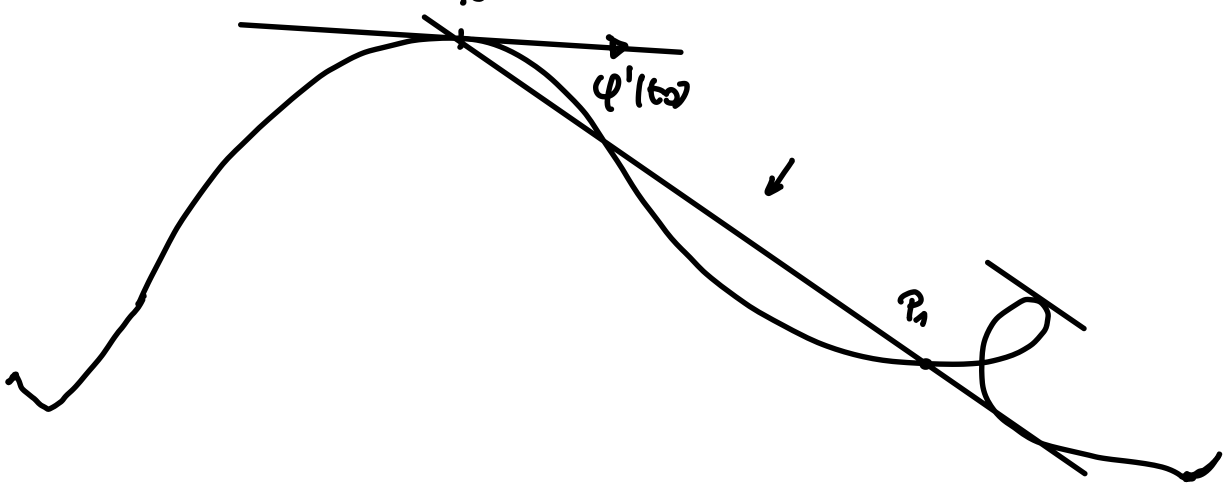


$\underline{\underline{n=2}}$

$\varphi(t) = (\varphi_1(t), \varphi_2(t))$

$t_0 \in]a,b[$

$P_0 = \varphi(t_0)$



$P_1 = \varphi(t_1)$

$2 : P_0, P_1$

$P_0 = (\varphi_1(t_0), \varphi_2(t_0))$, $P_1 = (\varphi_1(t_1), \varphi_2(t_1))$

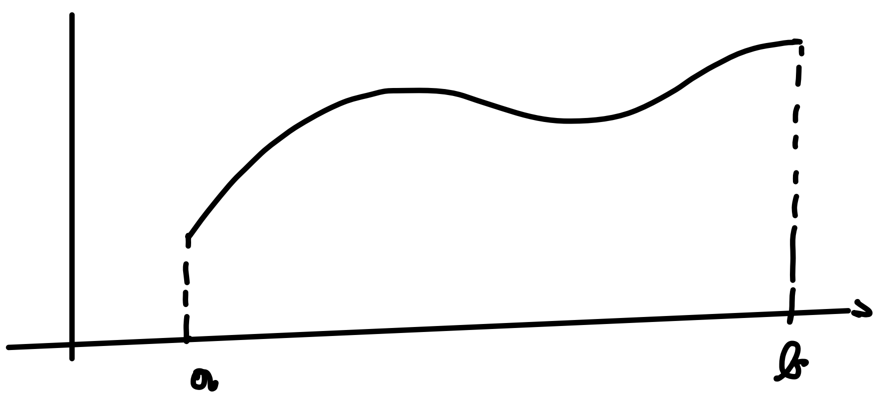
$$(1) \quad \begin{cases} x = \varphi_1(t_0) + \dfrac{\varphi_1(t_1) - \varphi_1(t_0)}{t_1 - t_0}(t - t_0) \\ \\ y = \varphi_2(t_0) + \dfrac{\varphi_2(t_1) - \varphi_2(t_0)}{t_1 - t_0}(t - t_0) \end{cases}$$

$t_1 \to t_0$ in (1) otteniamo le equazioni della retta tangente a φ in $P_0 = \varphi(t_0)$

$$\begin{cases} x = \varphi_1(t_0) + \varphi_1'(t_0)\,(t-t_0) \\ y = \varphi_2(t_0) + \varphi_2'(t_0)\,(t-t_0) \end{cases}$$

ES. (grafico di una funzione $f = f(x)$)

$f \in C^1([a,b])$ $\quad y = f(x)$ grafico di f



$$\{(x, f(x)) : x \in [a,b]\} = G_f \qquad y = f(x)$$

Rapp. parametrica :

$$\varphi \equiv \begin{cases} x = t = \varphi_1(t) & t \in [a,b] \\ y = f(t) = \varphi_2(t) \end{cases}$$

ha come sostegno G_f

$y = x^2$

$$\begin{cases} x = t \\ y = t^2 \end{cases}$$

$x_0 \in]a,b[$: $y = f(x_0) + f'(x_0)(x - x_0)$

$t_0 = x_0$:

$$\begin{cases} x = \varphi_1(t_0) + \varphi_1'(t_0)(t - t_0) \\ y = \varphi_2(t_0) + \varphi_2'(t_0)(t - t_0) \end{cases}$$

$\varphi_1(t) = t$, $\varphi_2(t) = f(t)$

$\varphi_1'(t) = 1$, $\varphi_2'(t) = f'(t)$

$$\begin{cases} x = \cancel{t_0} + t - \cancel{t_0} \\ y = f(t_0) + f'(t_0)(t-t_0) \end{cases}$$

$$\Downarrow$$

$$y = f(\underbrace{t_0}_{x_0}) + f'(\underbrace{t_0}_{x_0})(x-t_0)$$

<u>Def.</u> $\varphi'(t_0) = (\varphi_1'(t_0), \varphi_2'(t_0))$ dicesi vettore tangente a φ in $P_0 = \varphi(t_0)$ e

$$T(t_0) = \frac{\varphi'(t_0)}{\|\varphi'(t_0)\|}$$

dicesi versore tangente a φ in P_0.