Lezione del 02/12/2022

$m=3$ $\qquad \varphi(t) = (\varphi_1(t), \varphi_2(t), \varphi_3(t))$

$t \in [a,b]$ sia regolare

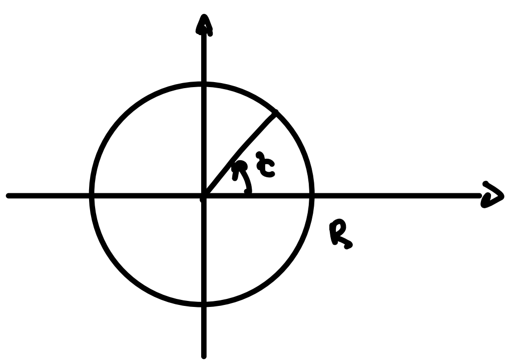
$t_0 \in]a,b[$

Eq. retta tangente a φ in $P_0 = \varphi(t_0)$:

$$\begin{cases} x = \varphi_1(t_0) + \varphi_1'(t_0)(t-t_0) \\ y = \varphi_2(t_0) + \varphi_2'(t_0)(t-t_0) \\ z = \varphi_3(t_0) + \varphi_3'(t_0)(t-t_0) \end{cases}$$

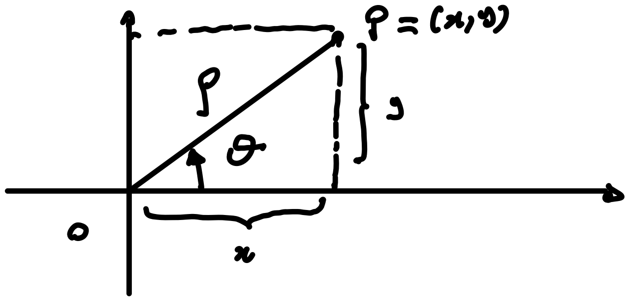
$\varphi'(t_0)$ = vettore tangente

$\underline{m=2}$ (curve piane) $\qquad x^2+y^2=R^2$ rapp. cartesiana



$$\begin{cases} x = R\cos t \\ y = R\sin t \end{cases} \quad t \in [0, 2\pi]$$

Equazione polare di una curva piana

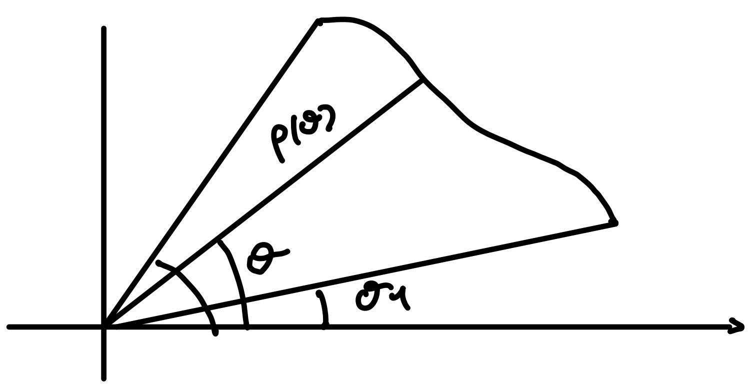


$$\rho = \sqrt{x^2 + y^2}$$

$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \end{cases}$$

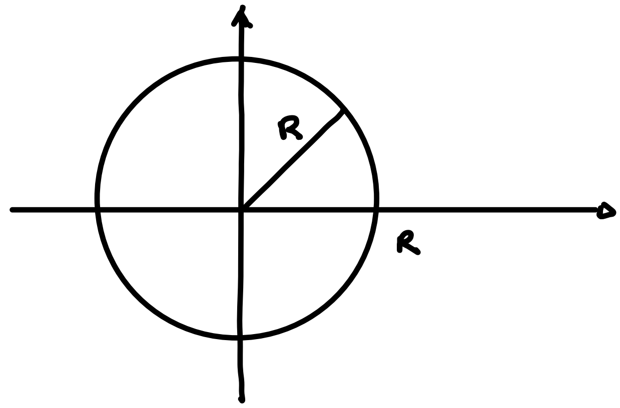
Equazione polare di una curva:

$$\rho = \rho(\theta) \qquad \theta \in [\theta_1, \theta_2]$$



$$\begin{cases} x = \rho\cos\theta = \rho(\theta)\cos\theta \\ y = \rho\sin\theta = \rho(\theta)\sin\theta \end{cases} \qquad \theta \in [\theta_1, \theta_2]$$

$\underbrace{\qquad}_{\text{rapp. parametrica}}$



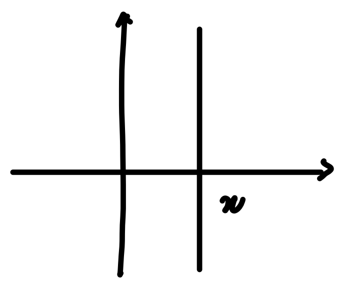
$\rho = R$

$x^2 + y^2 = R^2$

$\rho^2\cos^2\theta + \rho^2\sin^2\theta = R^2 \Leftrightarrow \rho^2 = R^2 \Leftrightarrow \rho = R$

$x = \rho\cos\theta$

$x = 1$



$\rho\cos\theta = 1$

$\rho = \cancel{\frac{1}{\cos\theta}}$

Regularitate? $\quad \rho = \rho(\theta) \quad \rho \in C^1([\theta_0, \theta_1])$

$$\begin{cases} x = \rho(\theta)\cos\theta \\ y = \rho(\theta)\sin\theta \end{cases} \quad \theta \in [\theta_0, \theta_1]$$

$$\begin{cases} x' = \rho'\cos\theta - \rho\sin\theta \\ y' = \rho'\sin\theta + \rho\cos\theta \end{cases}$$

$\underbrace{(x')^2 + (y')^2}\ > 0$

$(x')^2 + (y')^2 = (\rho')^2\cos^2\theta + \rho^2\sin^2\theta - 2\rho\rho'\cancel{\cos\theta\sin\theta}$

$$+ (\rho')^2 \sin^2\theta + \rho^2 \cos^2\theta + 2\rho\cancel{\rho'\cos\theta}\sin\theta$$

$$= \rho^2 + (\rho')^2 :$$

la curva è regolare $\iff [\rho(\theta)]^2 + [\rho'(\theta)]^2 > 0$

$\forall \theta \in \,]\theta_1, \theta_2[\,.$

$$\rho = a\underbrace{(1+\cos\theta)}_{\rho(\theta)} \quad , \theta \in [0, 2\pi] \qquad a > 0$$

<u>Cardioide</u>

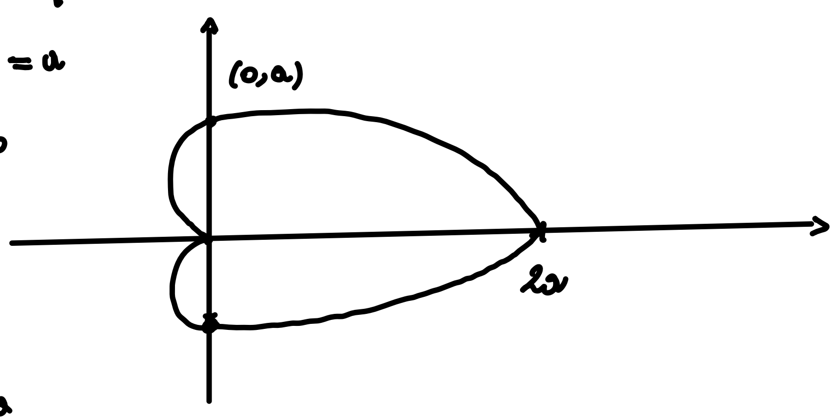
$\theta = 0 \longrightarrow \rho = 2a$

$\theta = \frac{\pi}{2} \longrightarrow \rho = a$

$\theta = \pi \longrightarrow \rho = 0$

$\theta = \frac{3}{2}\pi \longrightarrow \rho = a$

$\theta = 2\pi \longrightarrow \rho = 2a$



$$\rho' = -a\sin\theta$$

$$\rho^2 + (\rho')^2 = a^2\left[(1+\cos\theta)^2 + \sin^2\theta\right] =$$

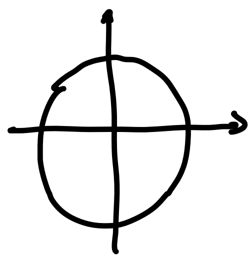
$$= a^2\left[1 + \cos^2\theta + 2\cos\theta + \sin^2\theta\right]$$

$= 2a^2 (1+\cos\theta) \qquad \theta \in [0, 2\pi]$

$= 0$ per $\theta = \pi$: non è regolare

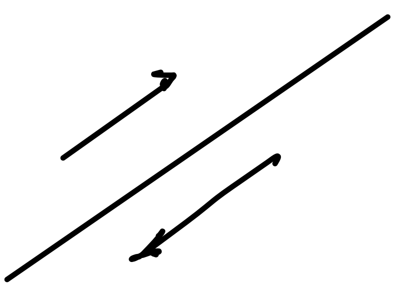
Se $\theta \in [-\pi, \pi]$: allora

$\rho^2 + (\rho')^2 = 2a^2 (1+\cos\theta) > 0$



$\theta \in]-\pi, \pi[$: $\cos\theta > -1$

Curve equivalenti - verso di percorrenza di una curva.



$$\begin{cases} x = x_0 + lt \\ y = y_0 + mt \\ z = z_0 + nt \end{cases}$$

$(\lambda l, \lambda m, \lambda n)$

<u>Def</u>. $\varphi : I \to \mathbb{R}^n$ $\varphi(t)$

$\psi : J \to \mathbb{R}^n$ $\psi(s)$

$\underbrace{\qquad\qquad\qquad\qquad}_{\text{di classe } C^1}$

Si dice che φ è equivalente a ψ se

$\exists \quad g = g(t), \quad g : I \longrightarrow J$ biettiva

di classe C^1 e tale che

$$g'(t) \neq 0 \quad \forall t \in I$$

ed inoltre $\left[\varphi(t) = \psi(g(t)) \quad \forall t \in I \right]$ ①

g = cambiamento ammissibile di parametro

$g' > 0$ oppure $g' < 0$

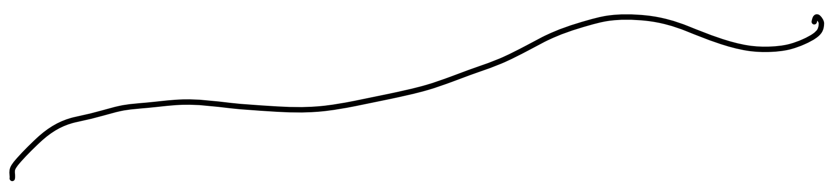
g str. crescente oppure è strett. decrescente

$g^{-1} : J \longrightarrow I$, quindi da

1 si ha

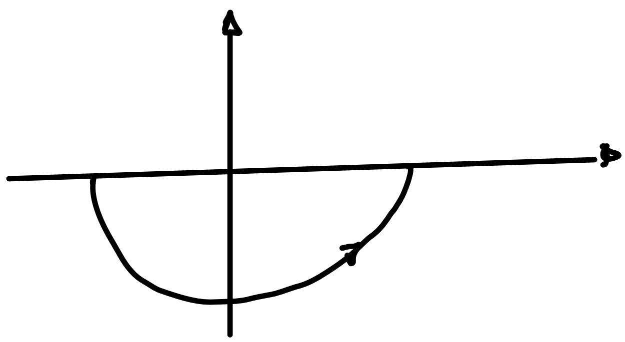
$$\psi(s) = \varphi(g^{-1}(s))$$

<u>Oss.</u> Due curve equivalenti hanno lo stesso sostegno!



<u>ES.</u> $\varphi(t) = (\cos t, \sin t)$ $\quad \begin{cases} x = \cos t \\ y = \sin t \end{cases}$

$t \in [-\pi, 0]$

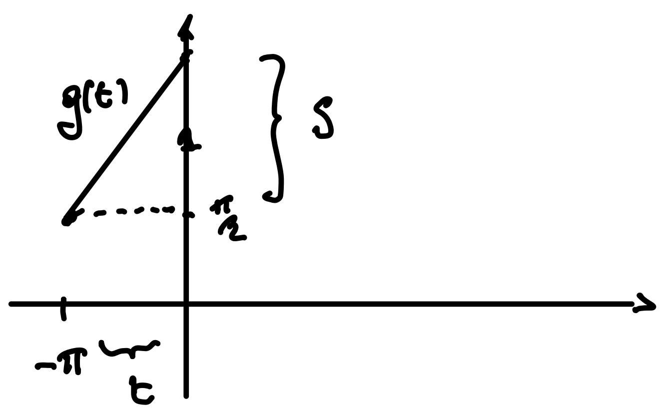


$$\psi(s) = (\cos(2s), \sin(2s)), \quad s \in [\tfrac{\pi}{2}, \pi]$$

$s = \frac{\pi}{2} \longrightarrow (\cos\pi, \sin\pi) = (-1, 0)$

$s = \pi \longrightarrow (1, 0)$

$$s = g(t) = \frac{t}{2} + \pi \quad , \quad t \in [-\pi, 0]$$



Dobbiamo verificare che $\psi(g(t)) = \varphi(t)$

$$\psi(g(t)) = \psi\left(\frac{t}{2}+\pi\right) = \left(\cos 2\left(\frac{t}{2}+\pi\right), \sin 2\left(\frac{t}{2}+\pi\right)\right)$$

$$= \left(\cos(t+2\pi), \sin(t+2\pi)\right)$$

$$= (\cos t, \sin t) = \varphi(t)$$

Se φ è equivalente a ψ

$\varphi \sim \psi$

"tilde"

<u>ES.</u>

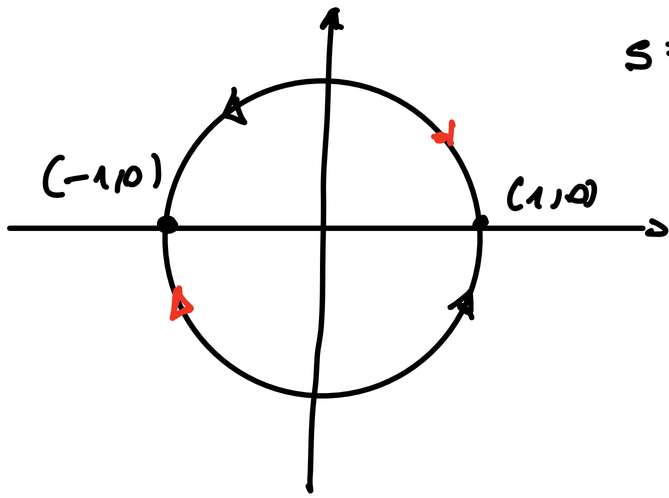
$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad t \in [0, 2\pi]$$

$s = -t = g(t)$

$t \leftrightarrow -t$

equivalenti

$$\begin{cases} x = \cos s \\ y = \sin(-s) = -\sin s \end{cases} \qquad s \in [-2\pi, 0]$$



$$s = -\pi \rightarrow \begin{cases} x = -1 \\ y = 0 \end{cases}$$

$$s = 0 \Rightarrow \begin{cases} x = 1 \\ y = 0 \end{cases}$$

$(-1,0)$ precede $(1,0)$

Def. (Verso di percorrenza o orientamento)

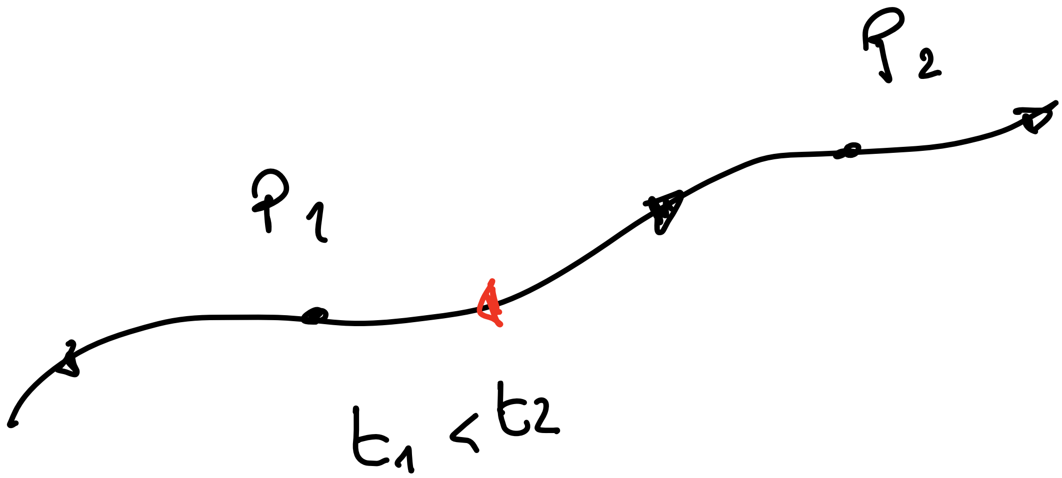
Ogni curva $\varphi : I \rightarrow \mathbb{R}^n$ induce una orientazione (o verso di percorrenza, orientamento) sul proprio sostegno, ossia il verso in cui il punto $\varphi(t)$ percorre il sostegno all'aumentare di t; si dice che il punto

$$P_1 = \varphi(t_1)$$

precede il punto $P_2 = \varphi(t_2)$

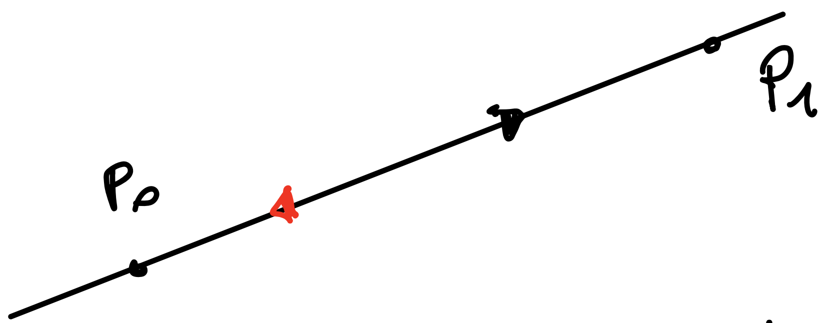
nel verso indotto dal parametro t (verso delle t crescenti) $\underline{\underline{se}}$

$$t_1 < t_2$$



<u>ES.</u>

$$\begin{cases} x = t x_1 + (1-t) x_0 \\ y = t y_1 + (1-t) y_0 \end{cases} \quad t \in [0,1]$$



$$\begin{cases} x = t x_0 + (1-t) x_1 \\ y = t y_0 + (1-t) y_1 \end{cases}$$

Se $\varphi \sim \psi$ e $g' > 0$: stesso verso ||

e $g' < 0$: verso contrario