Massimi e minimi assoluti

$f = f(x)$, $f : [a,b] \longrightarrow \mathbb{R}$ continua

$\min\limits_{x\in[a,b]} f(x)$ $\max\limits_{x\in[a,b]} f(x)$

1) $f(a)$, $f(b)$

2) $f'(x) = 0$: in ognuno di questi punti, si calcola $f(x)$

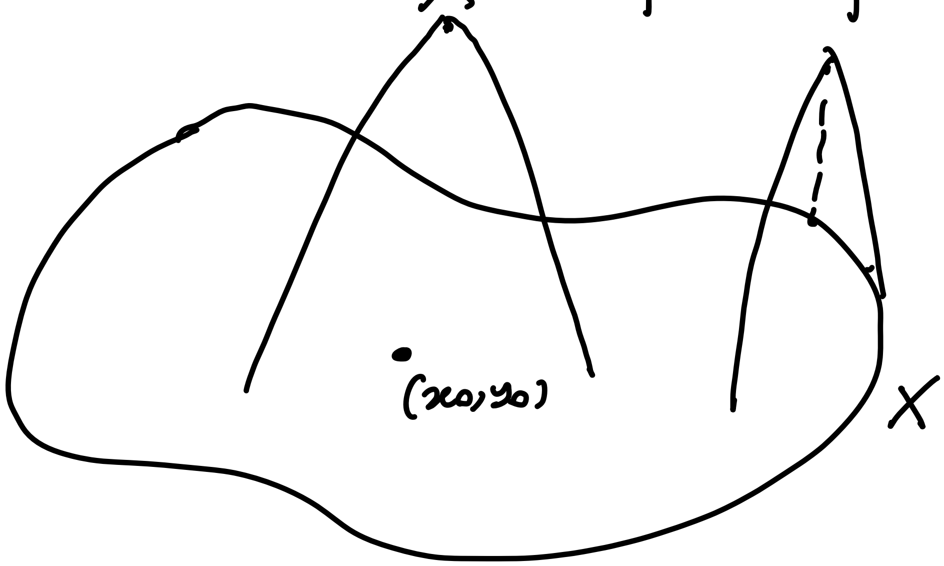
x_0 $f(x_0)$

3) nei punti x_0 dove f è continua ma non è derivabile

$f(x_0)$

$f(x,y)$ $\quad f: X \subseteq \mathbb{R}^2 \longrightarrow \mathbb{R}$

X compatto, f continua



$\exists \min_X f$, $\max_X f$ (per Weierstrass)

$f \in C^1(\mathring{X})$

1) Calcolare i punti critici di f, interni ad X

$$\nabla f(x,y) = \underline{0}$$

$$\Longleftrightarrow \begin{cases} f_x(x,y) = 0 \\ f_y(x,y) = 0 \end{cases}$$

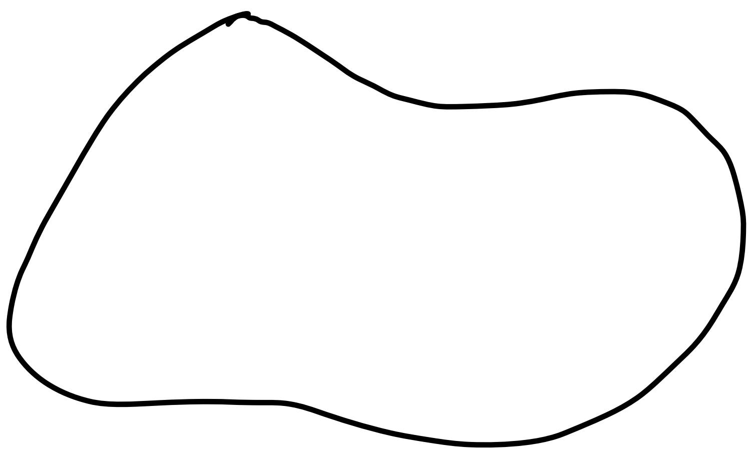
$f(x_0, y_0)$?

NON C'È BISOGNO DI CLASSIFICARE I PUNTI CRITICI (CIOÈ CALCOLARE

$Hf(x,y)$)

2) Studio di f su ∂X

$$\partial X \equiv \begin{cases} x = x(t) \\ y = y(t) \end{cases} \quad t \in [a,b]$$

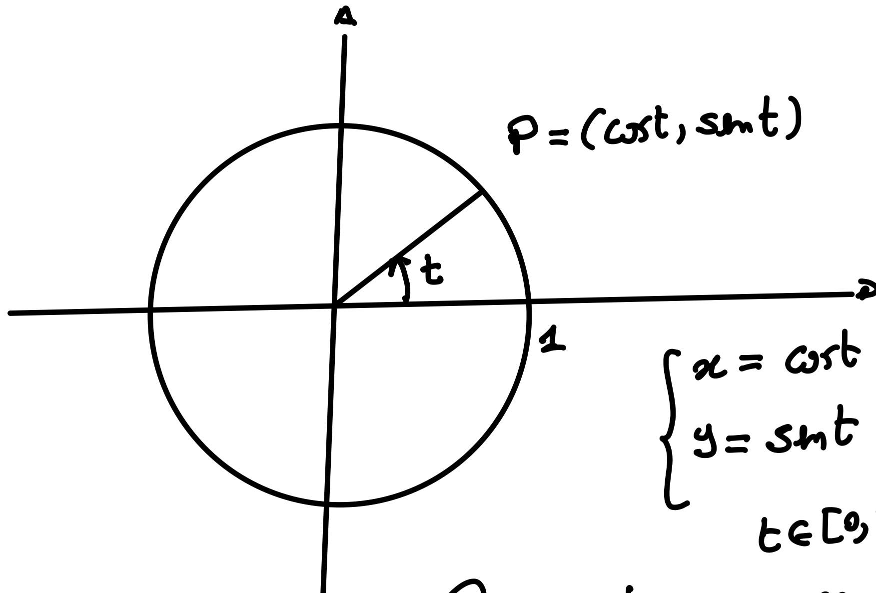


$g(t) = f(x(t), y(t)) \quad \forall\, t \in [a,b]$

$\downarrow$

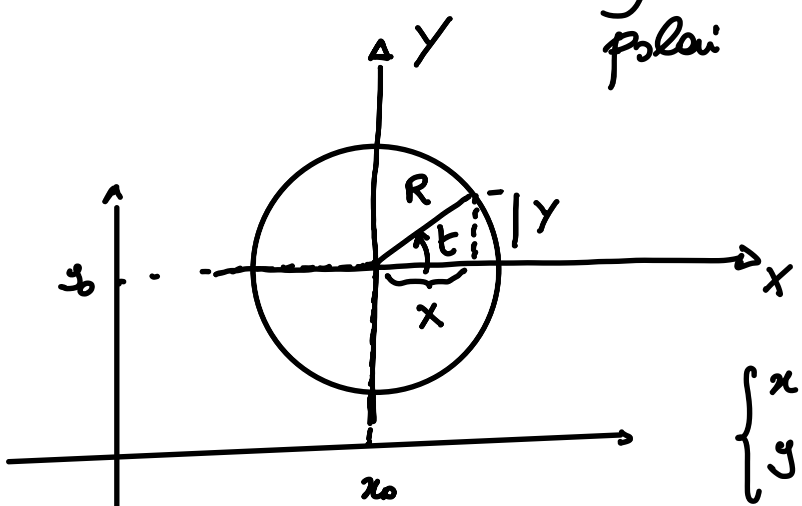
ricavare minimo e massimo

Circonferenza : $x^2+y^2=1$



$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad t \in [0, 2\pi]$$

Rappresentazione della circonferenza in coordinate polari



$$\begin{cases} x = X + x_0 \\ y = Y + y_0 \end{cases}$$

$X = R\cos t$, $Y = R\sin t$

$t \in [0, 2\pi]$

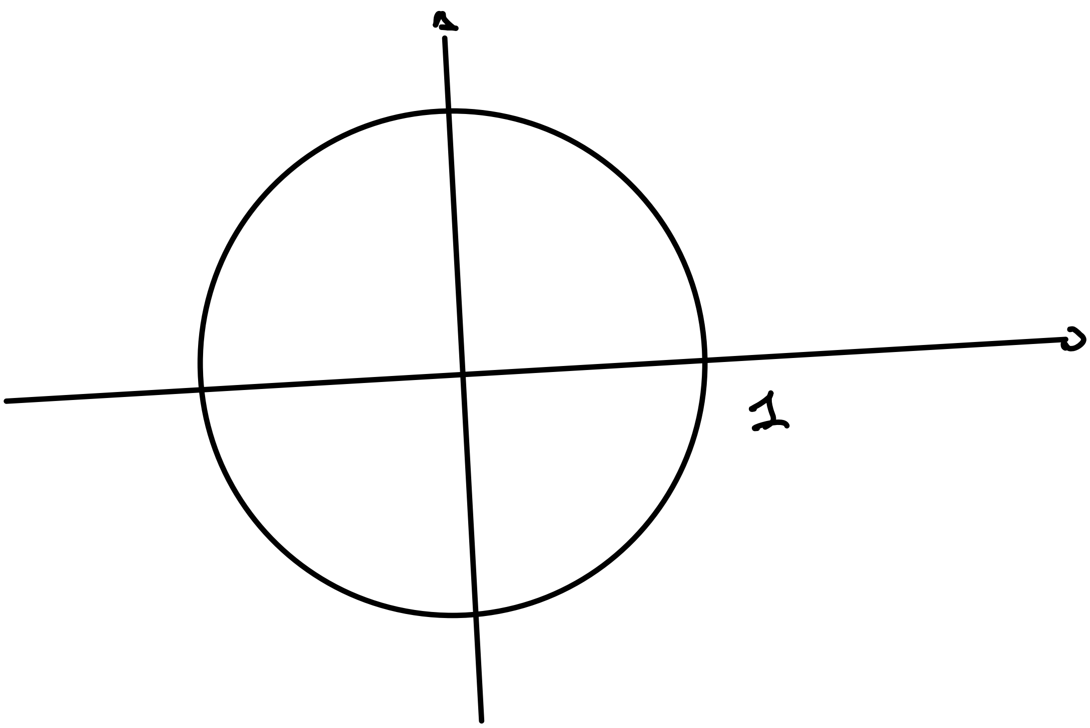
$$\begin{cases} x = x_0 + R\cos t \\ y = y_0 + R\sin t \end{cases}$$

ES.

$$f(x,y) = xy$$

minimo e massimo assoluto (estremi assoluti) in

$$C = \{(x,y) : x^2+y^2 \leq 1\}$$



$$f_x = y \quad , \quad f_y = x$$

$$\begin{cases} f_x = 0 \\ f_y = 0 \end{cases} \iff \begin{cases} y = 0 \\ x = 0 \end{cases}$$

$$\iff O = (0,0)$$

$$f(0,0) = 0$$

Studio di f su ∂C

$$x^2 + y^2 = 1 \qquad \begin{cases} x = \cos t \\ y = \sin t \end{cases} \quad t \in [0, 2\pi]$$

$$g(t) = f(x(t), y(t)) = f(\cos t, \sin t)$$

$$= \cos t \underbrace{\sin t}, \quad t \in [0, 2\pi]$$

$$g(t) = \frac{1}{2} \sin(2t)$$

$$\min_{t\in[0,2\pi]} g(t) = \frac{1}{2}(-1) = -\frac{1}{2}$$

$$\max_{t\in[0,2\pi]} g(t) = \frac{1}{2}\cdot 1 = \frac{1}{2}$$

$$-\frac{1}{2},\ 0,\ \frac{1}{2}$$

$$\min_{C} f = -\frac{1}{2},$$

$$\max_{C} f = 1/2$$

Minimi assoluti: si hanno quando

$$\sin(2t) = -1$$

$$\iff \quad 2t = \frac{3}{2}\pi + 2k\pi$$

$$\iff \quad t = \frac{3}{4}\pi + k\pi, \quad k \in \mathbb{Z}$$

$[0, 2\pi]$

$k = -1$: $\quad t = \frac{3}{4}\pi - \pi = -\frac{\pi}{4} \notin [0, 2\pi]$

$k = 0$ $\quad t_0 = \frac{3}{4}\pi$

$k = 1$: $\quad t_1 = \frac{3}{4}\pi + \pi = \frac{7}{4}\pi$

$$\begin{cases} x = \cos \frac{3}{4}\pi = -\frac{\sqrt{2}}{2} \\ y = \sin \frac{3}{4}\pi = \frac{\sqrt{2}}{2} \end{cases}$$

$A = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ p.to di minimo assoluto

$$\begin{cases} x = \cos \frac{7}{4}\pi = \frac{\sqrt{2}}{2} \\ y = \sin \frac{7}{4}\pi = -\frac{\sqrt{2}}{2} \end{cases}$$

$B = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ p.to di min. assoluto

Massimi : $\sin(2t) = 1$

$$2t = \frac{\pi}{2} + 2k\pi$$

$$t = \frac{\pi}{4} + k\pi$$

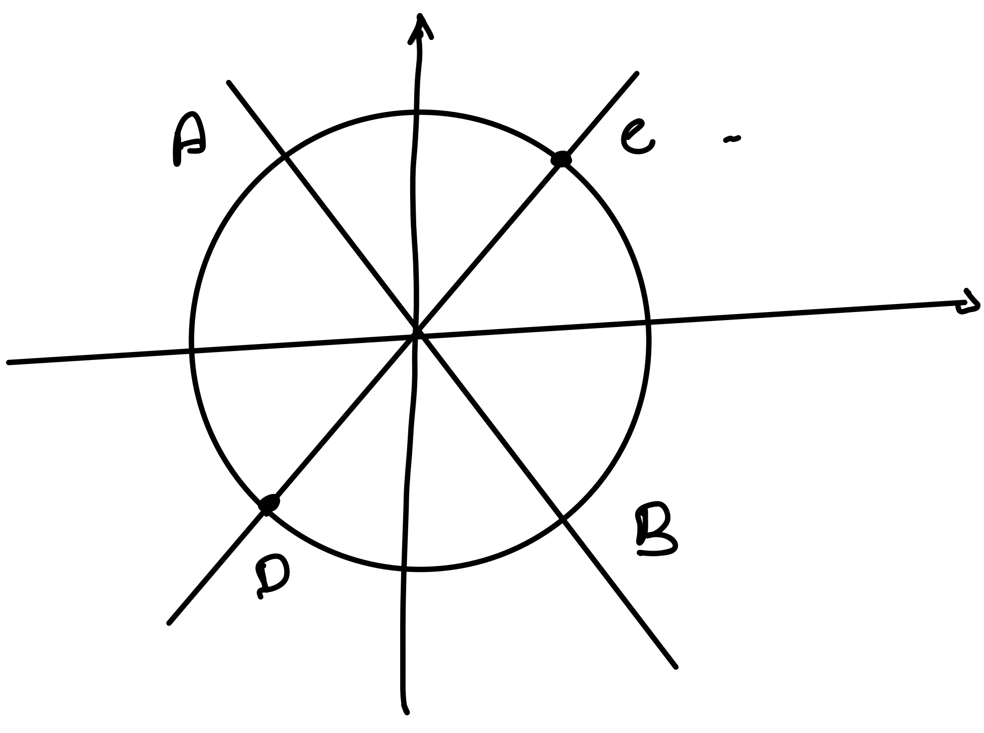
$t_2 = \frac{\pi}{4}$

$t_3 = \frac{\pi}{4} + \pi = \frac{5}{4}\pi$

$C = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right)$

$D = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$

p.ti di max assoluto

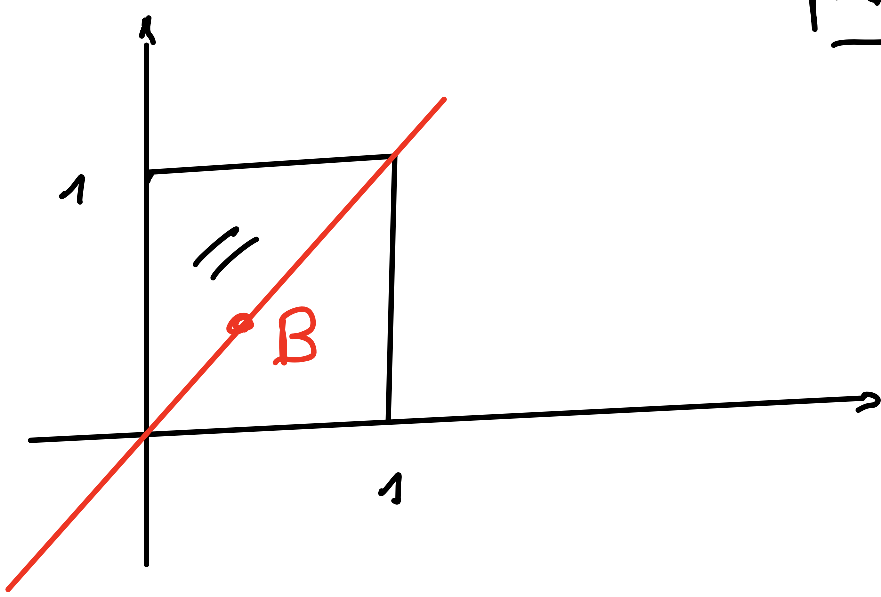


Estremi assoluti di

$$f(x,y) = 2xy - y^2 - x^4$$

nel quadrato $Q = [0,1] \times [0,1]$

prodotto cartesiano



$$f_x = 2y - 4x^3 \qquad f_y = 2x - 2y$$

$$\begin{cases} 2y - 4x^3 = 0 \\ 2x - 2y = 0 \end{cases}$$

$$\begin{cases} y - 2x^3 = 0 \\ y = x \end{cases} \iff \begin{cases} x - 2x^3 = 0 \\ y = x \end{cases}$$

$$\begin{cases} x(2x^2 - 1) = 0 \\ y = x \end{cases} \iff \begin{cases} x = 0 \\ y = 0 \end{cases}$$

$$\begin{cases} 2x^2 - 1 = 0 \\ y = x \end{cases} \quad \begin{cases} x = -\frac{\sqrt{2}}{2} \\ y = -\frac{\sqrt{2}}{2} \end{cases} \quad \begin{cases} x = +\frac{\sqrt{2}}{2} \\ y = +\frac{\sqrt{2}}{2} \end{cases}$$

$O = (0, 0)$

$\cancel{A = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)}$

$B = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

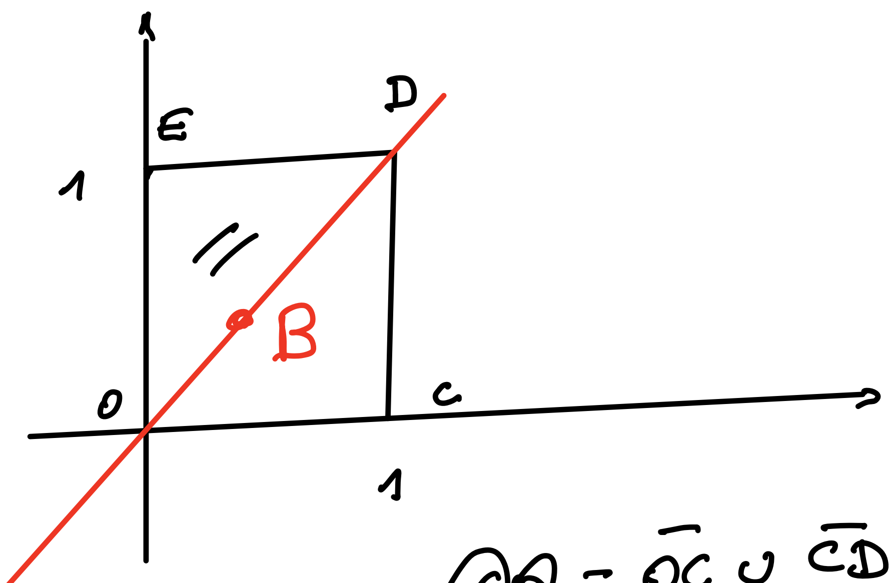
$A \notin Q$

$B \in Q$

$f(0,0) = 0$

$f\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \dots \quad \frac{1}{4}$

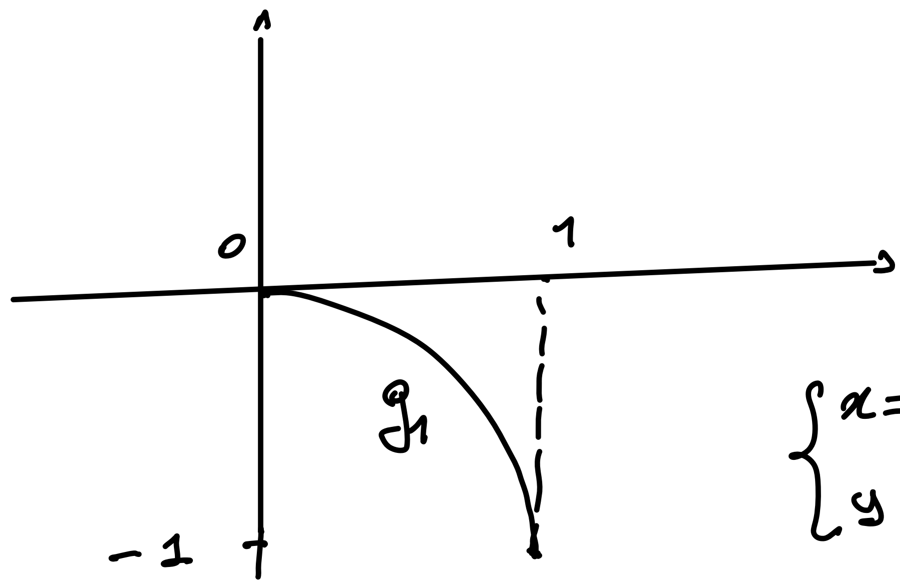
Studio di f lungo ∂Q



$\partial Q = \overline{OC} \cup \overline{CD} \cup \overline{DE} \cup \overline{OE}$

<u>Su $\overline{OC}$</u> : $y = 0$, $0 \le x \le 1$

$$g_1(x) = f(x, 0) = -x^4, \qquad 0 \le x \le 1$$



$$\begin{cases} x = 1 \\ y = 0 \end{cases}$$

$$\min_{[0,1]} g_1 = -1, \qquad \max_{[0,1]} g_1 = 0$$

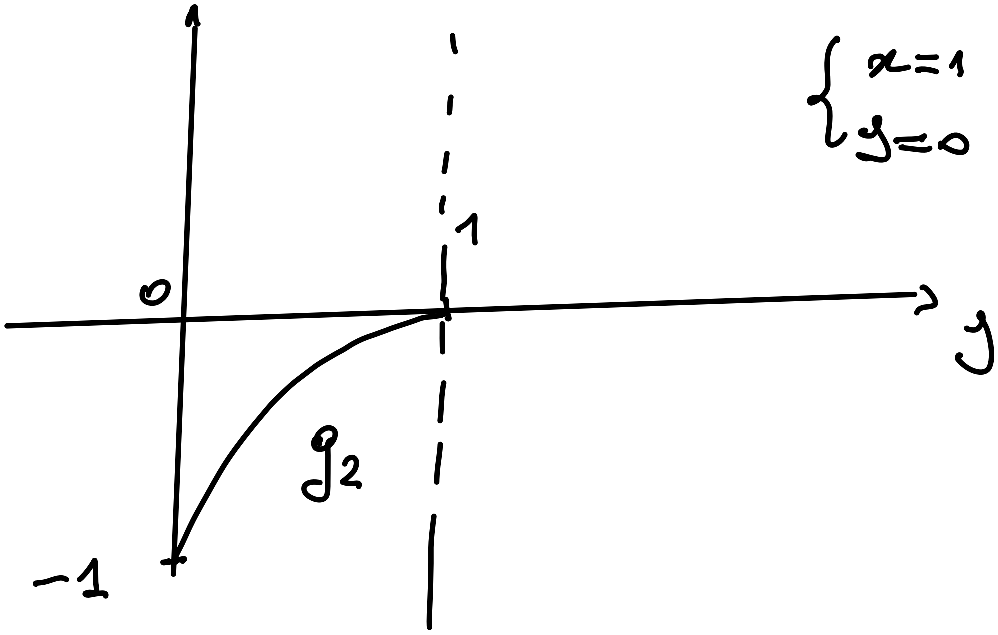
Su $\overline{CD}$: $\quad x = 1, \qquad 0 \le y \le 1$

$$g_2(y) = f(\underset{x}{\overset{\shortparallel}{1}}, y) = 2y - y^2 - 1 \qquad \forall y \in [0,1]$$

$$g_2(0) = -1, \qquad g_2(1) = 0$$

$$g_2'(y) = 2 - 2y \ge 0 \iff y \le 1$$

$\begin{cases} x=1 \\ y=0 \end{cases}$



$\min\limits_{[0,1]} g_2 = -1$, $\max\limits_{[0,1]} g_2 = 0$

Su $\overline{DE}$: $y = 1$, $0 \le x \le 1$

$$g_3(x) = f(x, 1) =$$

$$= 2x - x^4 - 1$$

$\forall x \in [0, 1]$

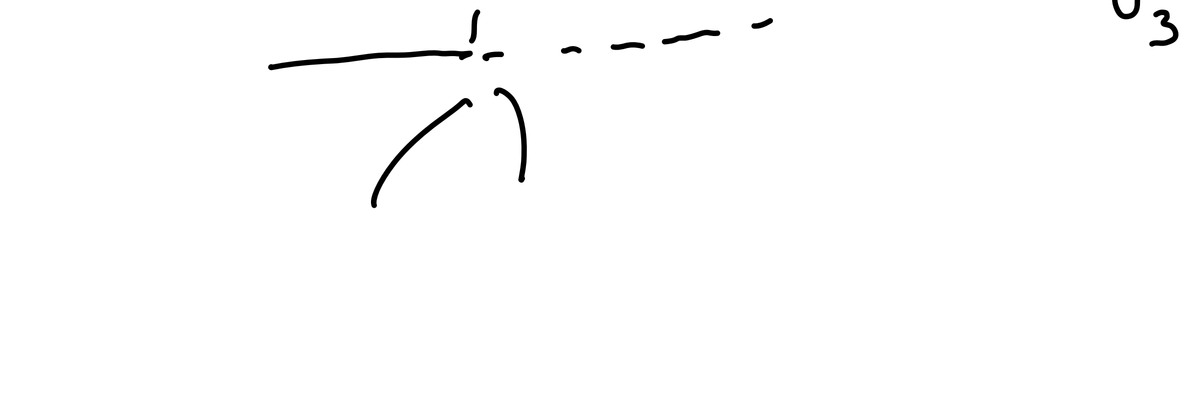
$g_3(0) = -1, \quad g_3(1) = 0$

$g_3'(x) = 2 - 4x^3 \geq 0 \iff$

$\iff 4x^3 - 2 \leq 0$

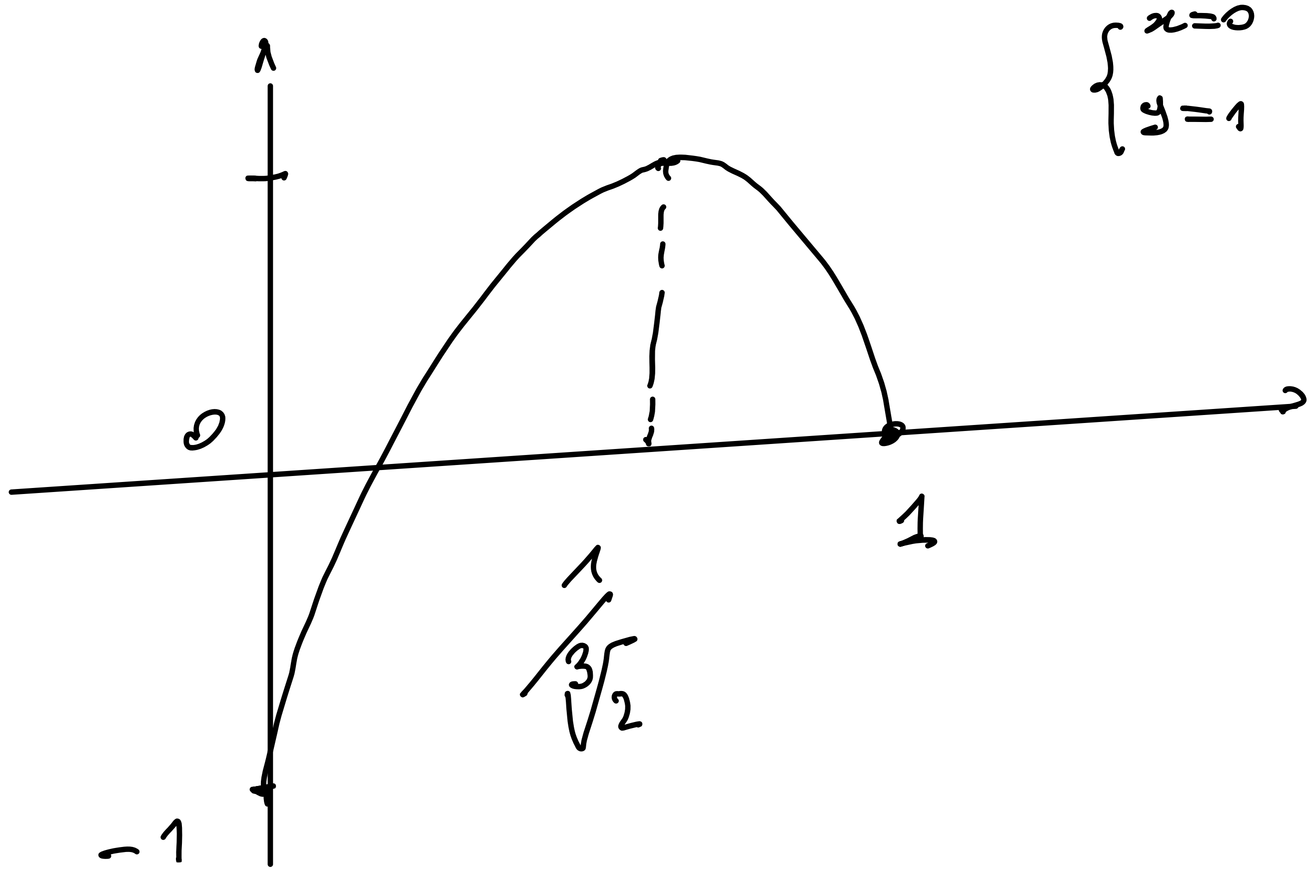
$\iff 2x^3 - 1 \leq 0$

$\iff x \leq \frac{1}{\sqrt[3]{2}}$



$x_0 = \frac{1}{\sqrt[3]{2}}$ punto di max. assoluto

$$g_3\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{3}{2\sqrt[3]{2}} - 1 > 0$$



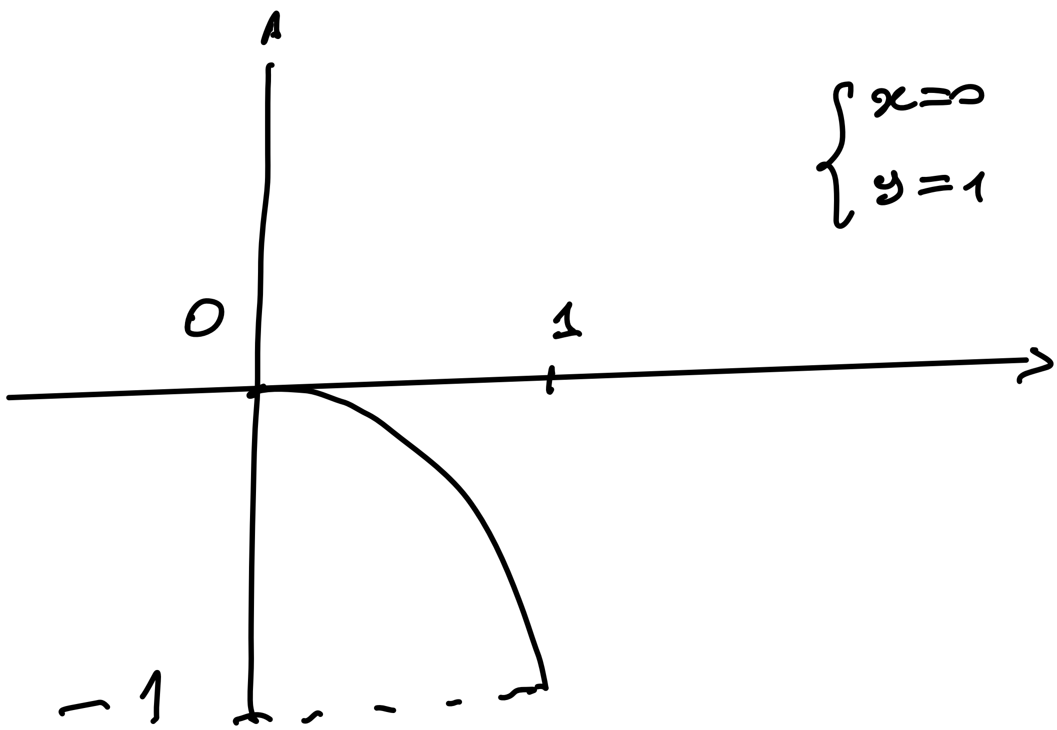
$\min\limits_{[0,1]} g_3 = -1, \quad \max\limits_{[0,1]} g_3 =$

$$= \frac{3}{2\sqrt[3]{2}} - 1$$

Su $\overline{OE}$: $x = 0$, $0 \le y \le 1$

$$g_4(y) = f(0, y) = -y^2$$

$$0 \le y \le 1$$



$$\begin{cases} x = 0 \\ y = 1 \end{cases}$$

$\min_{[0,1]} g_4 = -1$, $\max_{[0,1]} g_4 = 0$

-1 $\quad$ 0 , $\frac{3}{2\sqrt[3]{2}} - 1$, $\frac{1}{4}$

$\underbrace{\min_{Q} f = -1}$, $\underbrace{\max_{Q} f = \frac{1}{4}}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ pto di max. assoluto

$C = (1,0)$ minimo assoluto

$E = (0,1)$ " "

ES. $f(x,y) = x^2(2y-1) - 4y^2$

Estremi assoluti

$$T = \{ (x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1, \quad x \leq y \leq 1 \}$$