Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea "Triennale" - Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

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Phasors and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\hat{i}_x + f_y(x,y,z,t)\hat{i}_y + f_z(x,y,z,t,)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z) = F_x\,(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z\,(x,y,z)\hat{i}_z$$

$$f_x(x,y,z,t) = A_x\left(x,y,z\right)\cos\left(\omega_0\,t + \alpha_x\left(x,y,z\right)\right) \longrightarrow F_x\left(x,y,x\right) = A_x\left(x,y,z\right)e^{j\alpha_x(x,y,z)}$$

$$f_y(x,y,z,t) = A_y\left(x,y,z\right)\cos\left(\omega_0\,t + \alpha_y\left(x,y,z\right)\right) \longrightarrow F_y\left(x,y,z\right) = A_y\left(x,y,z\right)e^{j\alpha_y(x,y,z)}$$

$$f_z(x,y,z,t) = A_z\left(x,y,z\right)\cos\left(\omega_0\,t + \alpha_z\left(x,y,z\right)\right) \longrightarrow F_z\left(x,y,x\right) = A_z\left(x,y,z\right)e^{j\alpha_z(x,y,z)}$$

Phasors and vector functions of *n* variables

Time domain

$$\vec{\mathbf{f}}(x,y,z,t) = f_x(x,y,z,t)\,\hat{i}_x + f_y(x,y,z,t)\,\hat{i}_y + f_z(x,y,z,t,)\,\hat{i}_z$$

Phasor domain

$$\vec{\mathbf{F}}(x,y,z) = F_x(x,y,z)\hat{i}_x + F_y(x,y,z)\hat{i}_y + F_z(x,y,z)\hat{i}_z$$

$\vec{\mathbf{f}}(x,y,z,t)$ → **T-to-P** → $\vec{\mathbf{F}}(x,y,z)$

$\vec{\mathbf{F}}(x,y,z)$ → **P-to-T** → $\vec{\mathbf{f}}(x,y,z,t) = \mathrm{Re}\left\{\vec{\mathbf{F}}(x,y,z)e^{j\omega_0 t}\right\}$

$\dfrac{\partial \vec{\mathbf{f}}}{\partial t}$ → **T-to-P** → $j\omega_0\vec{\mathbf{F}}(x,y,z)$

$\nabla \cdot \vec{\mathbf{f}}$ → **T-to-P** → $\nabla \cdot \vec{\mathbf{F}}(x,y,z)$

$\nabla \times \vec{\mathbf{f}}$ → **T-to-P** → $\nabla \times \vec{\mathbf{F}}(x,y,z)$

Maxwell equations

Time domain	Frequency domain	Phasor domain
$\begin{cases} \nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla\cdot\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$	$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$	$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$
$\dfrac{\partial\rho(\vec{\mathbf{r}},t)}{\partial t} + \nabla\cdot\vec{\mathbf{j}}(\vec{\mathbf{r}},t) = 0$	$j\omega\rho(\vec{\mathbf{r}},\omega) + \nabla\cdot\vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) = 0$	$j\omega_0\rho(\vec{\mathbf{r}}) + \nabla\cdot\vec{\mathbf{J}}(\vec{\mathbf{r}}) = 0$

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

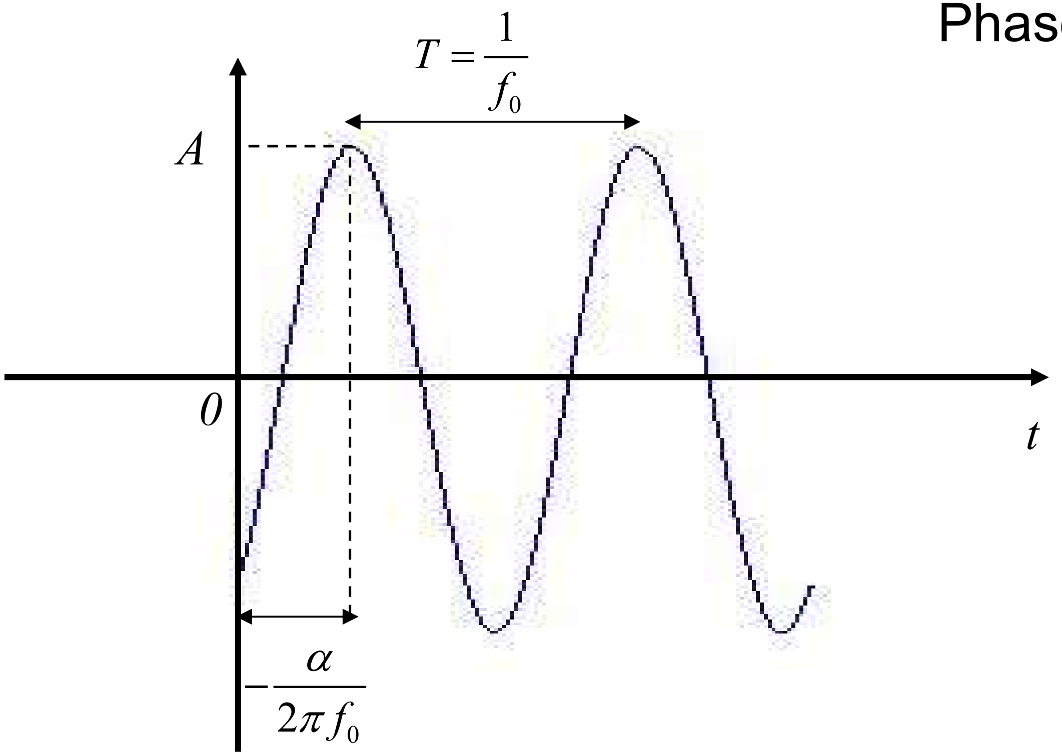
Mathematical tools to be exploited

Mathematics

Phasors

$$v(t) = A\cos\left(2\pi f_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



Phasors

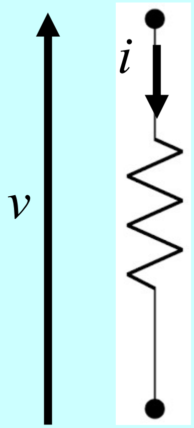
$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

Phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$



$$v(t) = R \cdot i(t)$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = R \cdot I$$

$$V = Z \cdot I$$

$$Z = R$$

Phasors

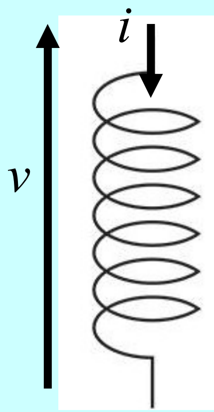
$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 A e^{j\alpha}$$

Phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 A e^{j\alpha}$$



$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = j\omega_0 LI$$

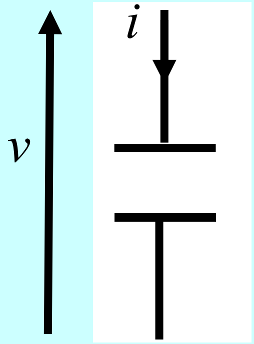
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

Phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 A e^{j\alpha}$$



$$i(t) = C\frac{dv(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$I = j\omega_0 C V$$

$$V = Z \cdot I$$

$$Z = -j\frac{1}{\omega_0 C}$$

Phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B\cos\left(\omega_0 t + \beta\right) \longrightarrow I = Be^{j\beta}$$

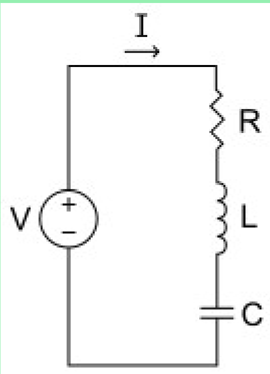
$$\left\langle v(t)\cdot i(t)\right\rangle = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt \; = \frac{1}{2}\mathrm{Re}\left\{V\cdot I^*\right\}$$

Phasors

$$v(t) = A\cos\left(\omega_0 t + \alpha\right) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B\cos\left(\omega_0 t + \beta\right) \longrightarrow I = Be^{j\beta}$$

$$\left\langle v(t)\cdot i(t)\right\rangle = \frac{1}{T}\int_0^T v(t)\cdot i(t)\,dt \;\; = \frac{1}{2}\mathrm{Re}\left\{V\cdot I^*\right\}$$



$$P = \frac{1}{2}V\cdot I^* = P_1 + jP_2$$

$$P = \frac{1}{2}V\cdot I^* = \frac{1}{2}\left(Z_R + Z_L + Z_C\right)I\cdot I^* = \frac{1}{2}\left(R + j\omega_0 L - \frac{j}{\omega_0 C}\right)\left|I\right|^2$$

$$P_1 = \frac{1}{2}R\left|I\right|^2 \;\; ; \quad P_2 = \frac{1}{2}\left(\omega_0 L - \frac{1}{\omega_0 C}\right)\left|I\right|^2$$

Phasors and vector functions of *n* variables

$$\vec{\mathbf{f}}_1(x,y,z,t) \longrightarrow \vec{\mathbf{F}}_1(x,y,z)$$

$$\vec{\mathbf{f}}_2(x,y,z,t) \longrightarrow \vec{\mathbf{F}}_2(x,y,z)$$

Phasors and vector functions of *n* variables

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t) \right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\cdot\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t) \right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\times\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

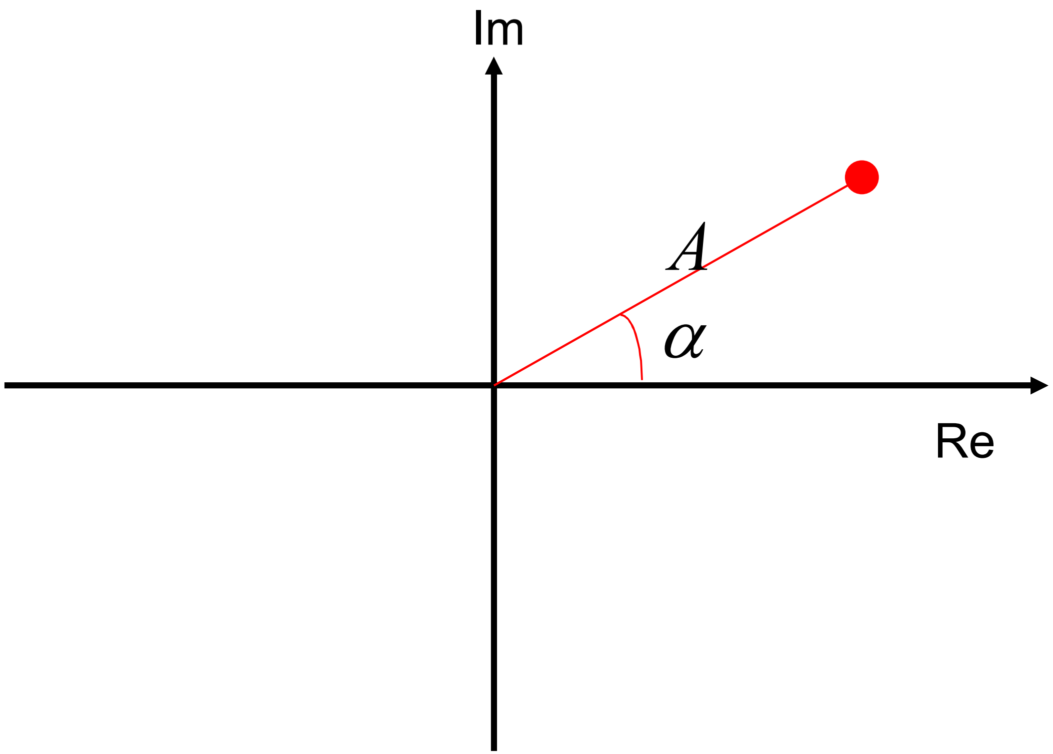
Complex numbers

&

Complex vectors

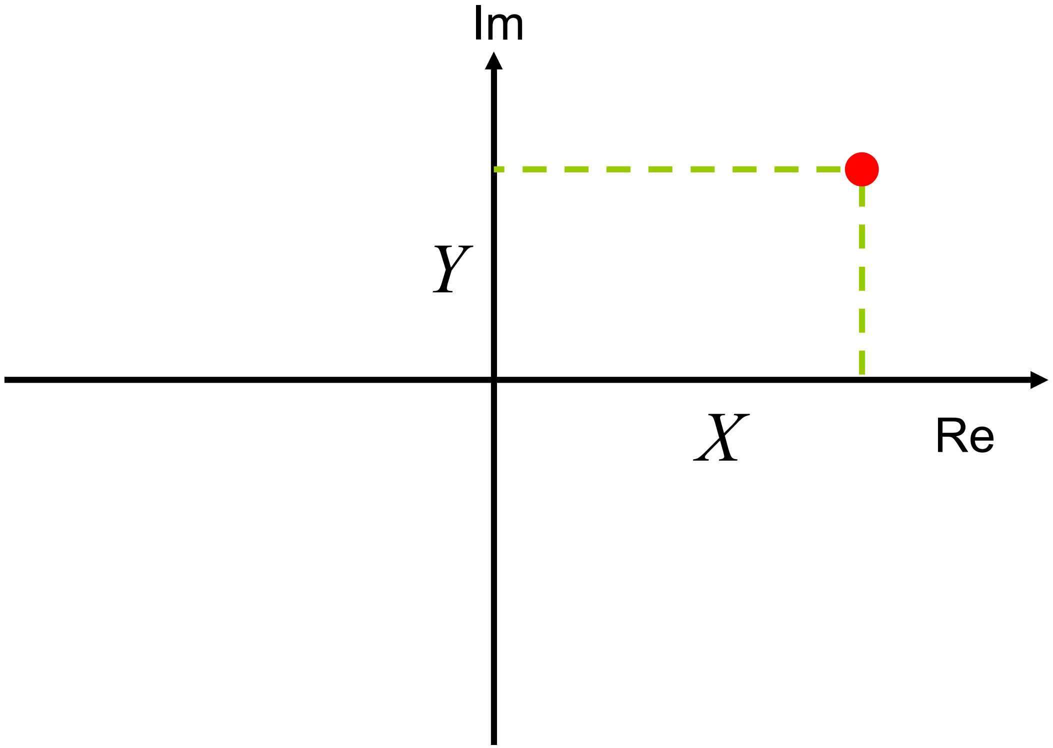
Complex numbers

$$Ae^{j\alpha}$$



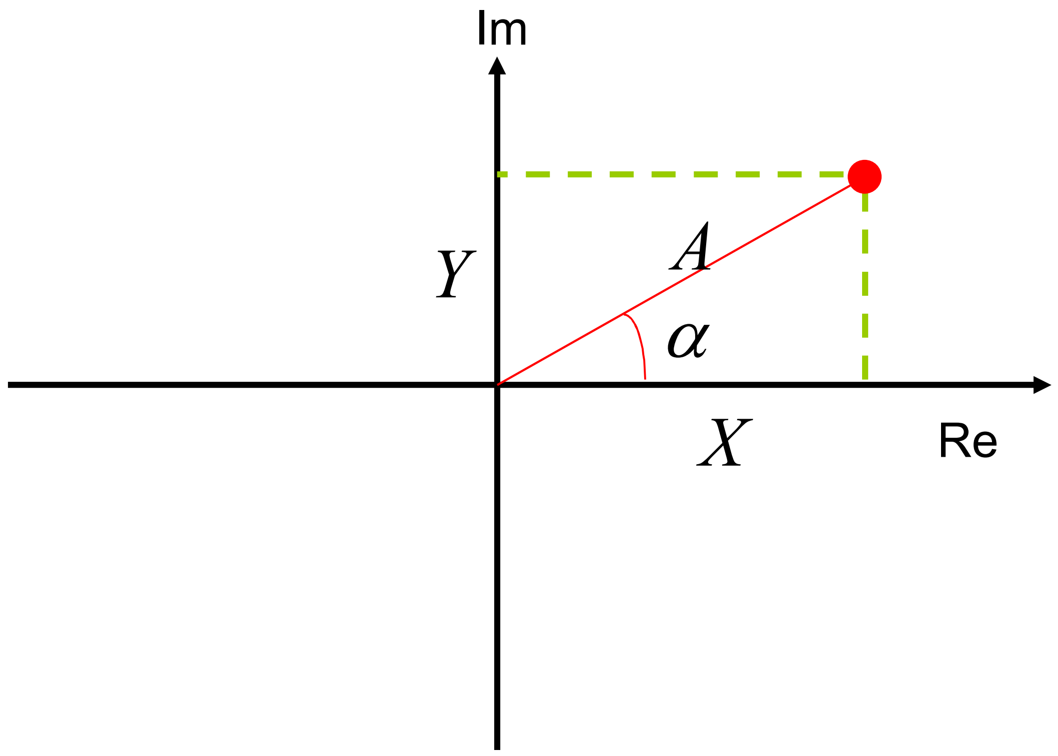
Complex numbers

$$Ae^{j\alpha} = X + jY$$



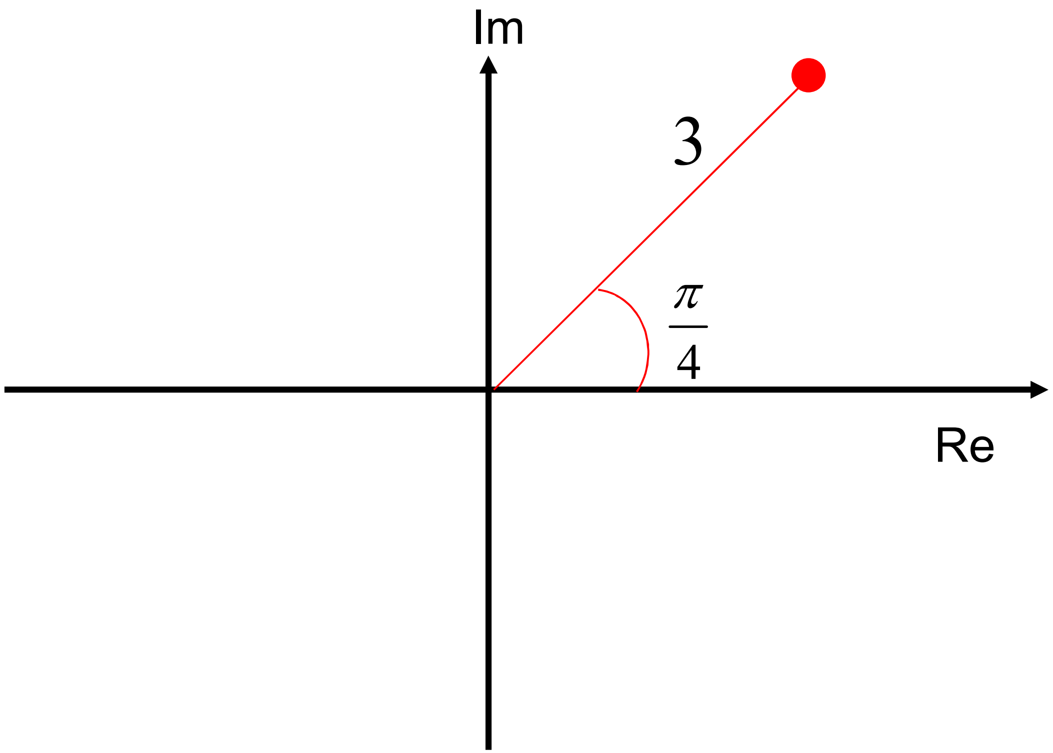
Complex numbers

$$Ae^{j\alpha} = X + jY$$



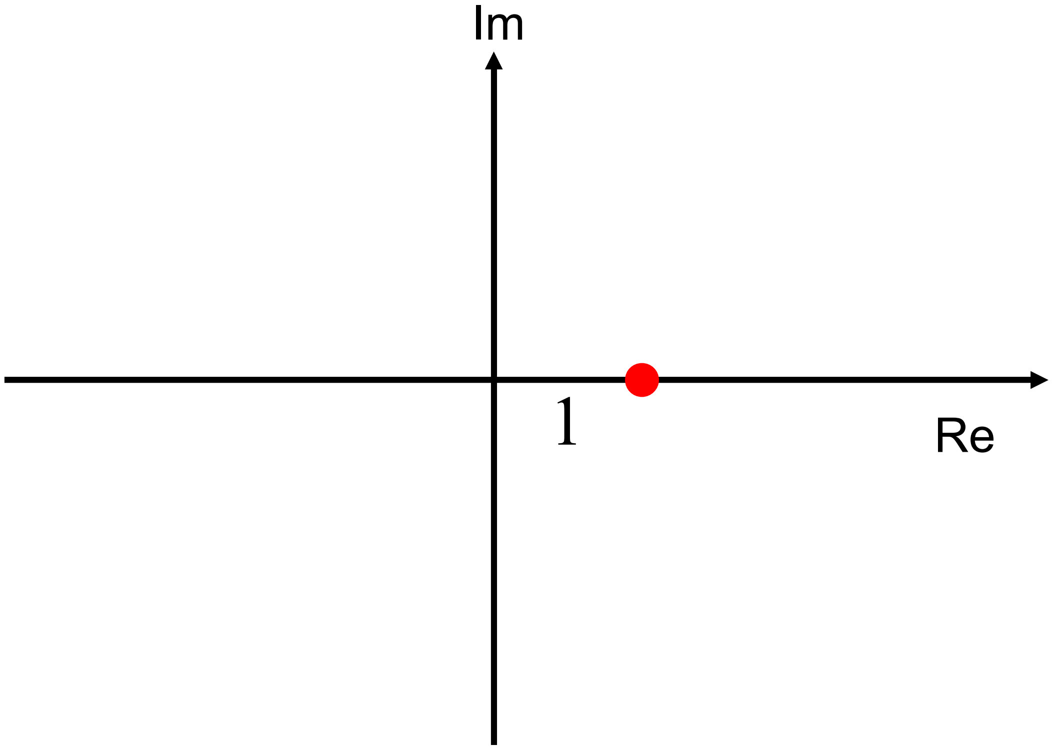
Complex numbers: some examples

$$3e^{j\frac{\pi}{4}}$$



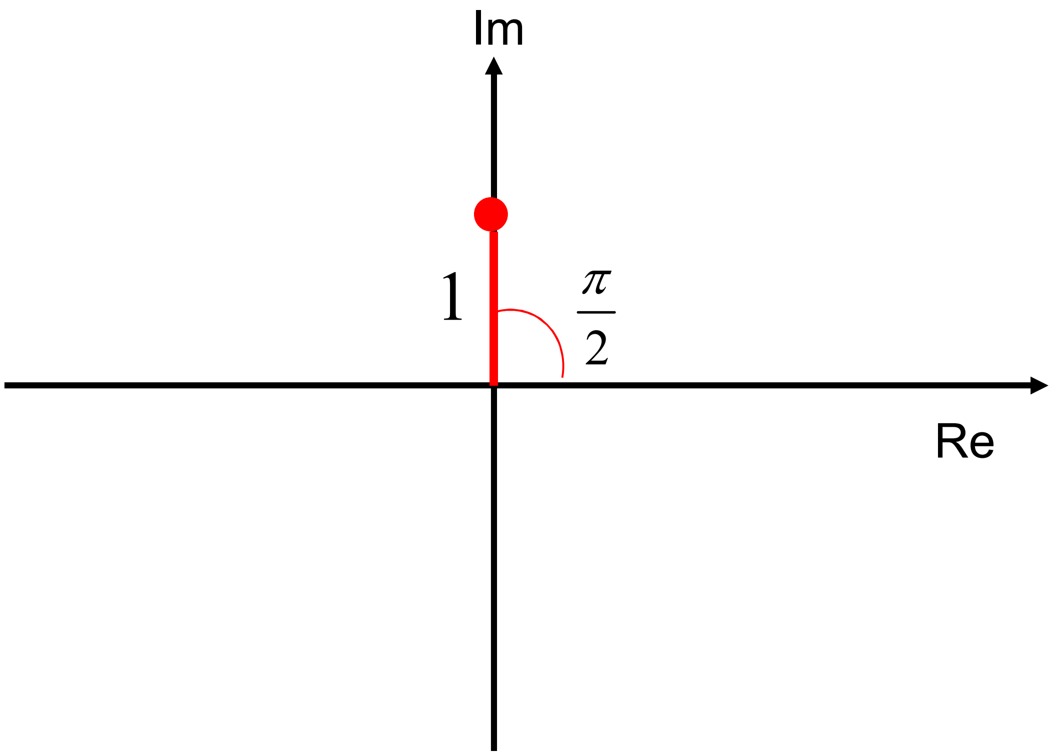
Complex numbers: some examples

$$1$$



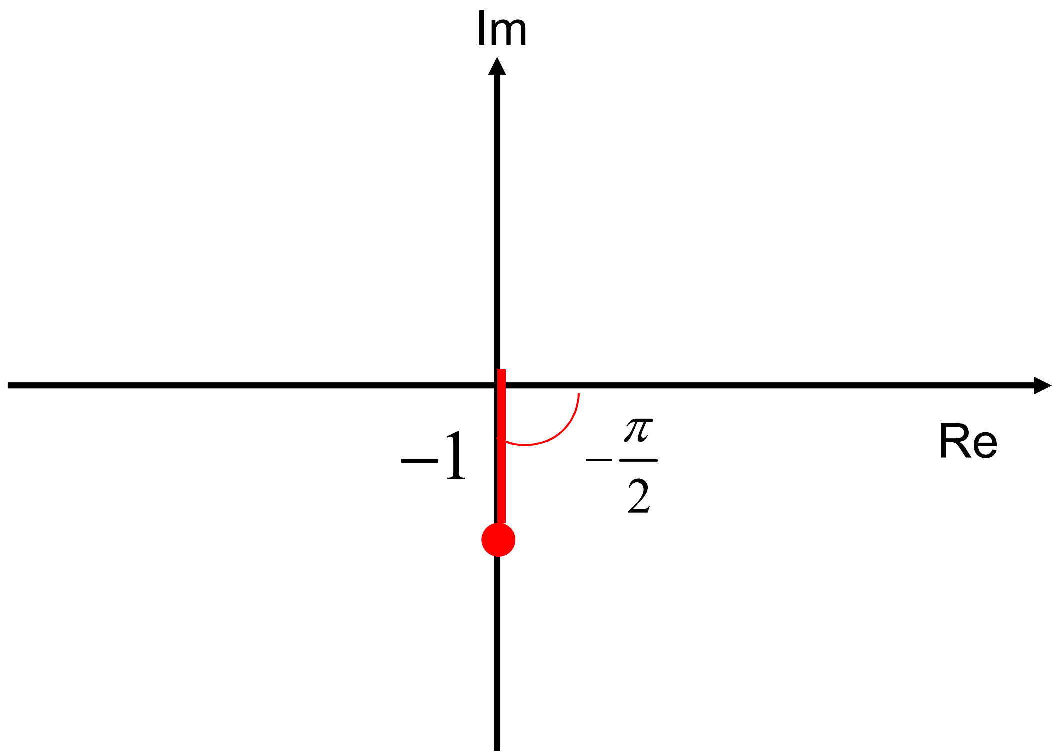
Complex numbers: some examples

$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2}+2\pi\right)}$$



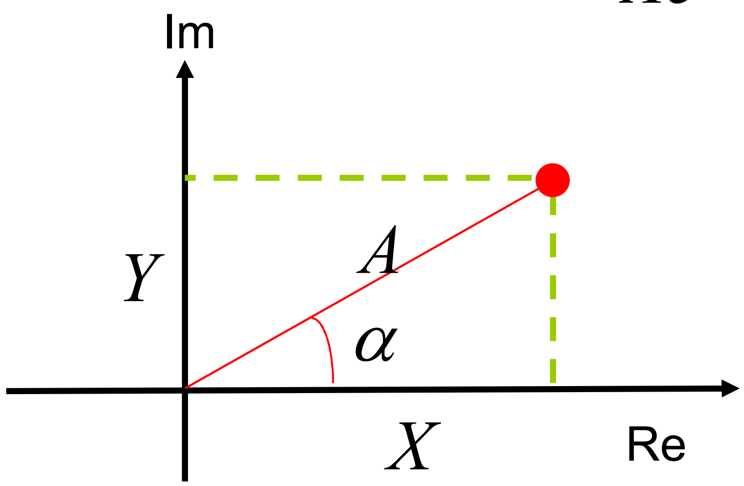
Complex numbers: some examples

$$-j = e^{-j\frac{\pi}{2}}$$



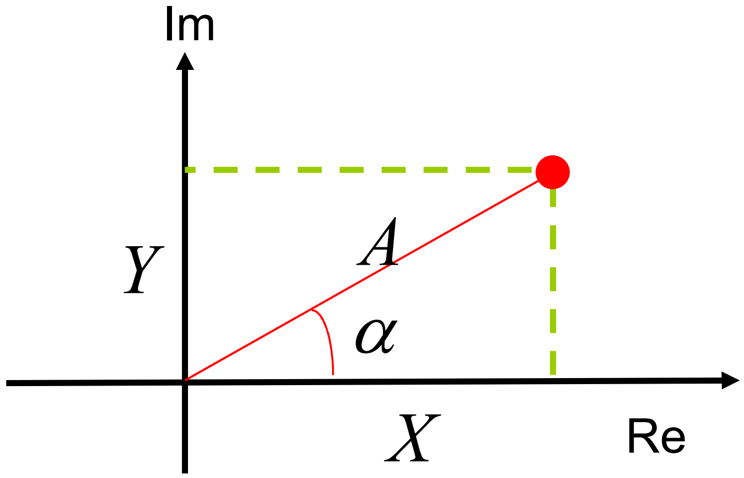
Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



Complex numbers: conversion formulas

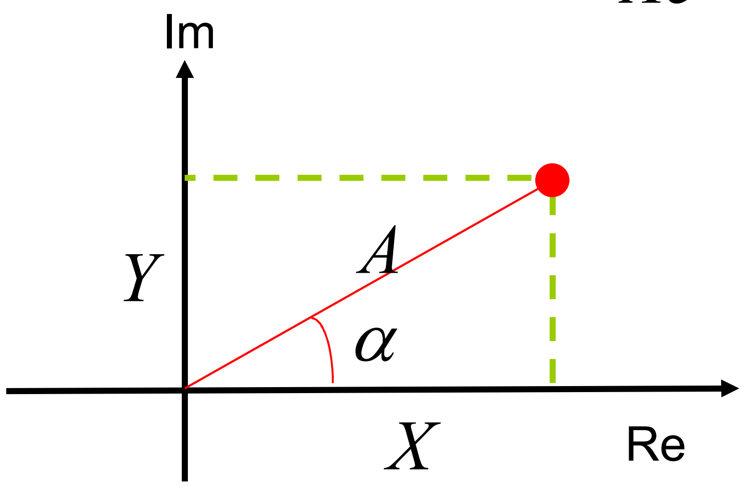
$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



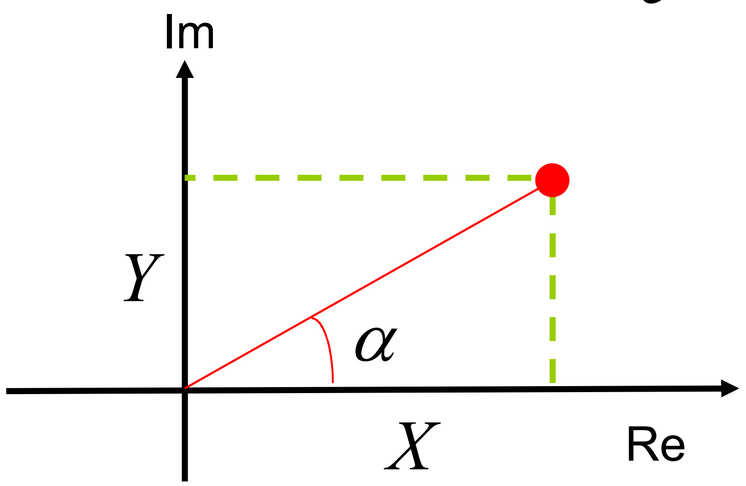
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

Complex numbers: conversion formulas

$$e^{j\alpha} = \cos\alpha + j\sin\alpha$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

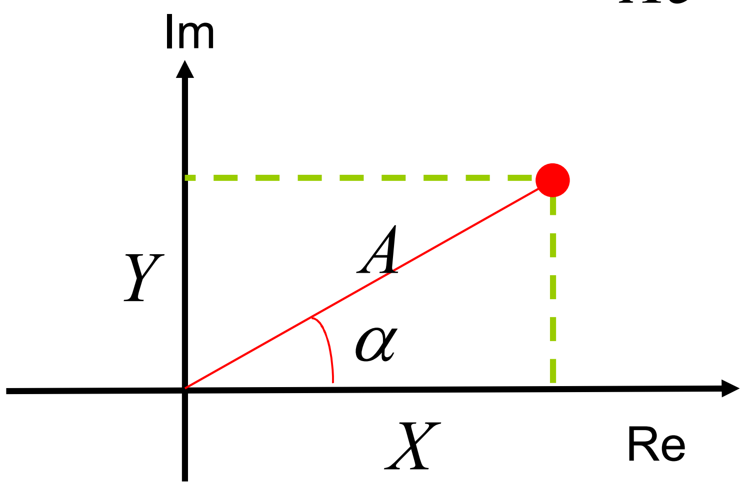
Eulero's formulas

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



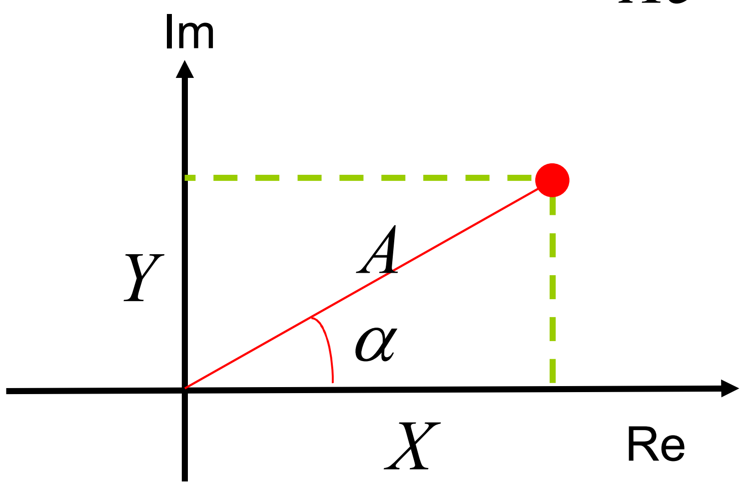
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

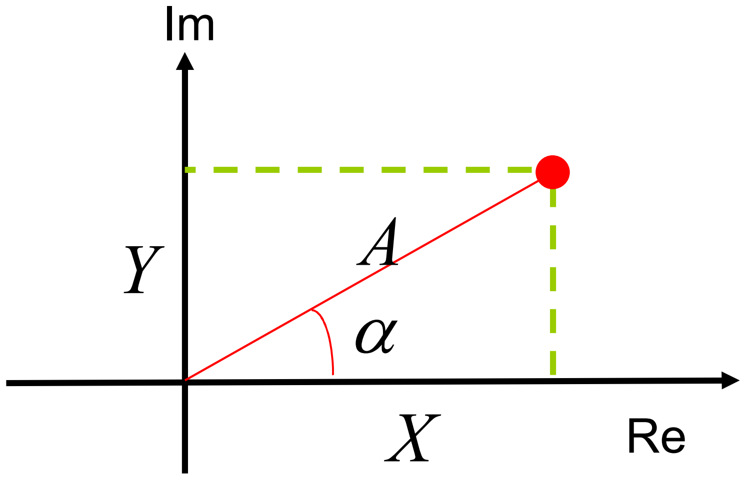
$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

$$X + jY \longrightarrow Ae^{j\alpha}$$

Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

$$X + jY \longrightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \text{acos}\left(\dfrac{X}{A}\right) & se\ Y \geq 0 \\ \alpha = -\text{acos}\left(\dfrac{X}{A}\right) & se\ Y < 0 \end{cases}$$

Complex numbers: conversion formulas

Some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A\cos\alpha$$

$$Y = A\sin\alpha$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

Complex numbers: conversion formulas

Some examples

$$X + jY \longrightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \operatorname{acos}\left(\dfrac{X}{A}\right) \ \ se\ Y \geq 0 \\ \alpha = -\operatorname{acos}\left(\dfrac{X}{A}\right) \ \ se\ Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$

Complex numbers: summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = \left(X_1 + X_2\right) + j\left(Y_1 + Y_2\right)$$

Complex numbers: product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = \left(X_1 \cdot X_2 - Y_1 \cdot Y_2\right) + j\left(X_1 \cdot Y_2 + Y_1 \cdot X_2\right)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j\left(\alpha_1 + \alpha_2\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6}+\frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = \sqrt{3} + j \quad = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} \quad = 2e^{j\frac{3}{4}\pi}$$

$$z_1 \cdot z_2 = \quad 4e^{j\left(\frac{\pi}{6}+\frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

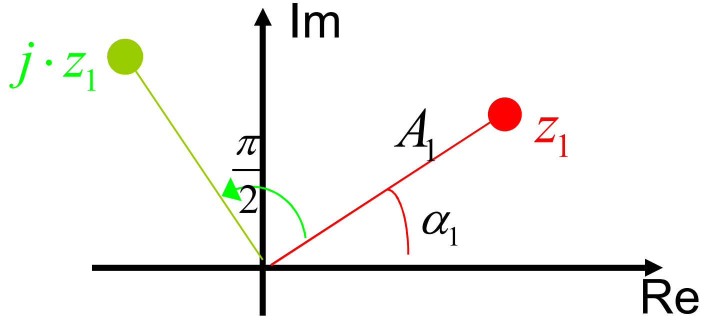
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1+\alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1 + \frac{\pi}{2}\right)}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

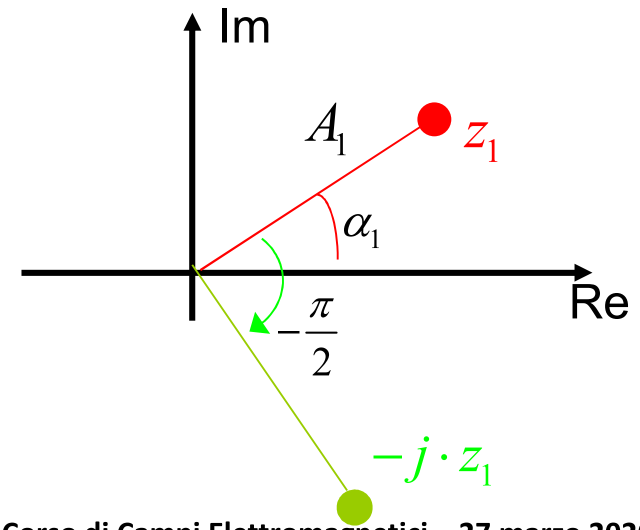
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$



Complex numbers: product

Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-z_1 = A_1 e^{j(\alpha_1 + \pi)}$$

$$z_2 = -1 = e^{j\pi}$$