

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica, Biomedica e delle
Telecomunicazioni**

a.a. 2019–2020 – Laurea “Triennale” – Secondo semestre – Secondo anno

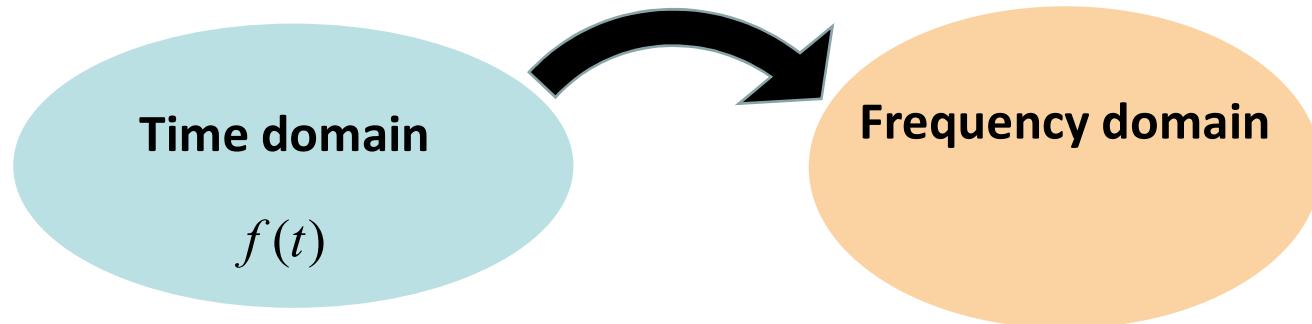
Università degli Studi di Napoli “Parthenope”

Stefano Perna

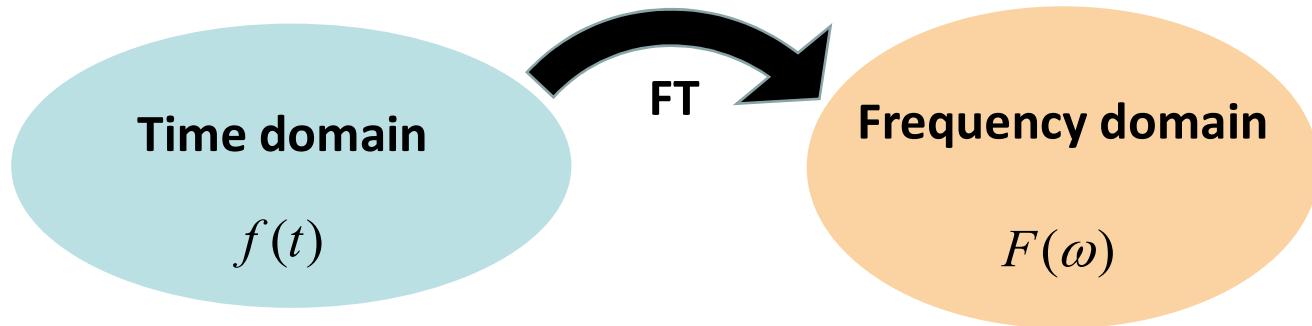
Maxwell equations: Time domain, Frequency domain, Phasors



Frequency domain



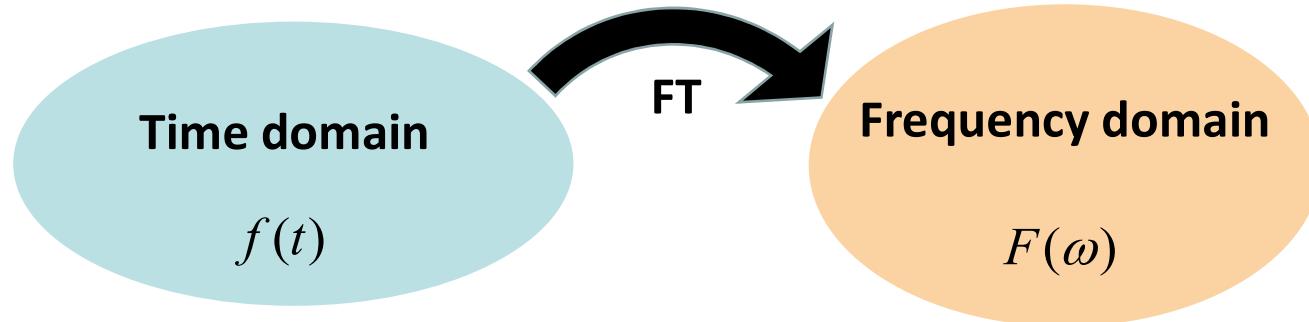
Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

Frequency domain

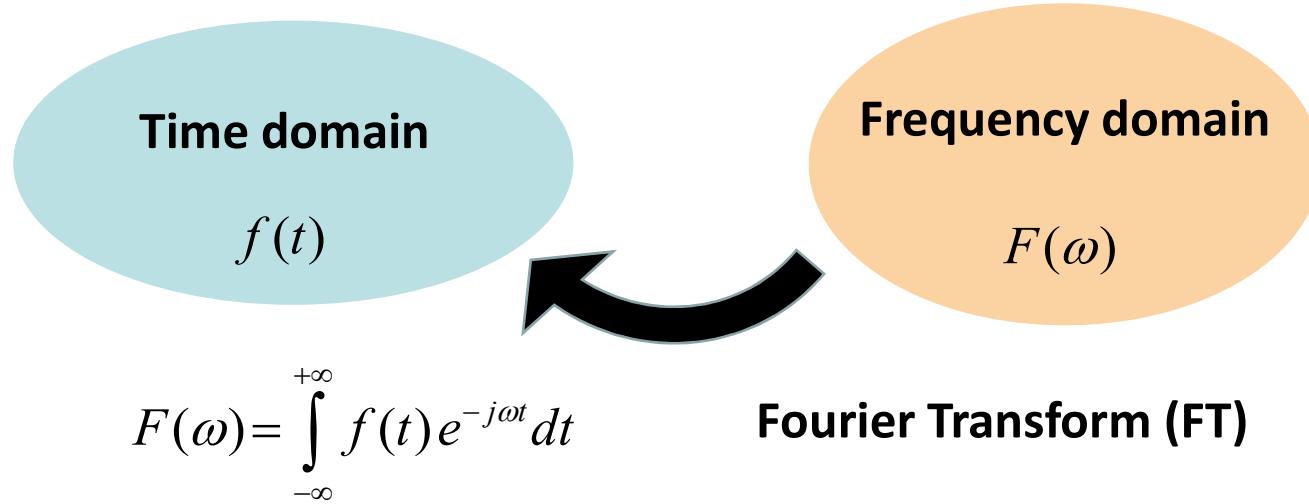


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Fourier Transform (FT)

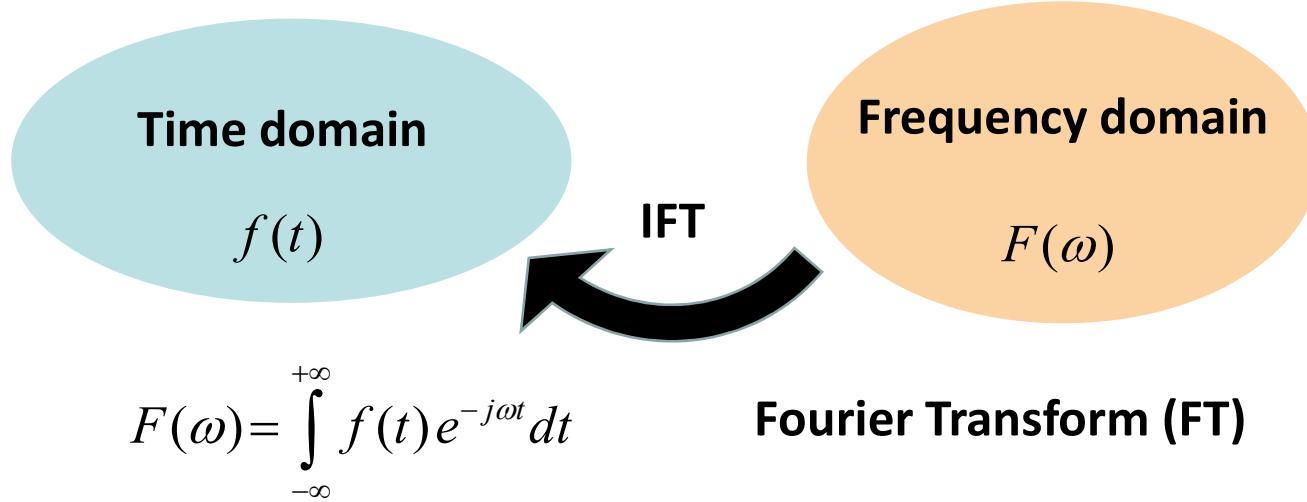
- 1) How to jump back from the Frequency domain to the Time domain**
- 2) Time domain derivative and Fourier Transform**

Frequency domain



1) How to jump back from the Spectral domain to the Time domain

Frequency domain

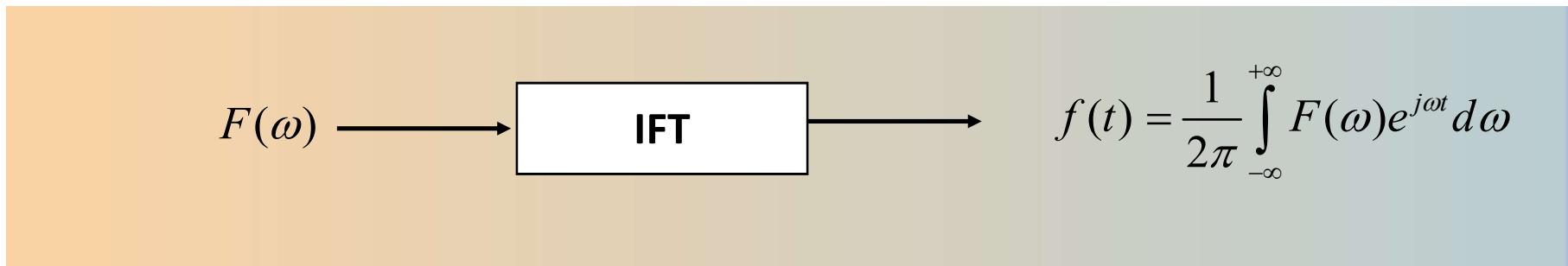
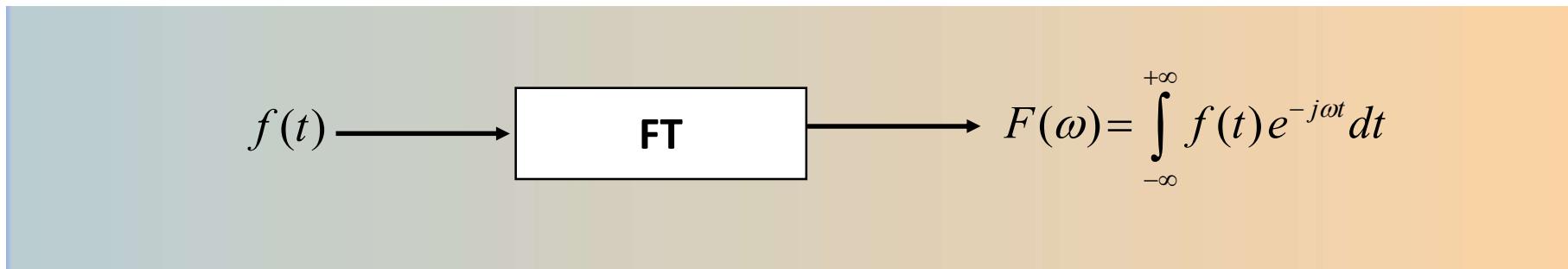
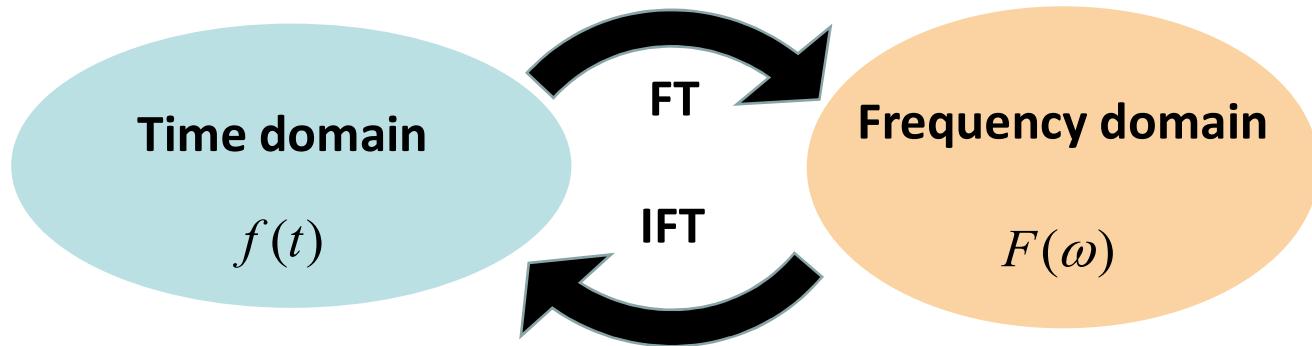


1) How to jump back from the Spectral domain to the Time domain

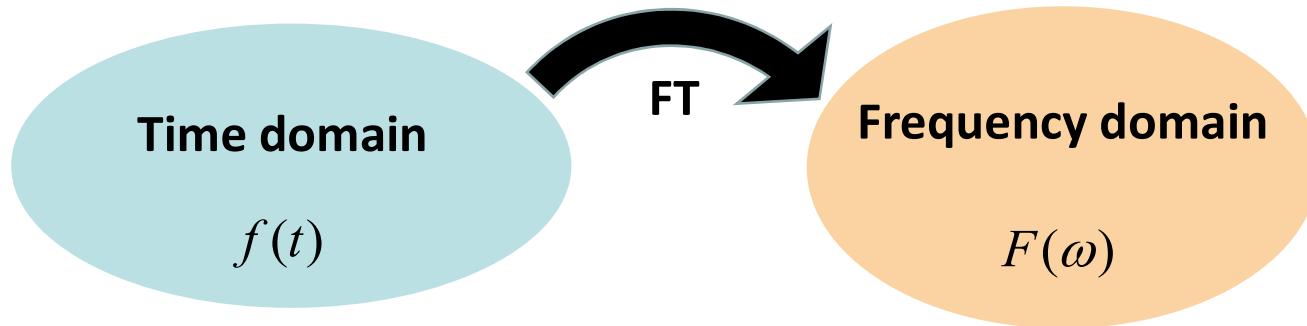
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega$$

Inverse Fourier Transform (IFT)

Frequency domain



Frequency domain



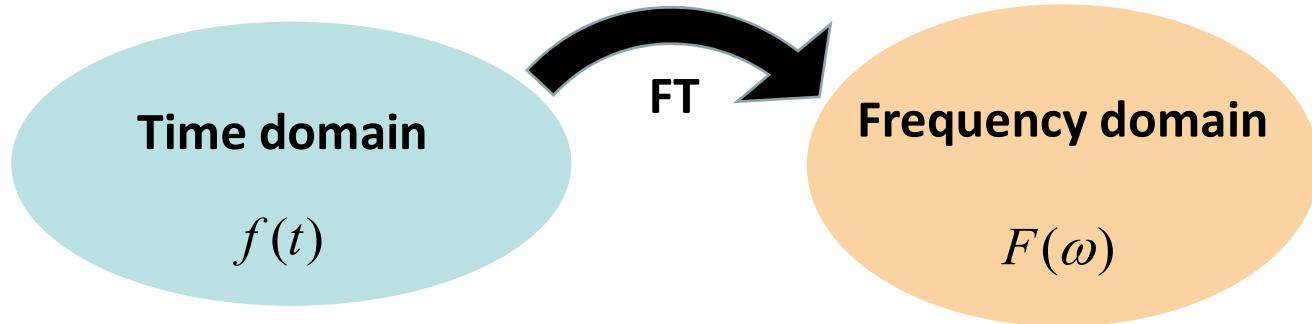
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



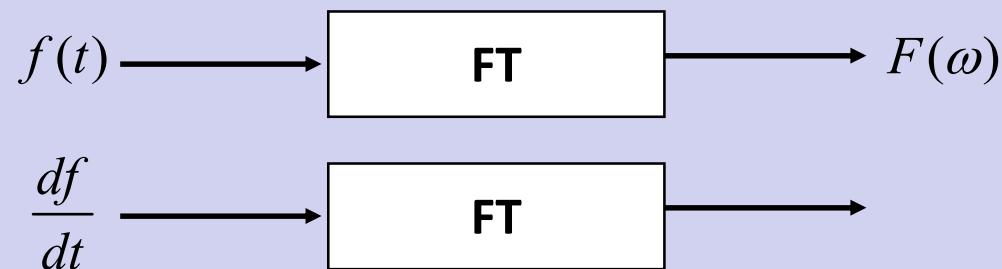
Frequency domain



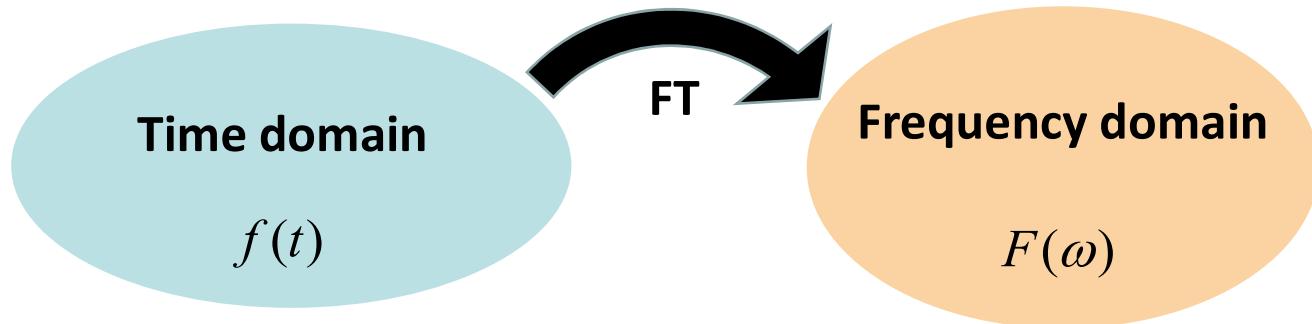
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



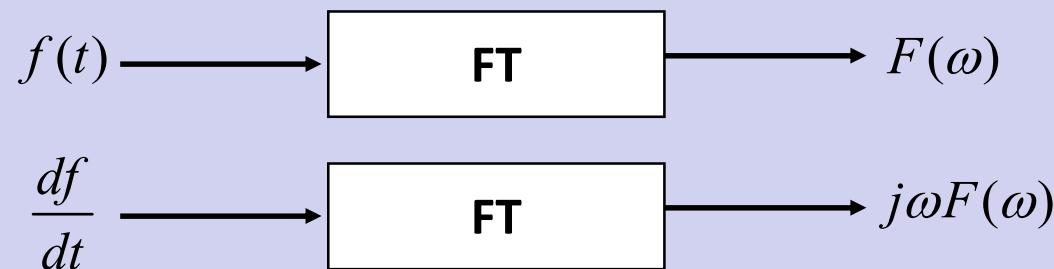
Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

Fourier Transform (FT)

2) Time-domain derivative and Fourier Transform



Frequency domain

- Fourier Transform and functions of n variables
- Fourier Transform and vector functions
- Fourier Transform and vector functions of n variables

Frequency domain

- Fourier Transform and functions of n variables
- Fourier Transform and vector functions
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1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

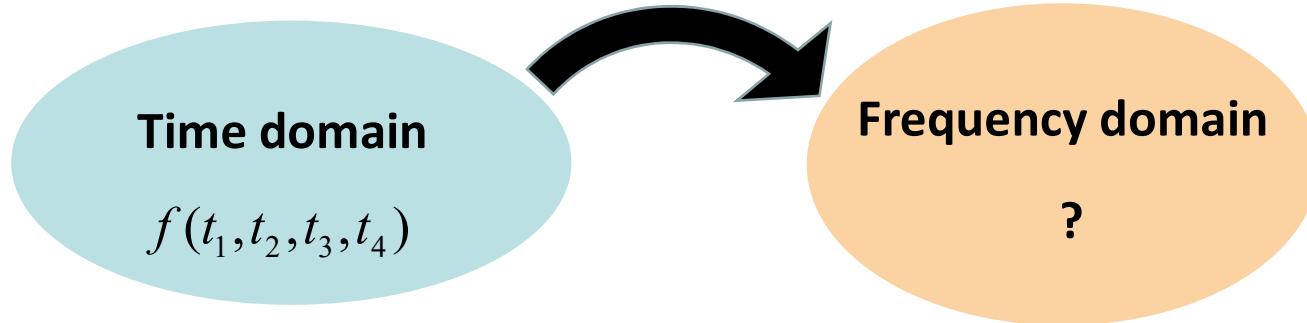
Frequency domain

- Fourier Transform and functions of n variables
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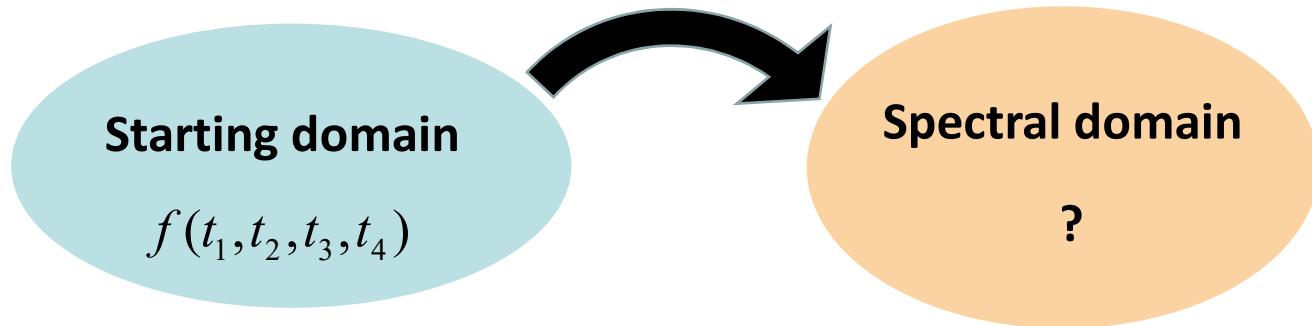
1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform

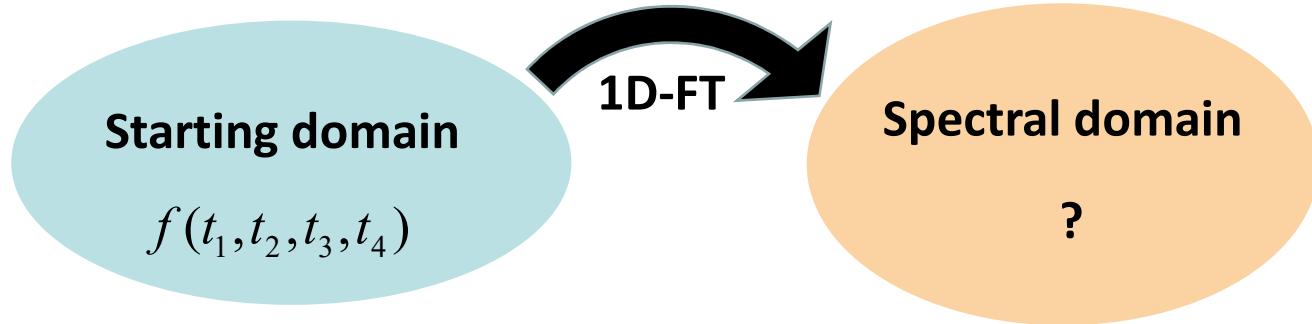
Fourier Transform and functions of n variables



Fourier Transform and functions of n variables

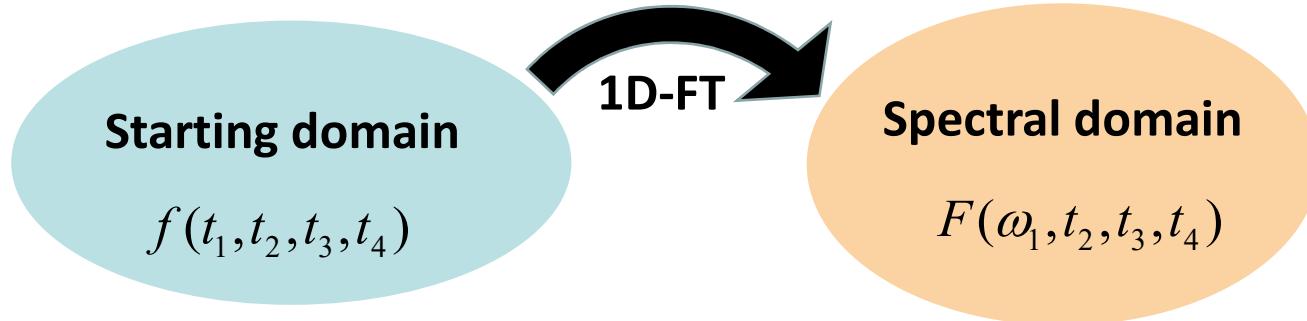


Fourier Transform and functions of n variables



One Dimensional Fourier Transform (1D-FT)

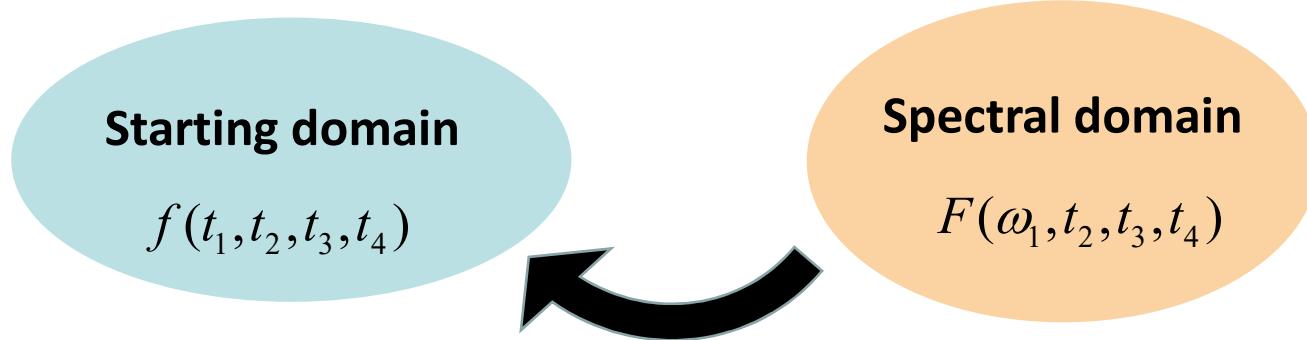
Fourier Transform and functions of n variables



One Dimensional Fourier Transform (1D-FT)

$$F(\omega_1, t_2, t_3, t_4) = \int_{-\infty}^{+\infty} f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} dt_1$$

Fourier Transform and functions of n variables

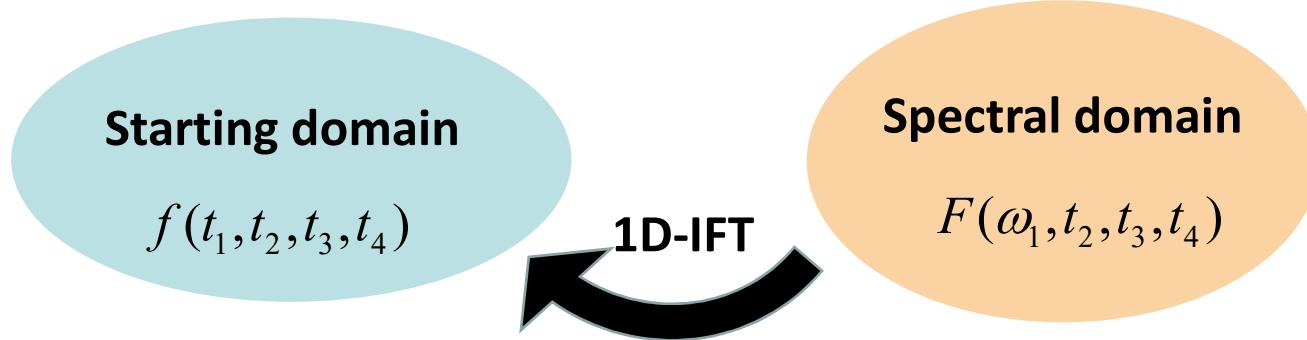


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1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of n variables



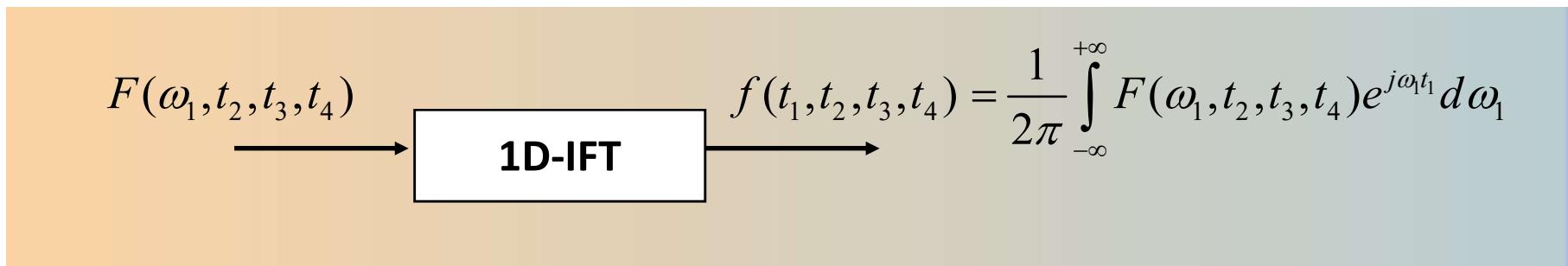
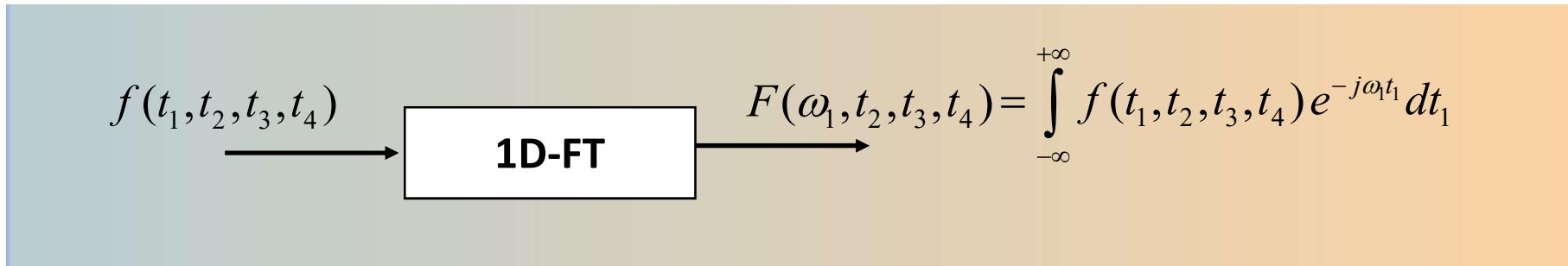
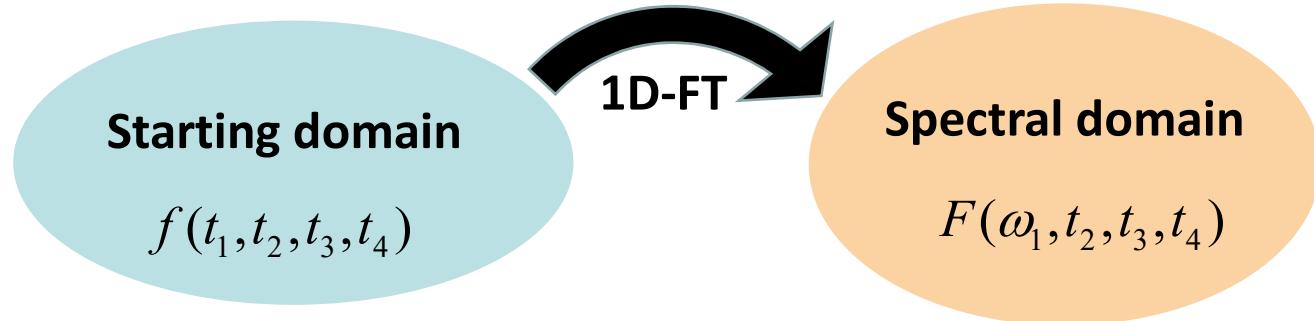
One Dimensional Fourier Transform (1D-FT)

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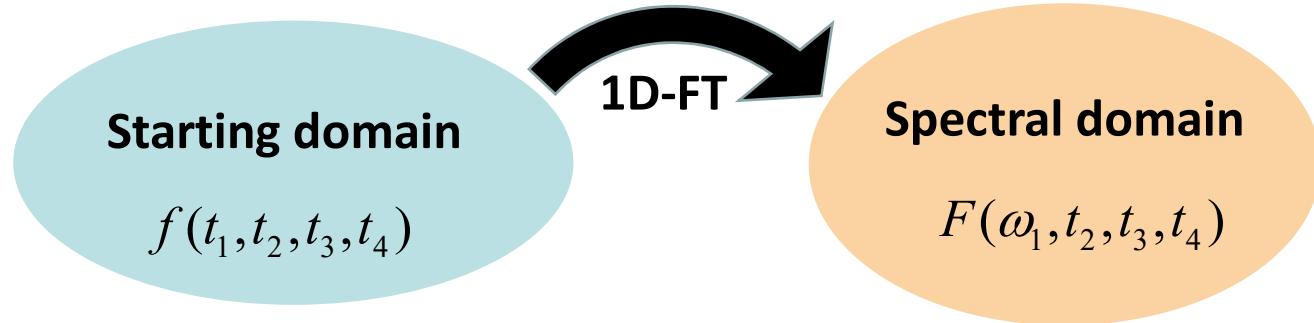
1) How to jump back from the Spectral domain to the Time domain

$$f(t_1, t_2, t_3, t_4) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega_1, t_2, t_3, t_4) e^{j\omega_1 t_1} d\omega_1 \quad \text{1D-IFT}$$

Fourier Transform and functions of n variables

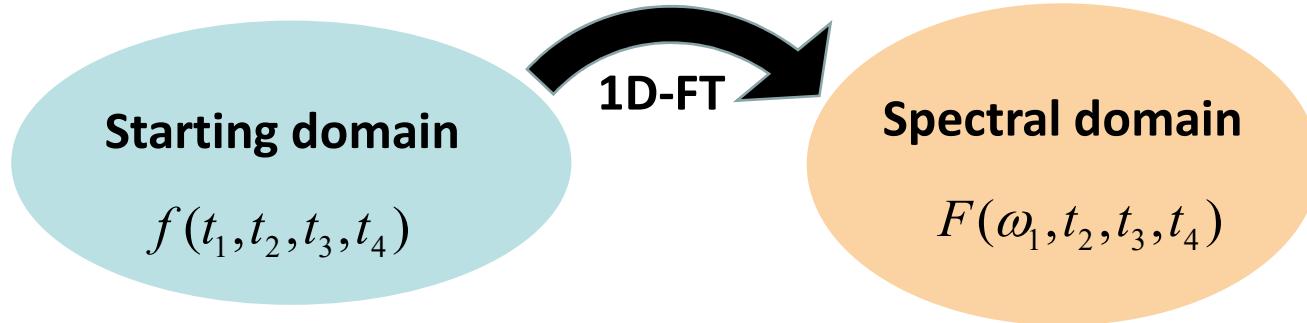


Fourier Transform and functions of n variables



2) Time domain derivative and Fourier Transform

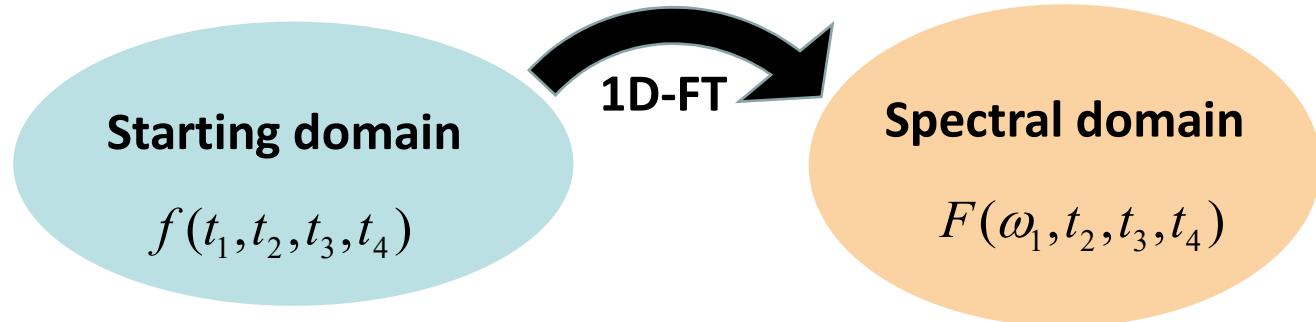
Fourier Transform and functions of n variables



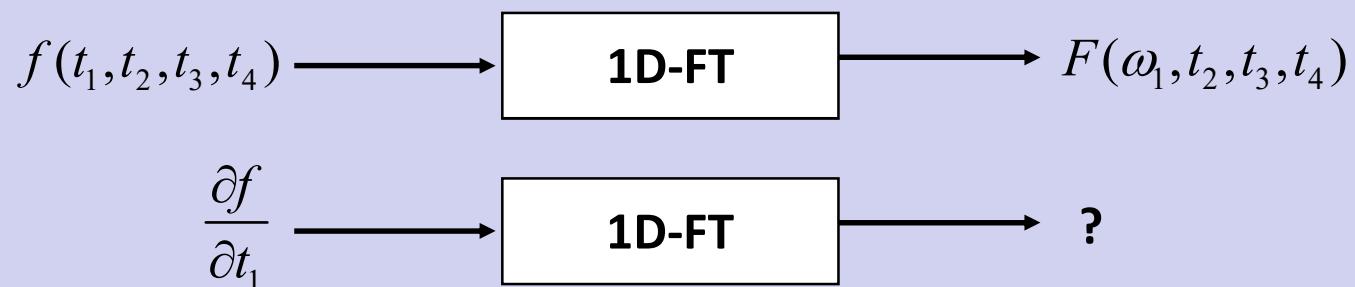
2) Time domain derivative and Fourier Transform



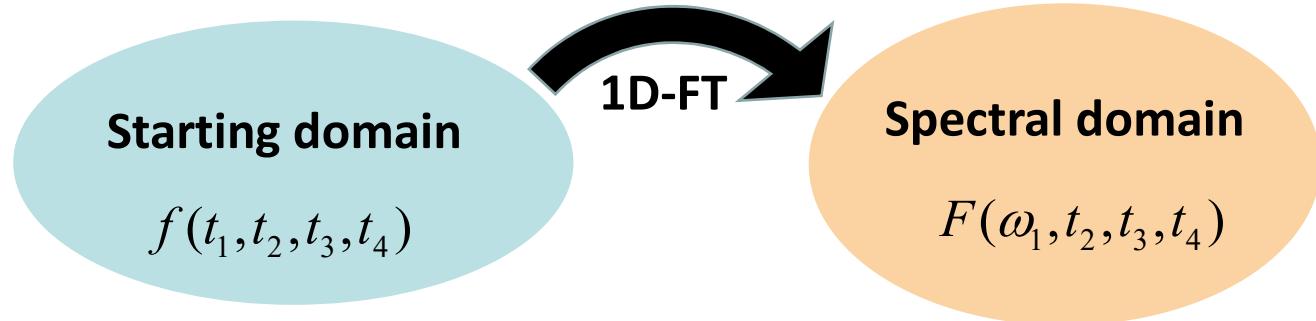
Fourier Transform and functions of n variables



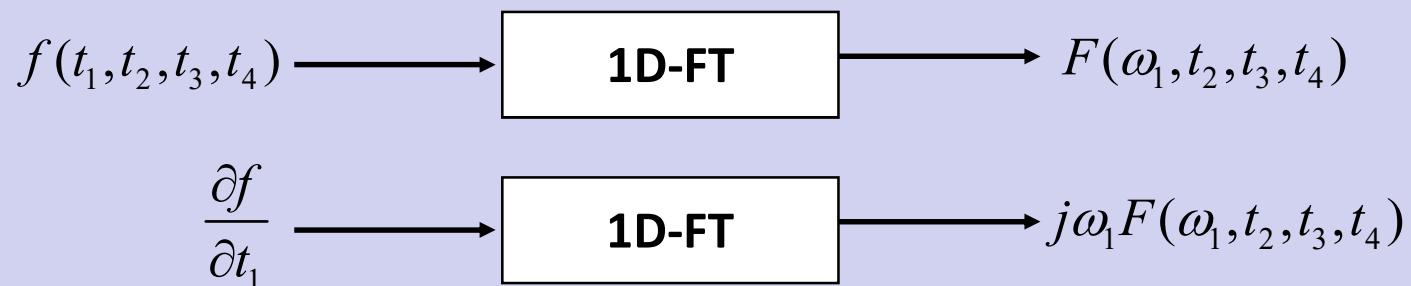
2) Time domain derivative and Fourier Transform



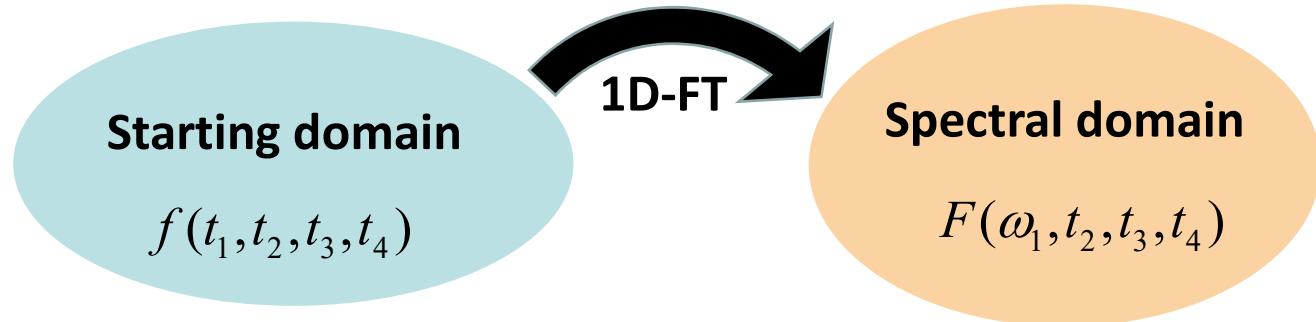
Fourier Transform and functions of n variables



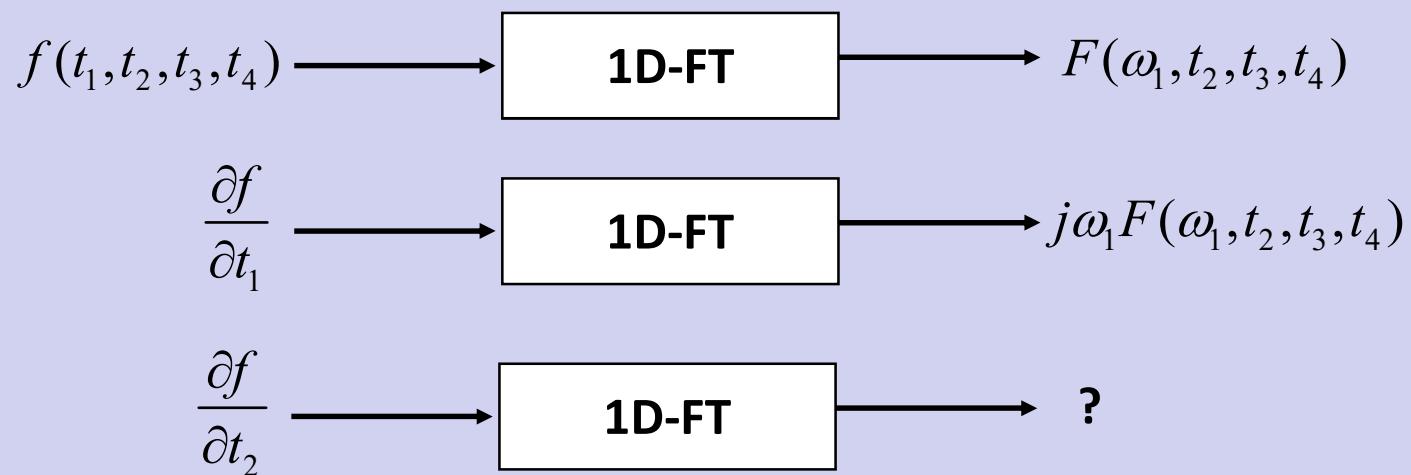
2) Time domain derivative and Fourier Transform



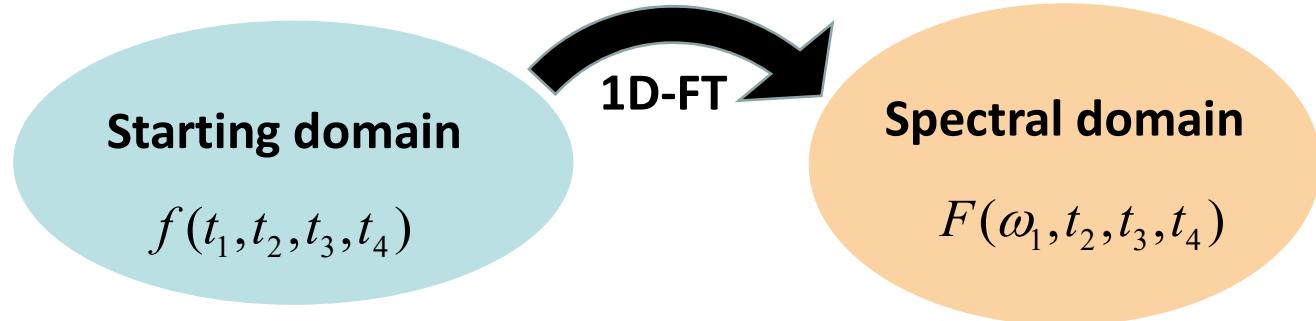
Fourier Transform and functions of n variables



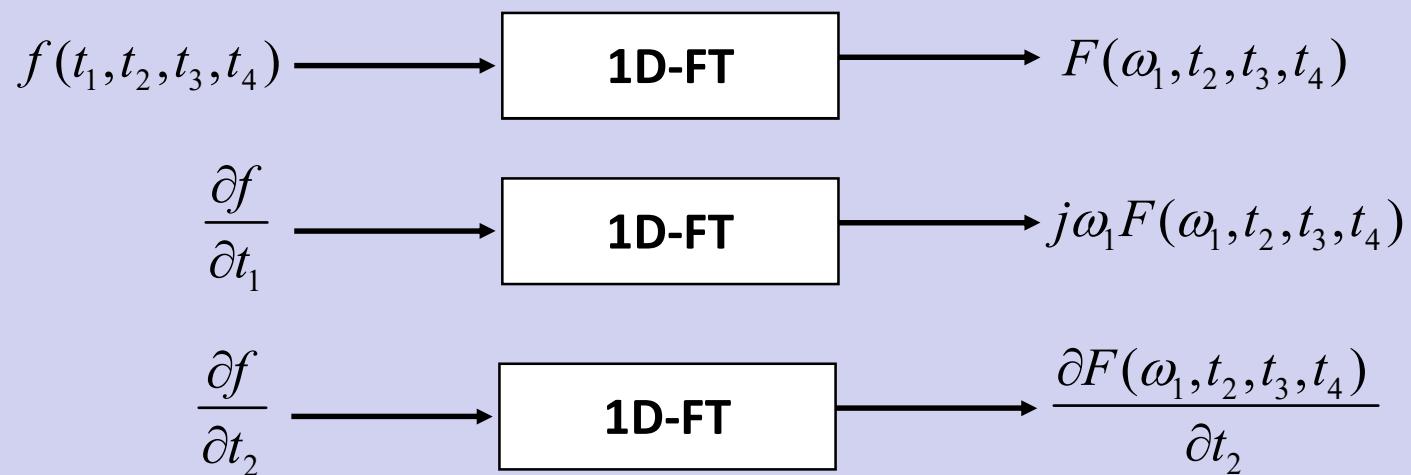
2) Time domain derivative and Fourier Transform



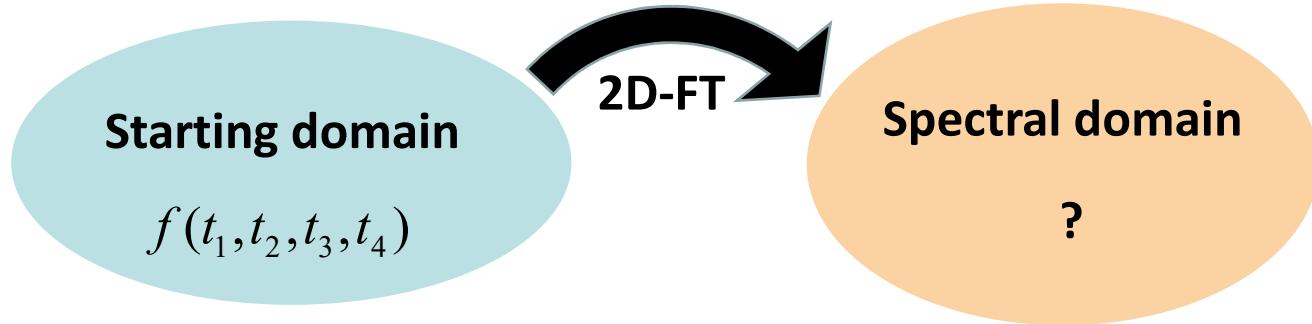
Fourier Transform and functions of n variables



2) Time domain derivative and Fourier Transform

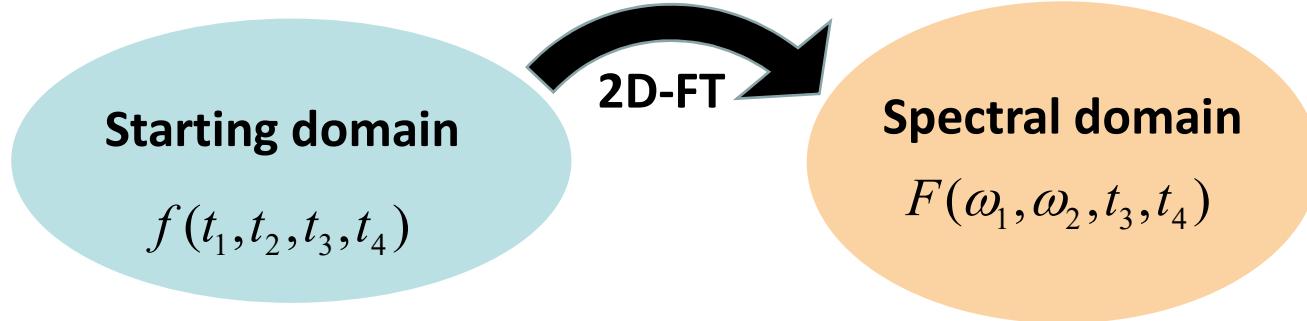


Fourier Transform and functions of n variables



Two Dimensional Fourier Transform (2D-FT)

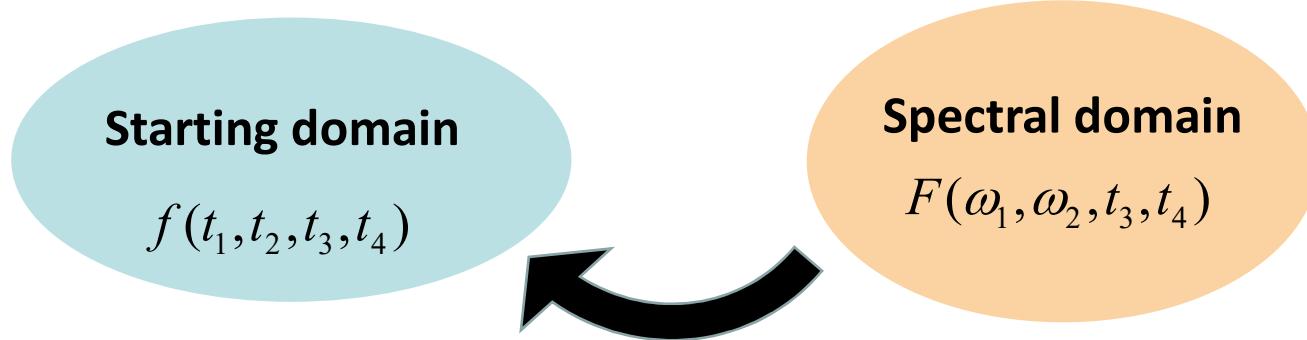
Fourier Transform and functions of n variables



Two Dimensional Fourier Transform (2D-FT)

$$F(\omega_1, \omega_2, t_3, t_4) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dt_1 dt_2 f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} e^{-j\omega_2 t_2}$$

Fourier Transform and functions of n variables

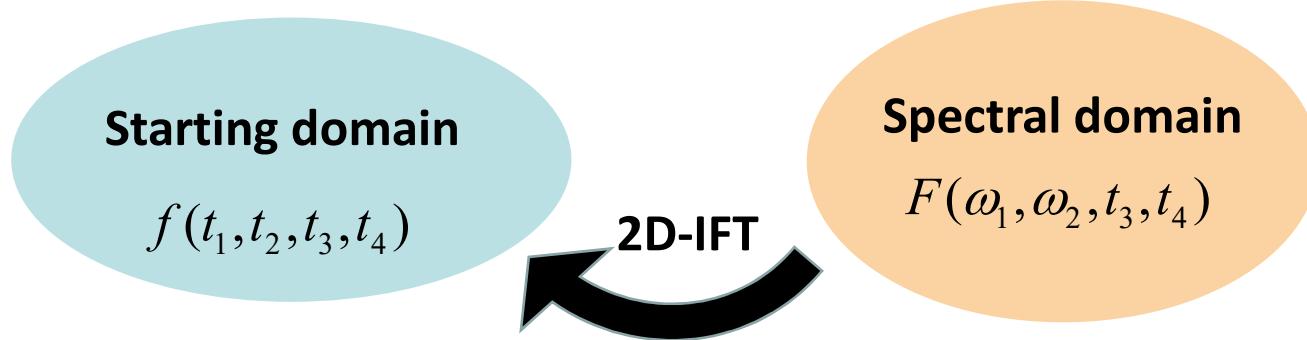


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1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of n variables



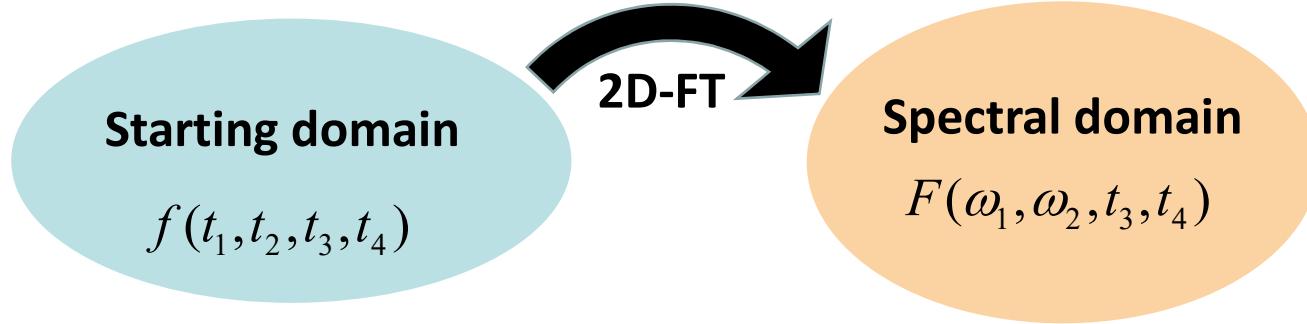
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1) How to jump back from the Spectral domain to the Time domain

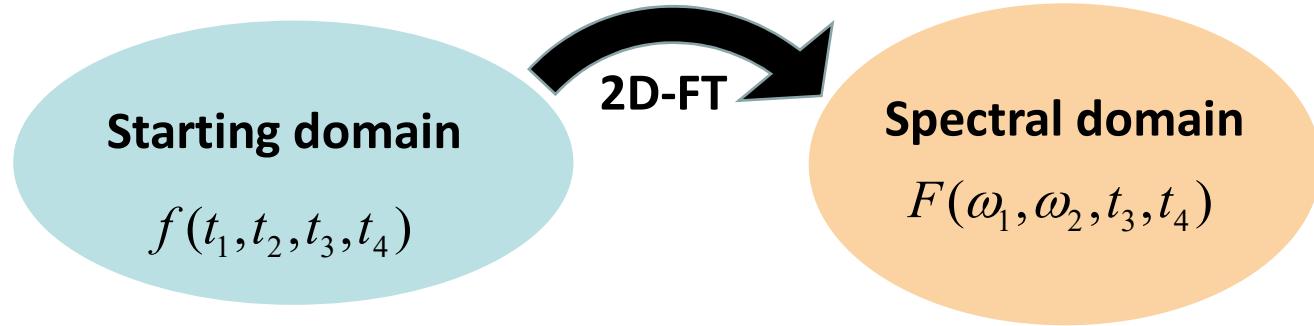
$$f(t_1, t_2, t_3, t_4) = \left(\frac{1}{2\pi} \right)^2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\omega_1, \omega_2, t_3, t_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} d\omega_1 d\omega_2 \text{ 2D-IFT}$$

Fourier Transform and functions of n variables

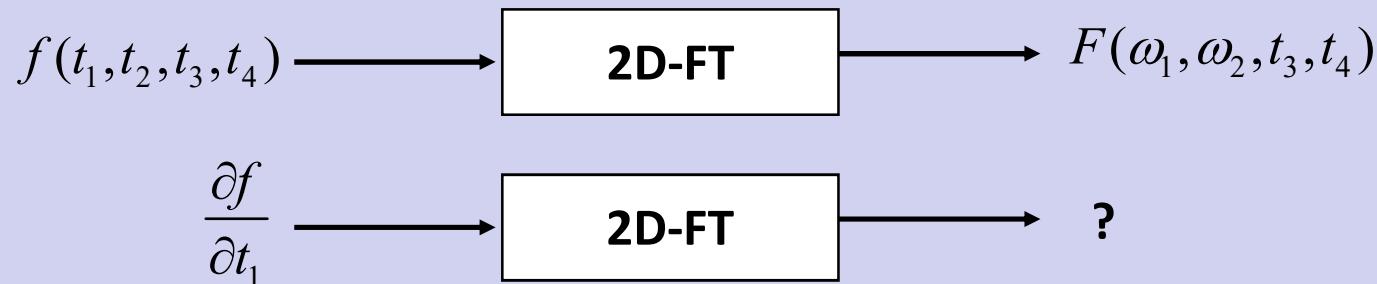


2) Time domain derivative and Fourier Transform

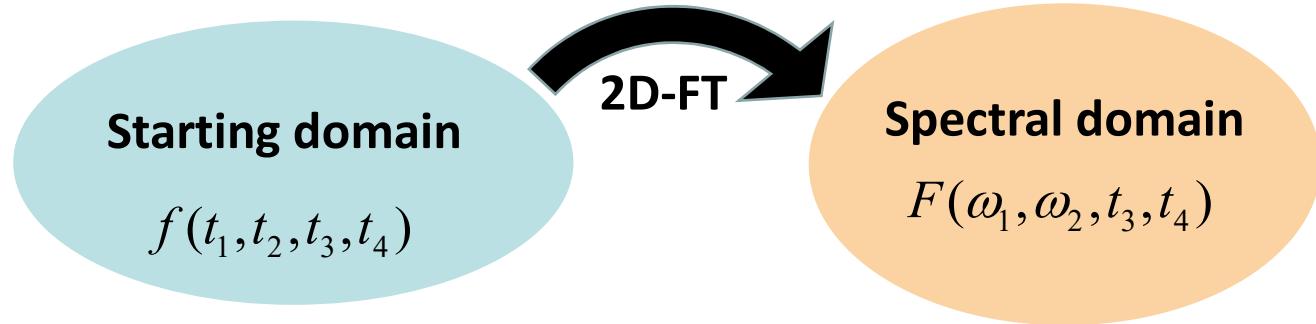
Fourier Transform and functions of n variables



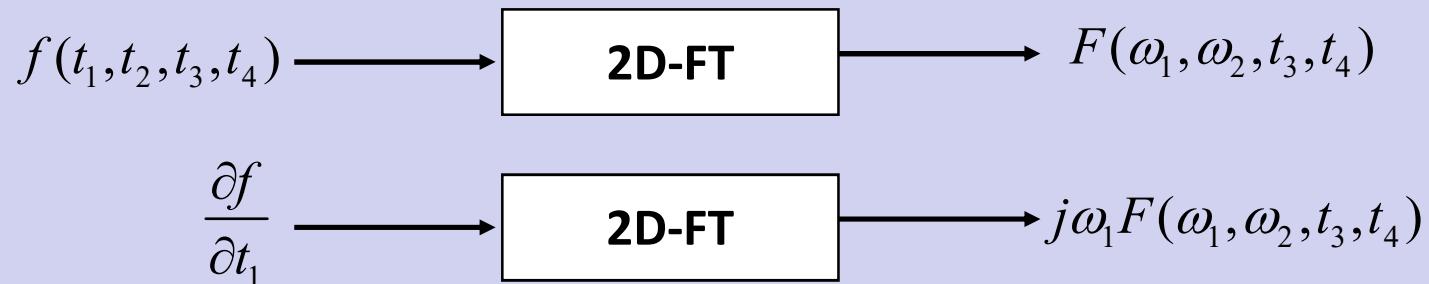
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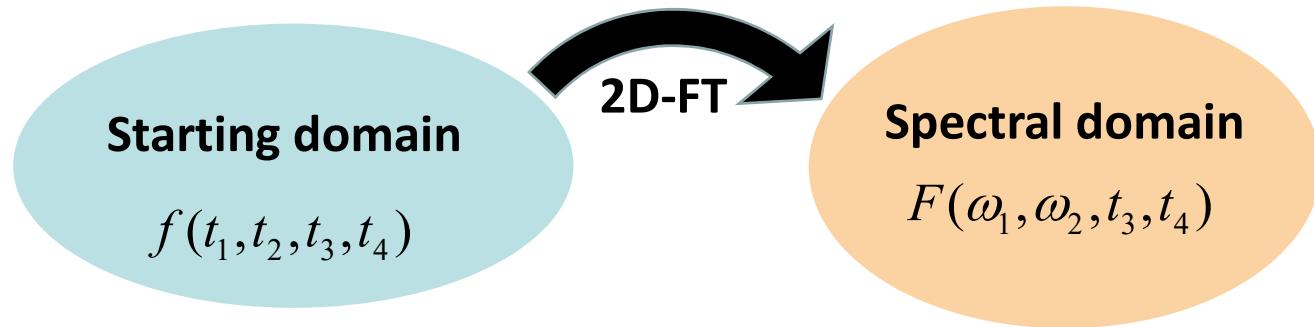
Fourier Transform and functions of n variables



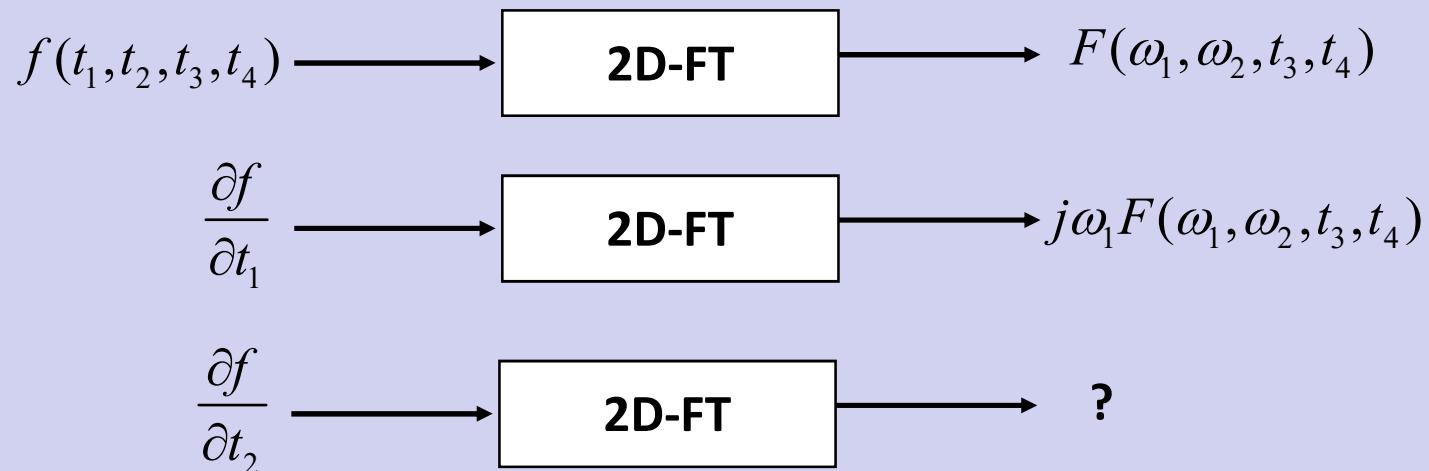
2) Time domain derivative and Fourier Transform



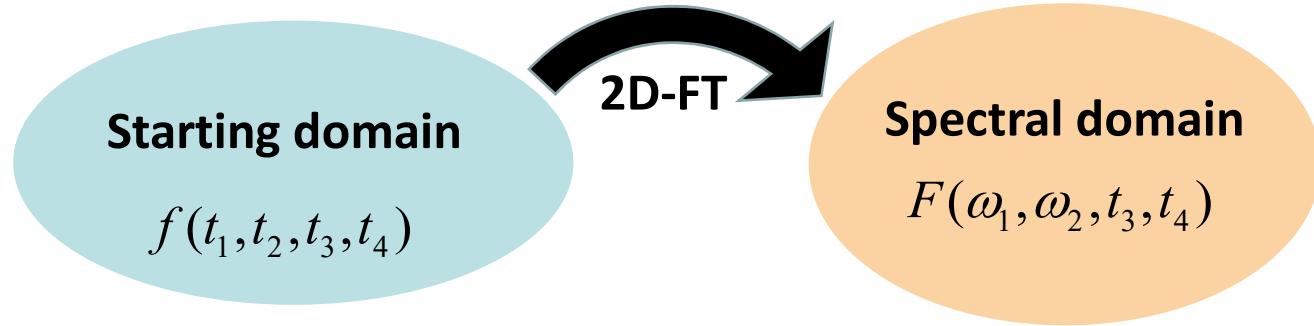
Fourier Transform and functions of n variables



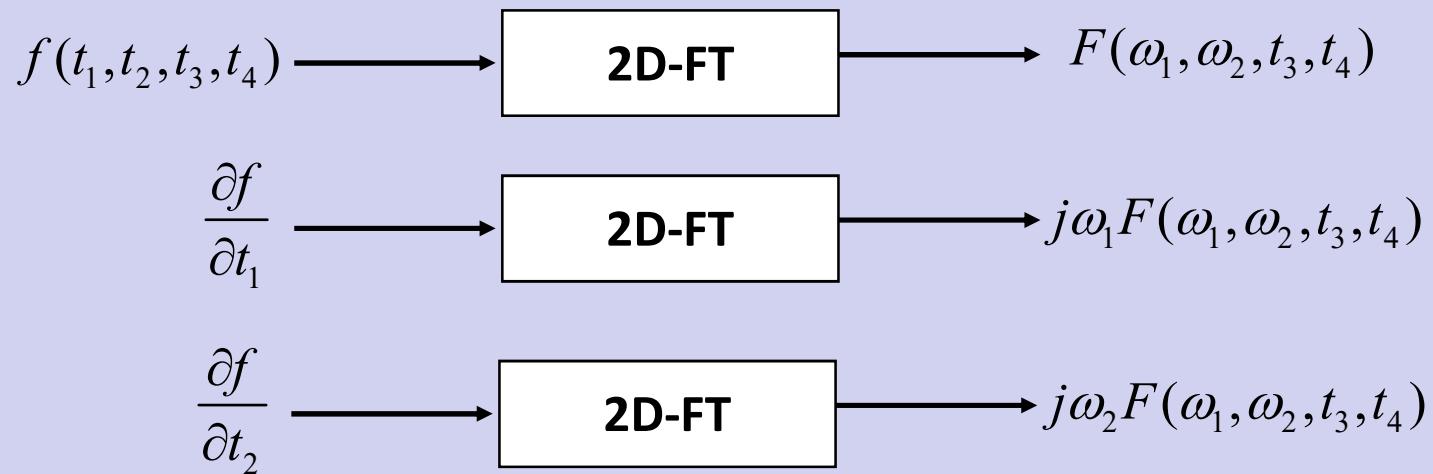
2) Time domain derivative and Fourier Transform



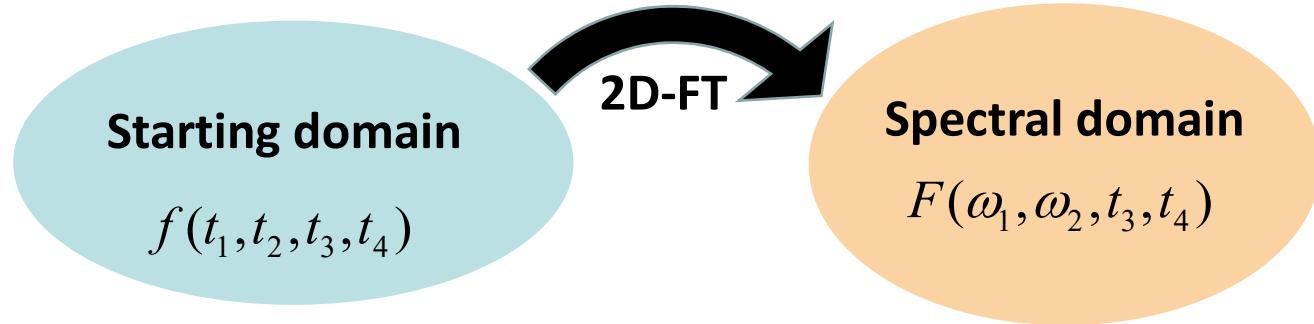
Fourier Transform and functions of n variables



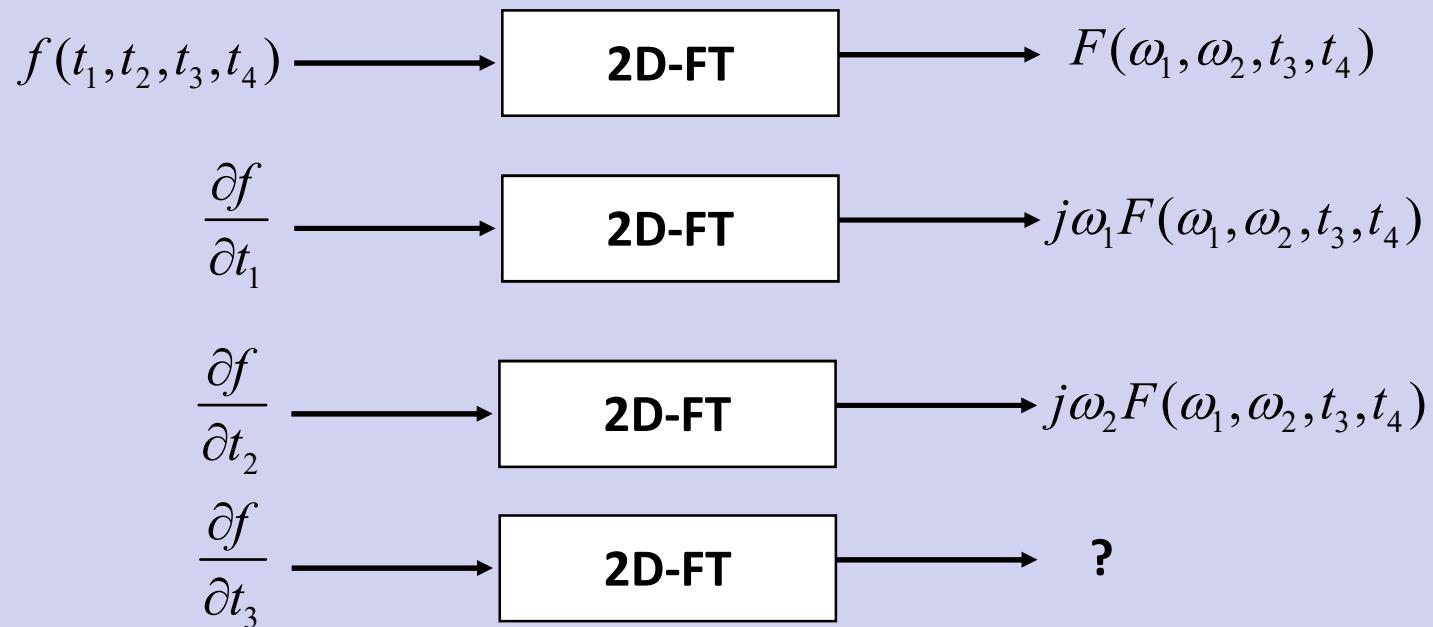
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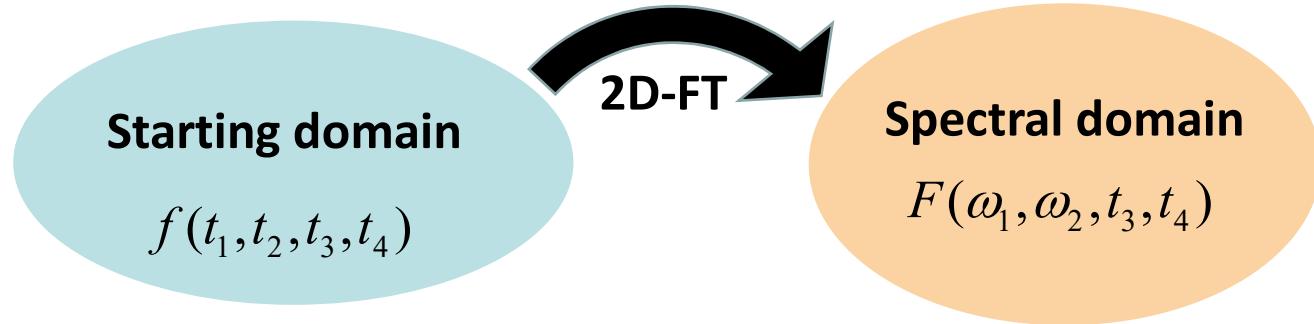
Fourier Transform and functions of n variables



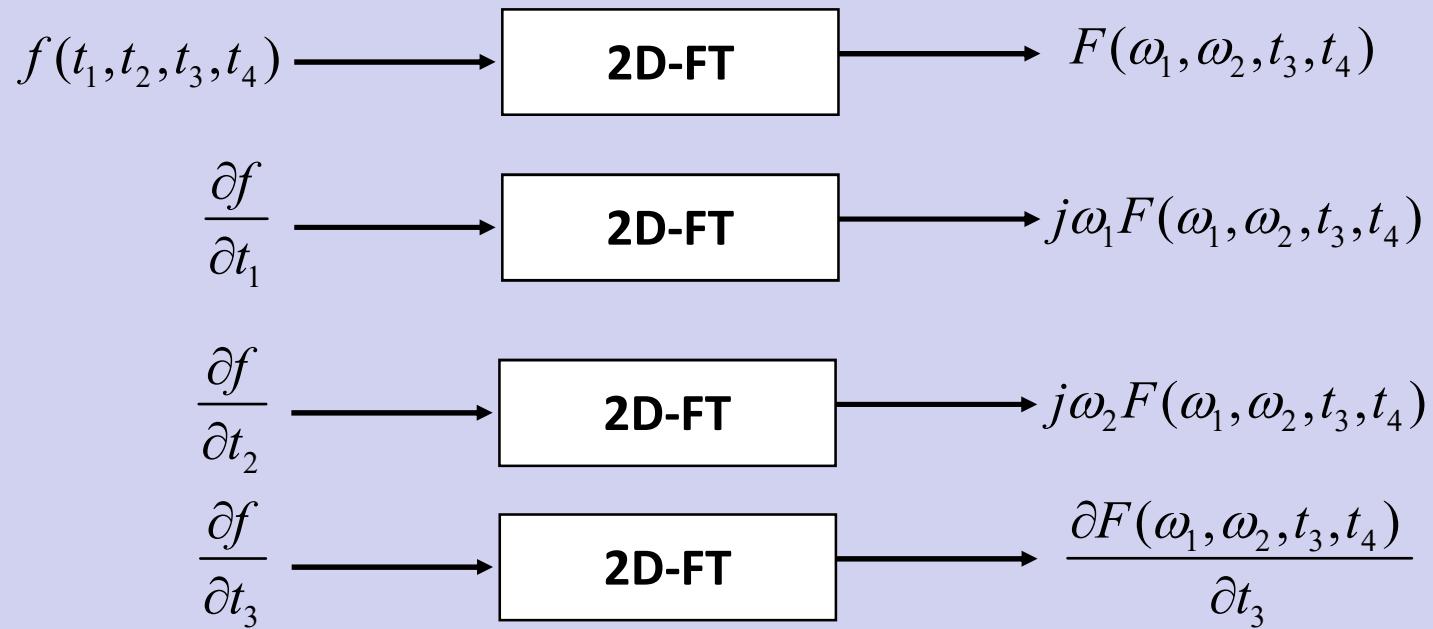
2) Time domain derivative and Fourier Transform



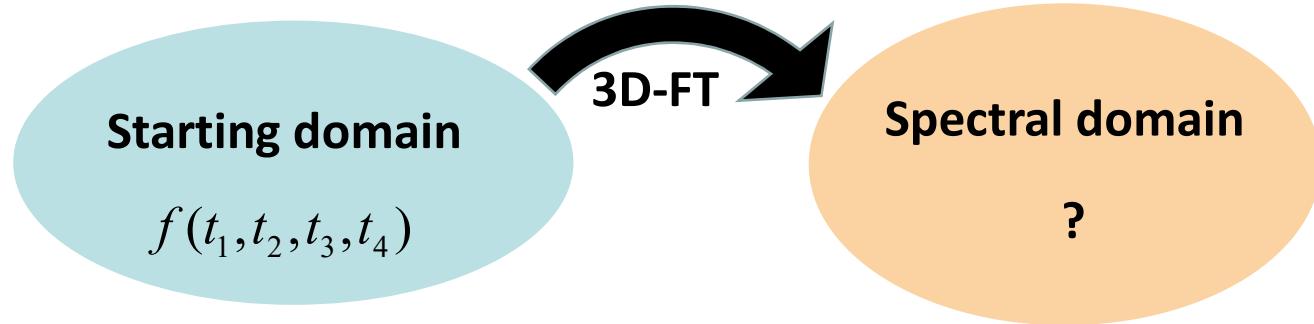
Fourier Transform and functions of n variables



2) Time domain derivative and Fourier Transform

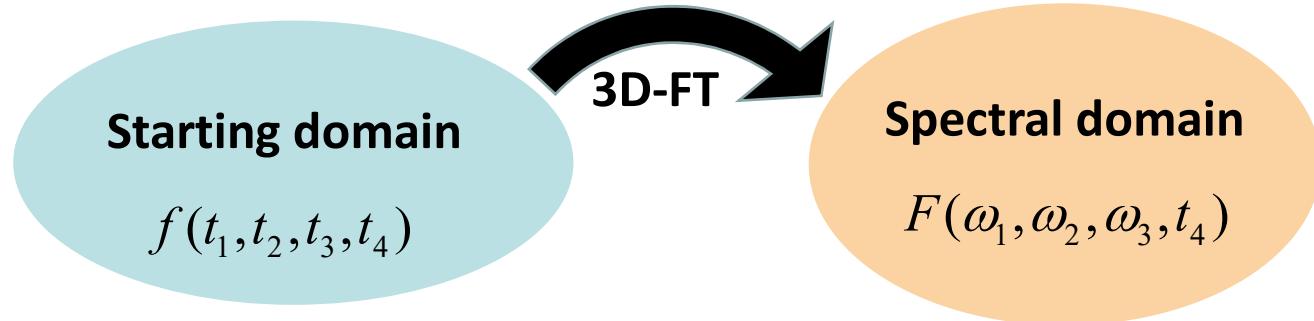


Fourier Transform and functions of n variables



Three Dimensional Fourier Transform (3D-FT)

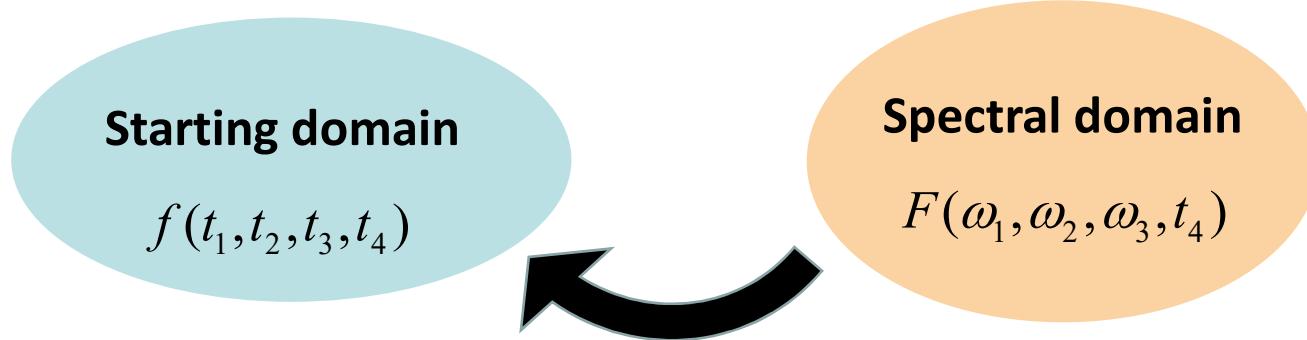
Fourier Transform and functions of n variables



Three Dimensional Fourier Transform (3D-FT)

$$F(\omega_1, \omega_2, \omega_3, t_4) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3}$$

Fourier Transform and functions of n variables

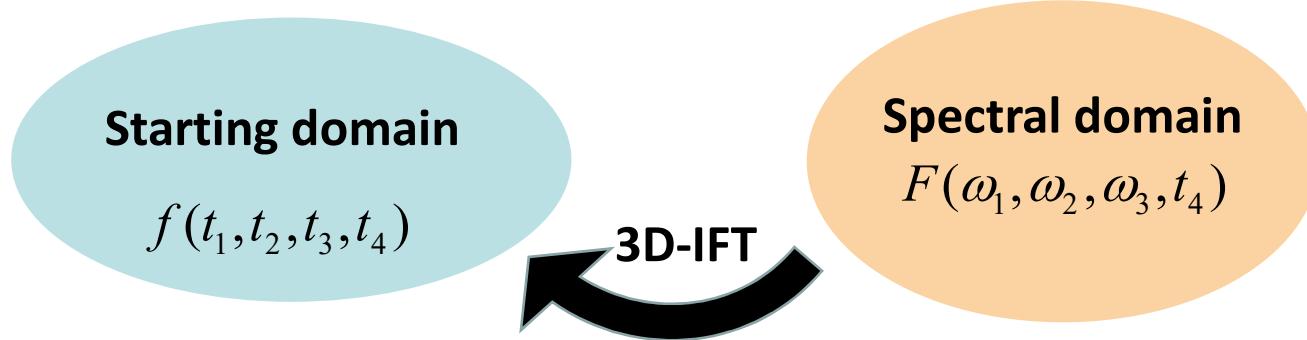


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1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of n variables



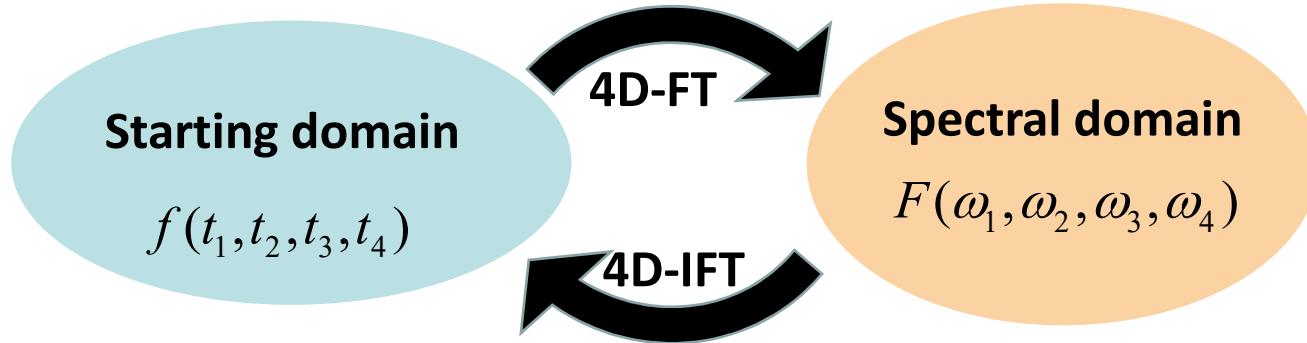
Three Dimensional Fourier Transform (3D-FT)

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1) How to jump back from the Spectral domain to the Time domain

$$f(t_1, t_2, t_3, t_4) = \left(\frac{1}{2\pi} \right)^3 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\omega_1, \omega_2, \omega_3, t_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} e^{j\omega_3 t_3} d\omega_1 d\omega_2 d\omega_3 \quad \text{3D-IFT}$$

Fourier Transform and functions of n variables



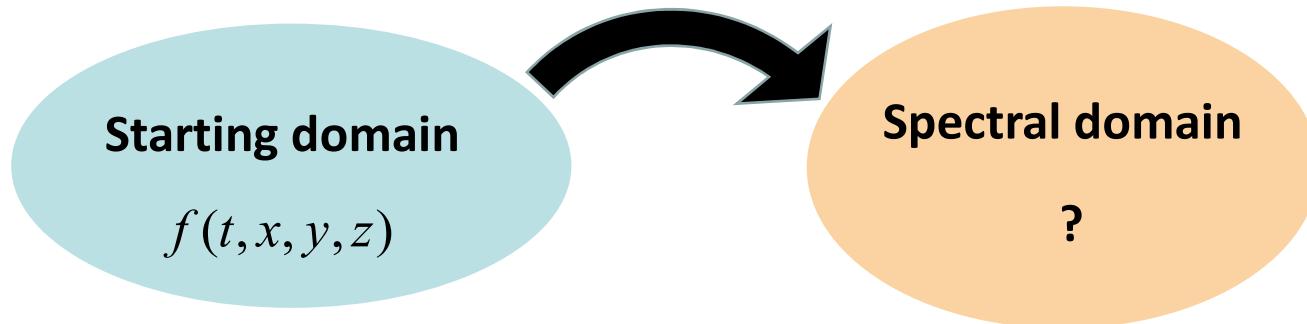
Four Dimensional Fourier Transform (4D-FT)

$$F(\omega_1, \omega_2, \omega_3, \omega_4) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dt_1 dt_2 dt_3 dt_4 f(t_1, t_2, t_3, t_4) e^{-j\omega_1 t_1} e^{-j\omega_2 t_2} e^{-j\omega_3 t_3} e^{-j\omega_4 t_4}$$

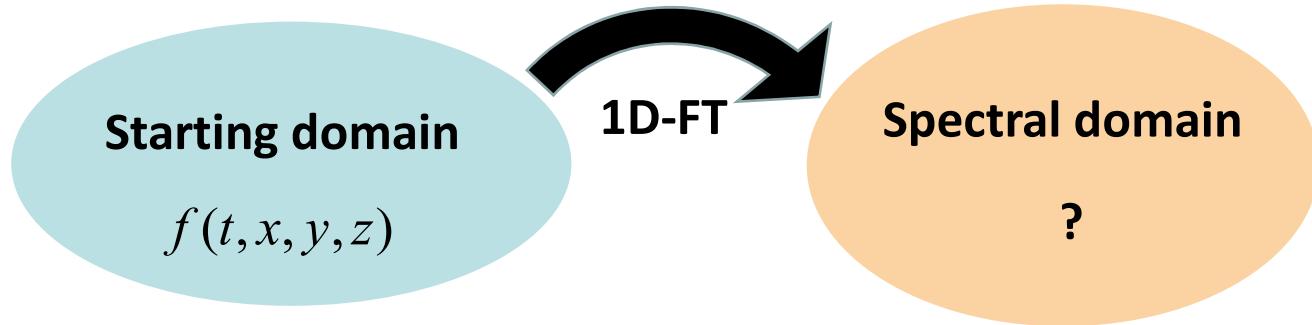
1) How to jump back from the Spectral domain to the Time domain

$$f(t_1, t_2, t_3, t_4) = \left(\frac{1}{2\pi} \right)^4 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\omega_1, \omega_2, \omega_3, \omega_4) e^{j\omega_1 t_1} e^{j\omega_2 t_2} e^{j\omega_3 t_3} e^{j\omega_4 t_4} d\omega_1 d\omega_2 d\omega_3 d\omega_4 \text{ 4D-IFT}$$

Fourier Transform and functions of n variables

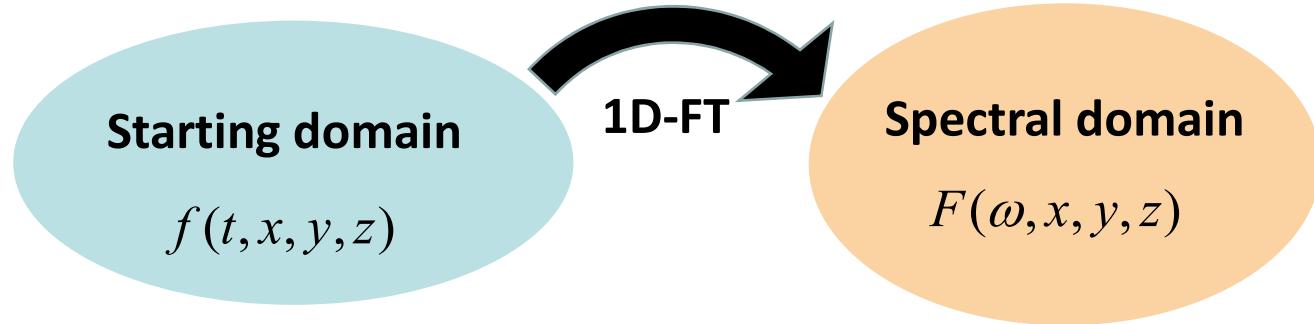


Fourier Transform and functions of n variables



One Dimensional Fourier Transform (1D-FT)

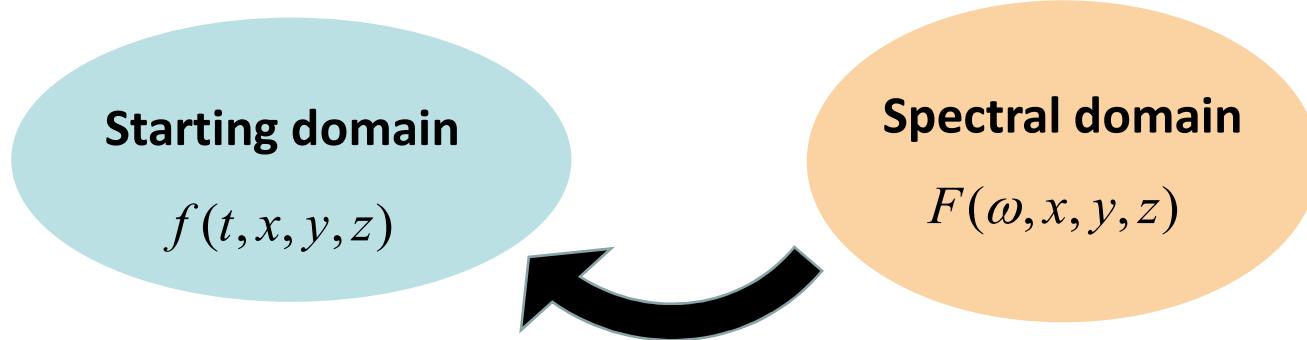
Fourier Transform and functions of n variables



One Dimensional Fourier Transform (1D-FT)

$$F(\omega, x, y, z) = \int_{-\infty}^{+\infty} f(t, x, y, z) e^{-j\omega t} dt$$

Fourier Transform and functions of n variables

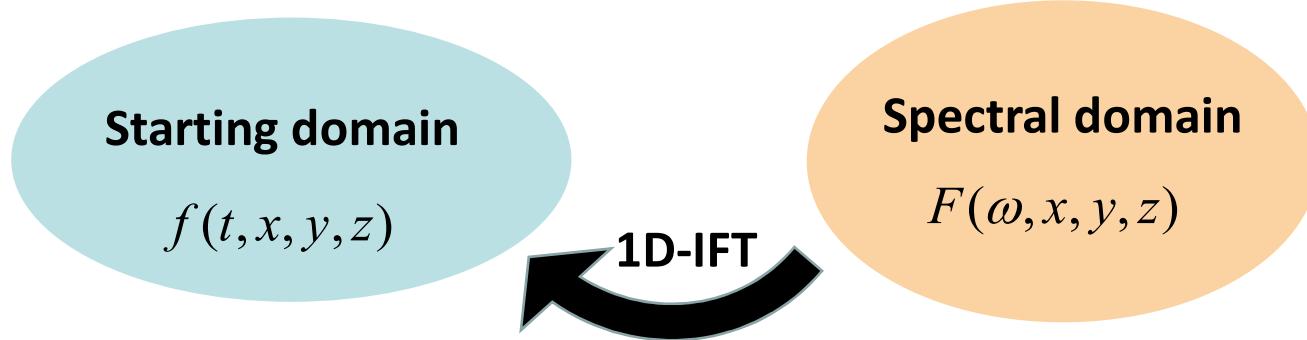


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1) How to jump back from the Spectral domain to the Time domain

Fourier Transform and functions of n variables



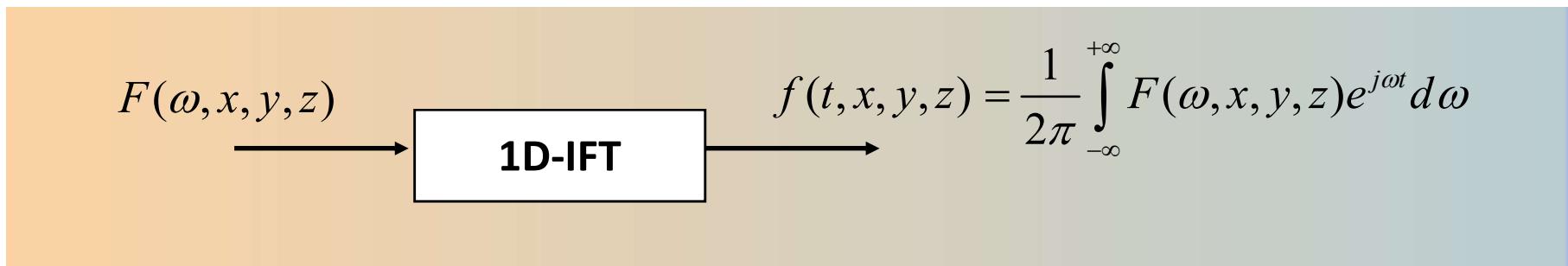
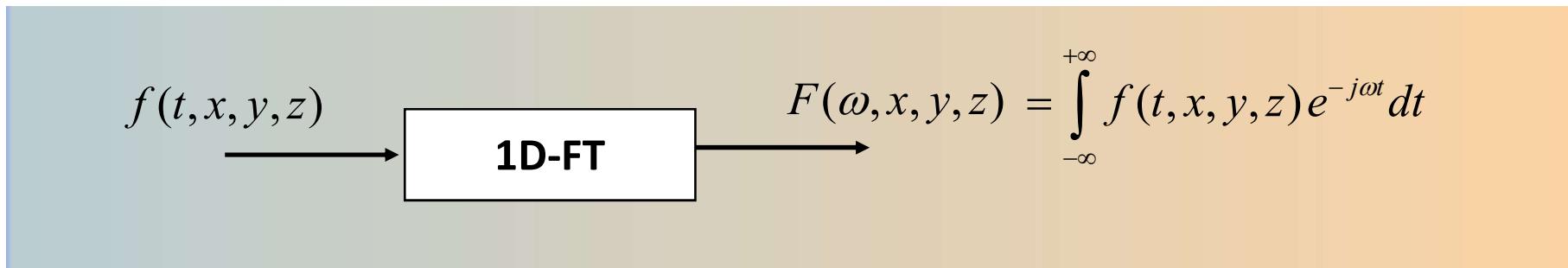
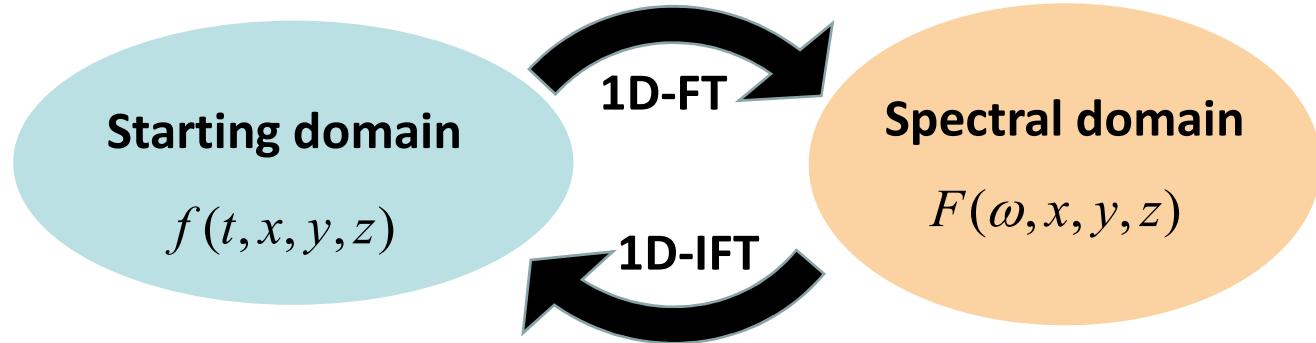
One Dimensional Fourier Transform (1D-FT)

$$F(\omega, x, y, z) = \int_{-\infty}^{+\infty} f(t, x, y, z) e^{-j\omega t} dt$$

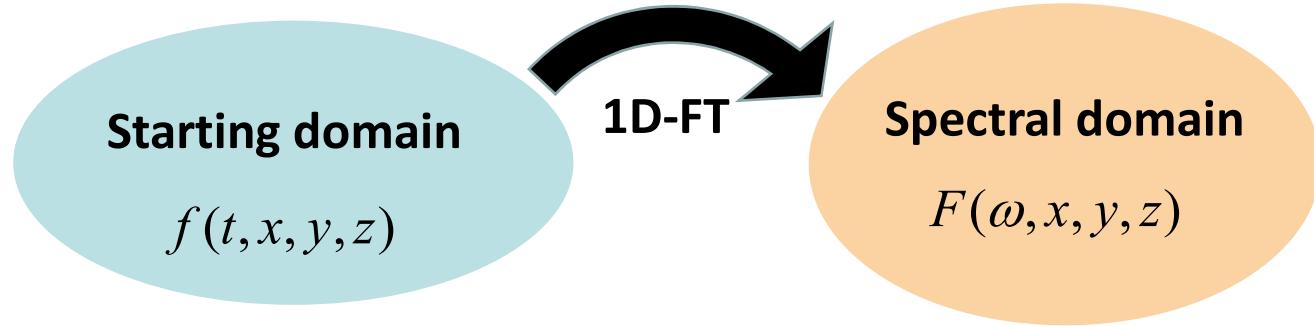
1) How to jump back from the Spectral domain to the Time domain

$$f(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega, x, y, z) e^{j\omega t} d\omega \quad \text{1D-IFT}$$

Fourier Transform and functions of n variables

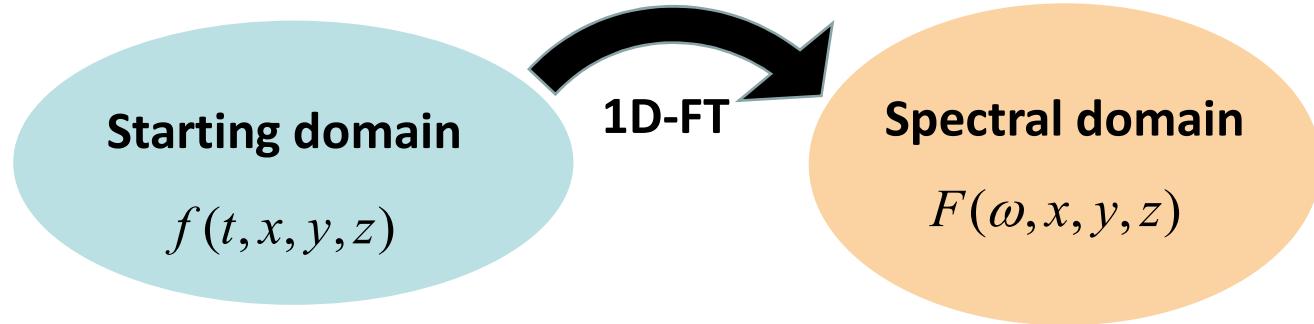


Fourier Transform and functions of n variables

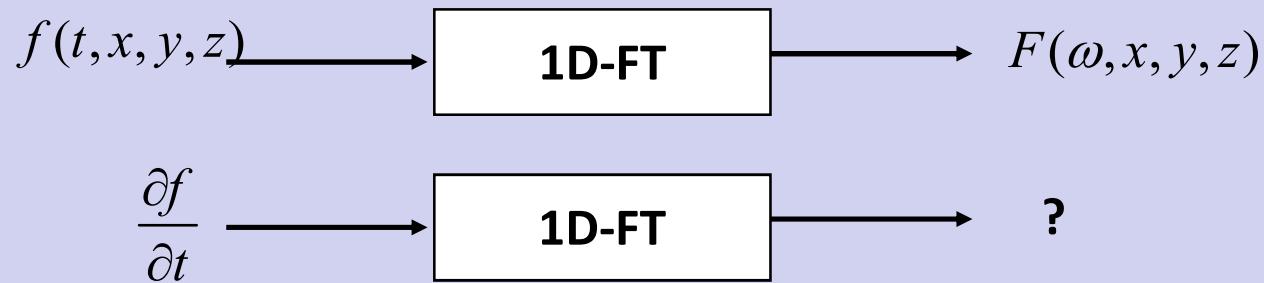


2) Time domain derivative and Fourier Transform

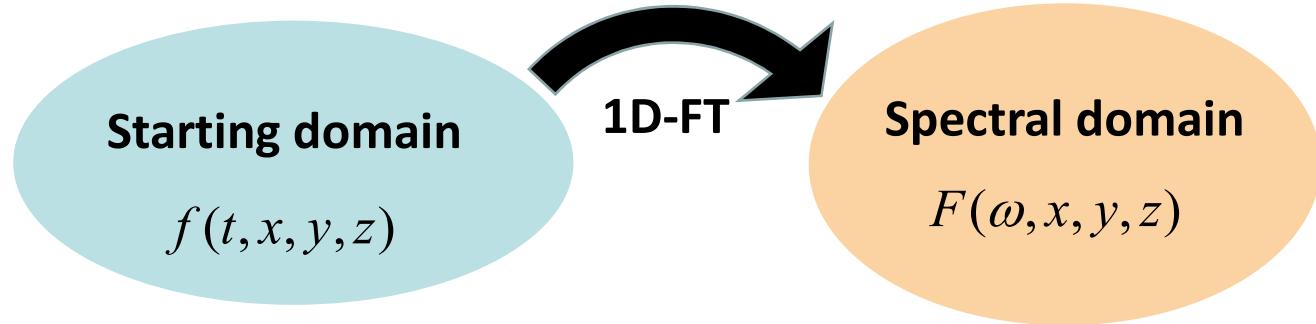
Fourier Transform and functions of n variables



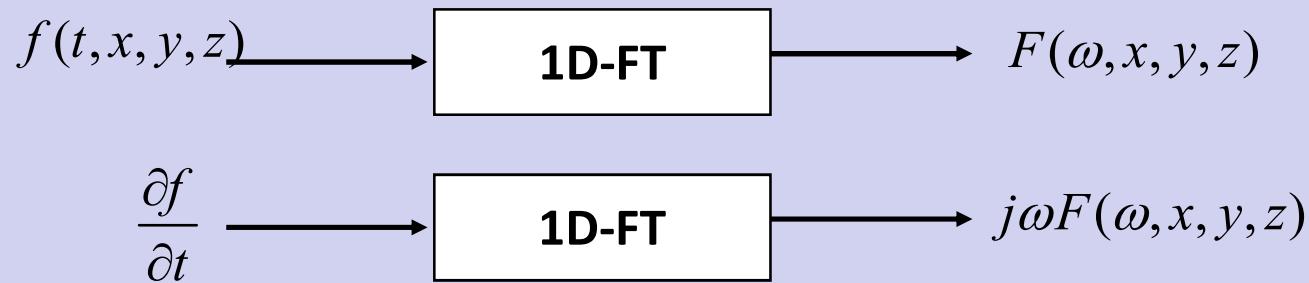
2) Time domain derivative and Fourier Transform



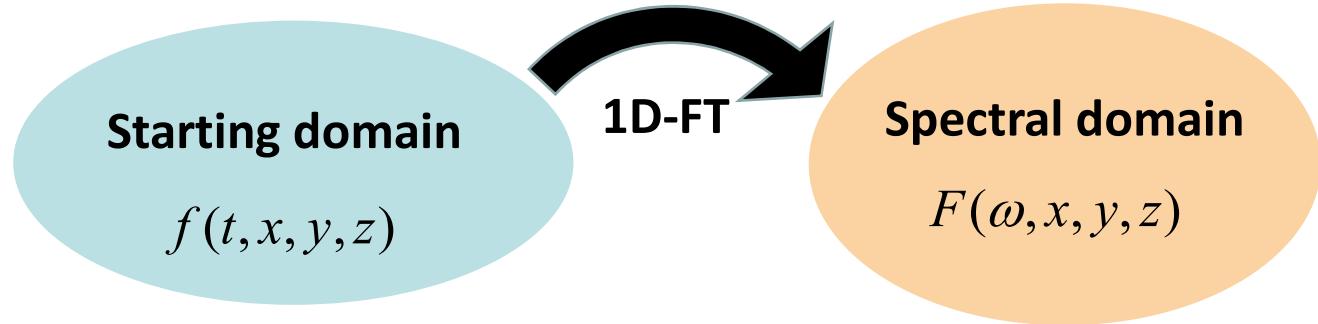
Fourier Transform and functions of n variables



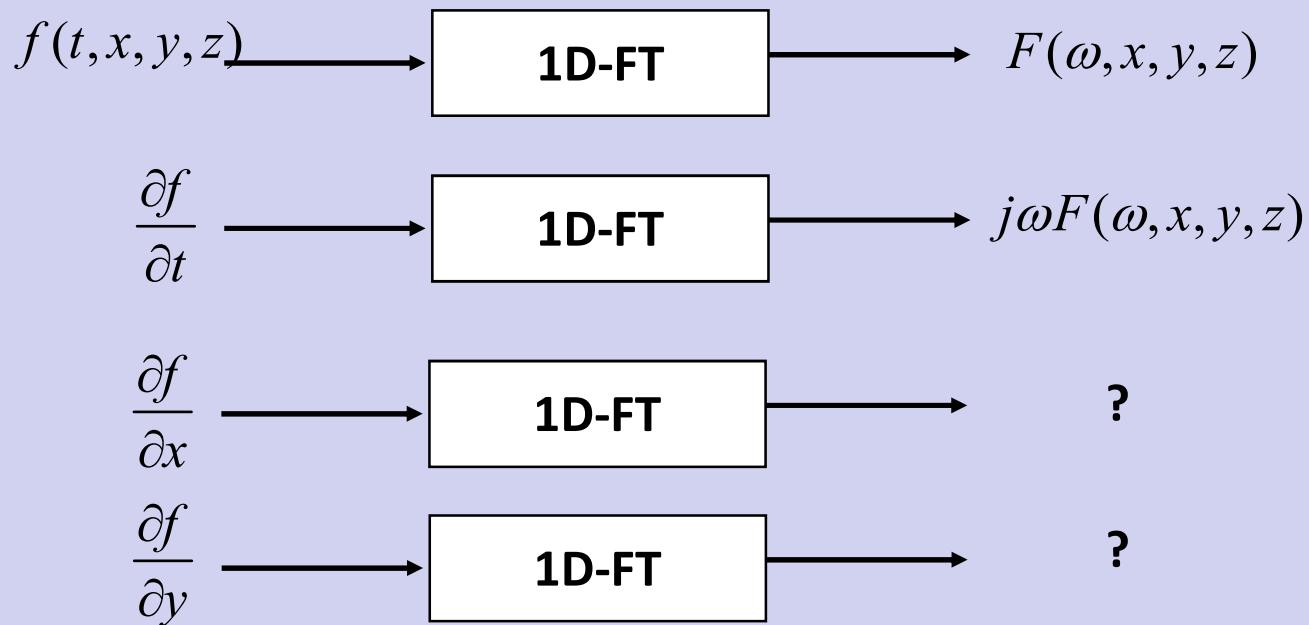
2) Time domain derivative and Fourier Transform



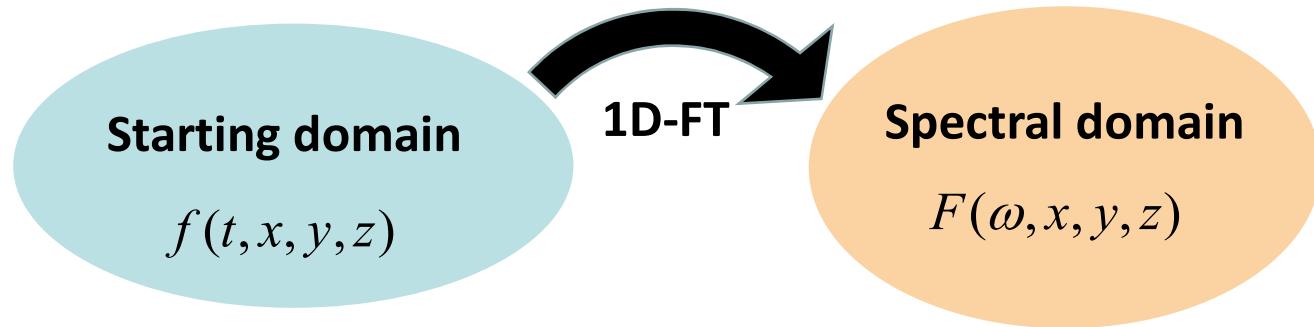
Fourier Transform and functions of n variables



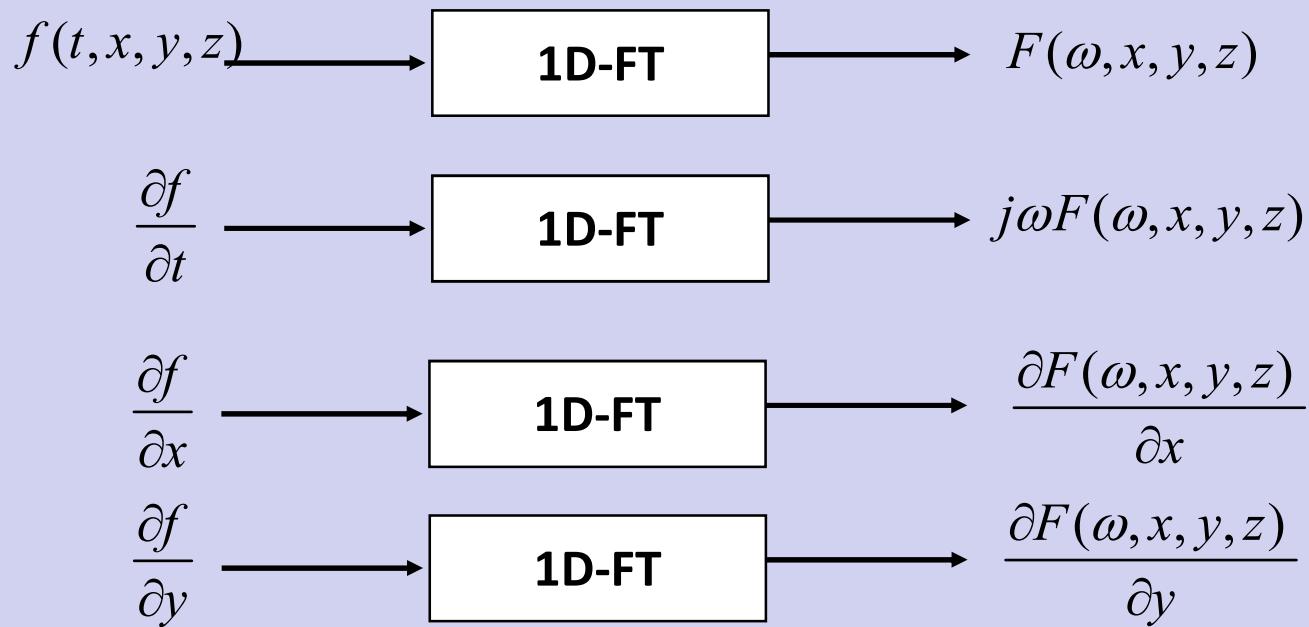
2) Time domain derivative and Fourier Transform



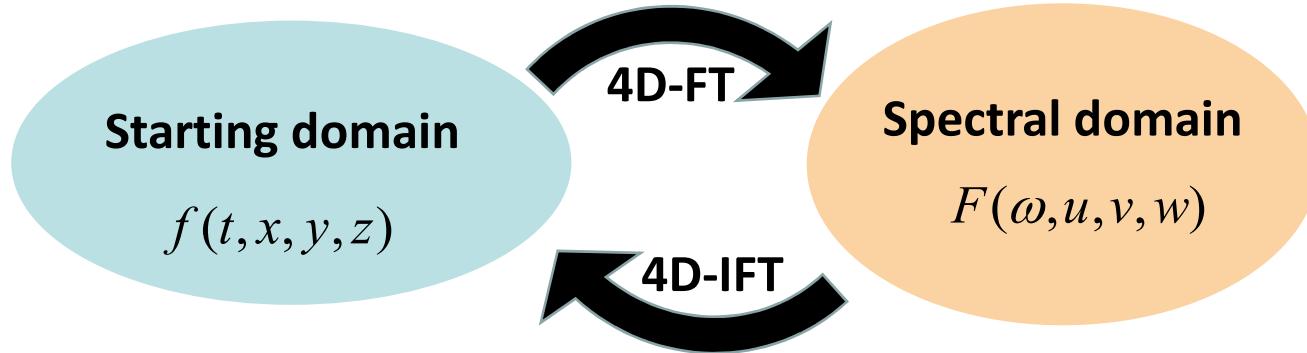
Fourier Transform and functions of n variables



2) Time domain derivative and Fourier Transform



Fourier Transform and functions of n variables



Four Dimensional Fourier Transform (4D-FT)

$$F(\omega, u, v, w) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(t, x, y, z) e^{-j\omega t} e^{-jux} e^{-jvy} e^{-jwz} dt dx dy dz$$

1) How to jump back from the Spectral domain to the Time domain

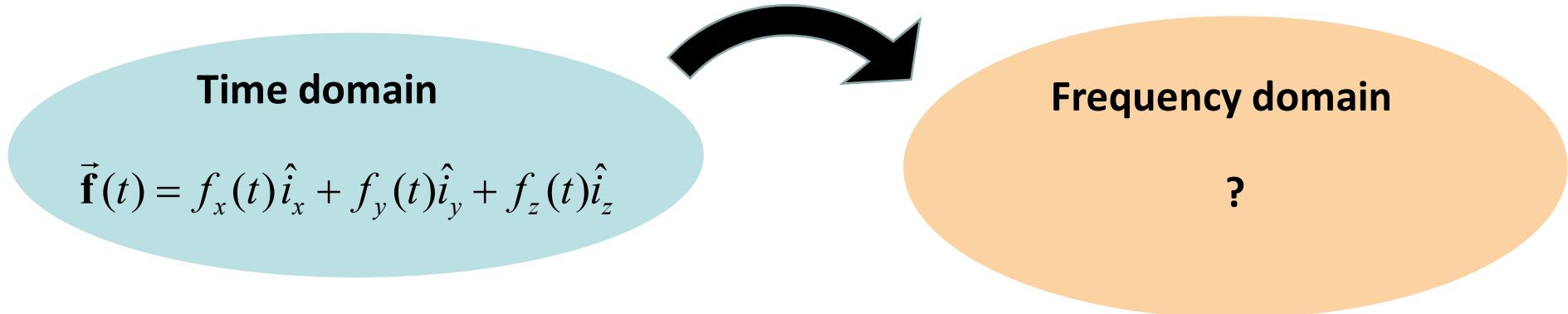
$$f(t, x, y, z) = \left(\frac{1}{2\pi} \right)^4 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(\omega, u, v, w) e^{j\omega t} e^{jux} e^{jvy} e^{jwz} d\omega du dv dw \quad \text{4D-IFT}$$

Frequency domain

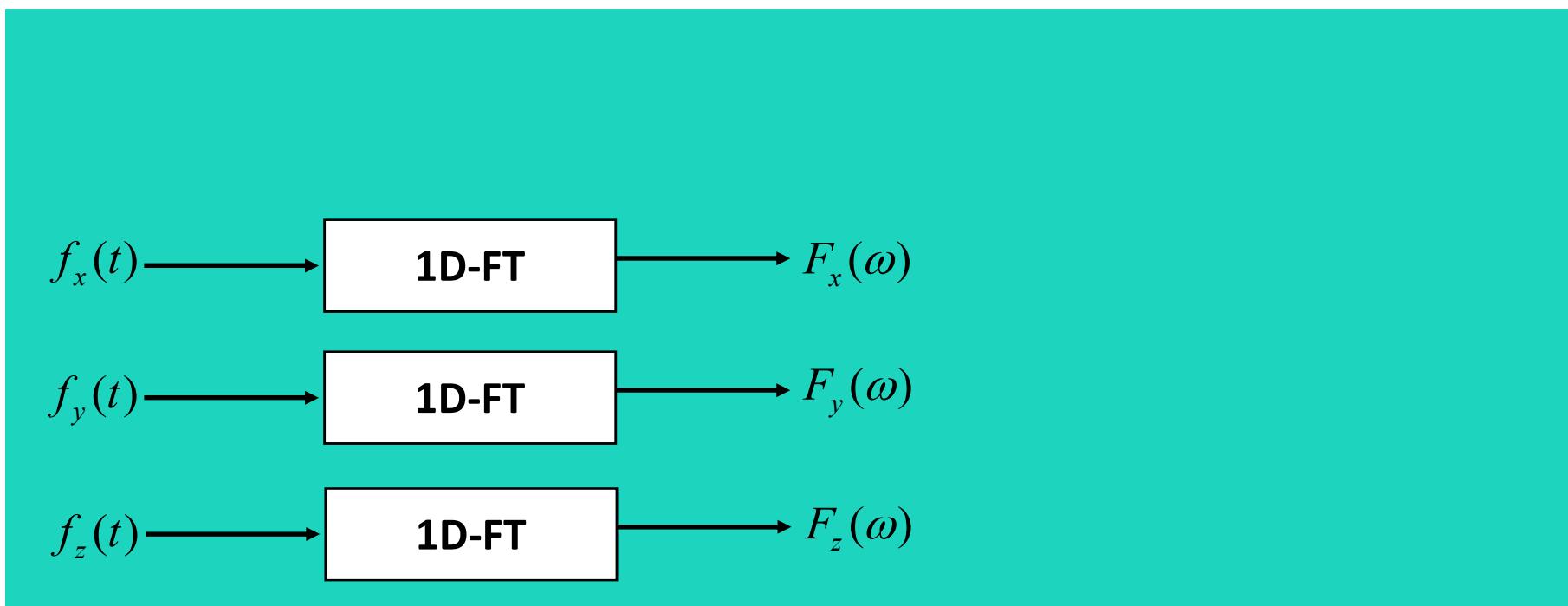
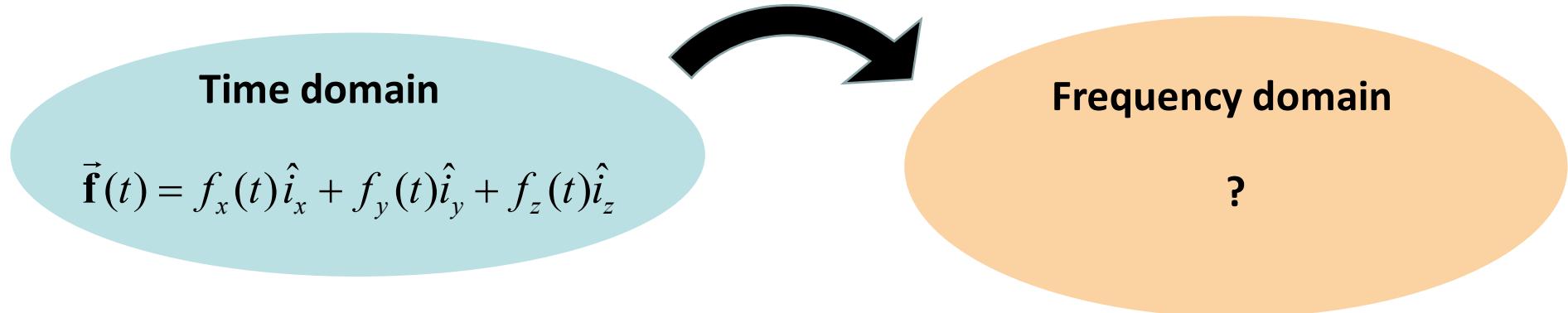
- Fourier Transform and functions of n variables
- Fourier Transform and vector functions
- Fourier Transform and vector functions of n variables

- 1) How to jump back from the Frequency domain to the Time domain
- 2) Time domain derivative and Fourier Transform

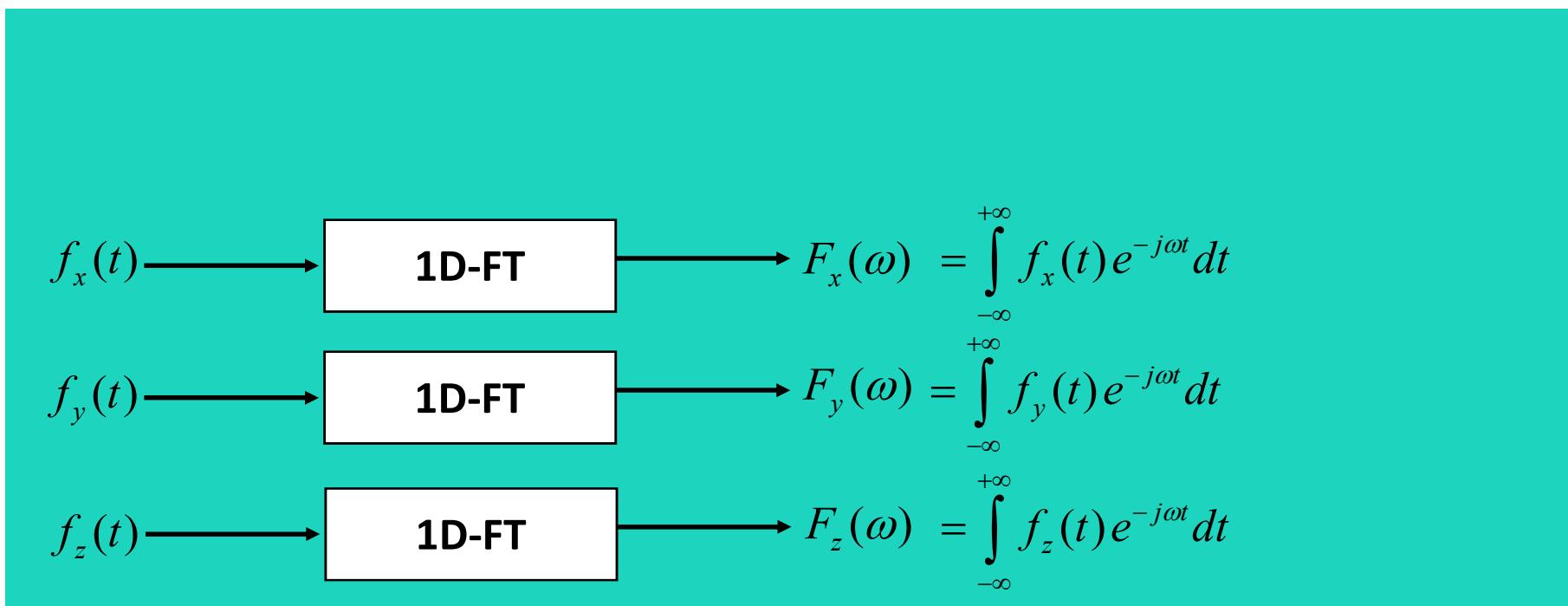
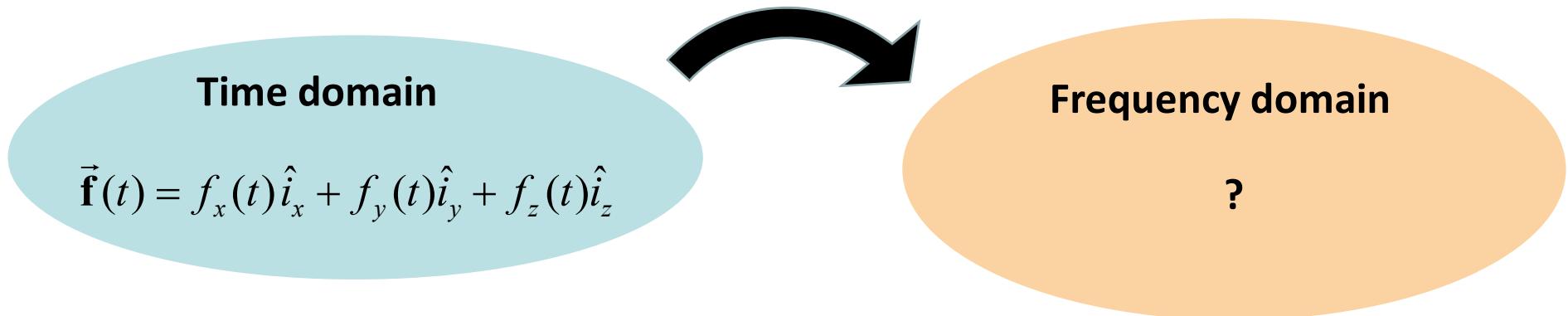
Fourier Transform and vector functions



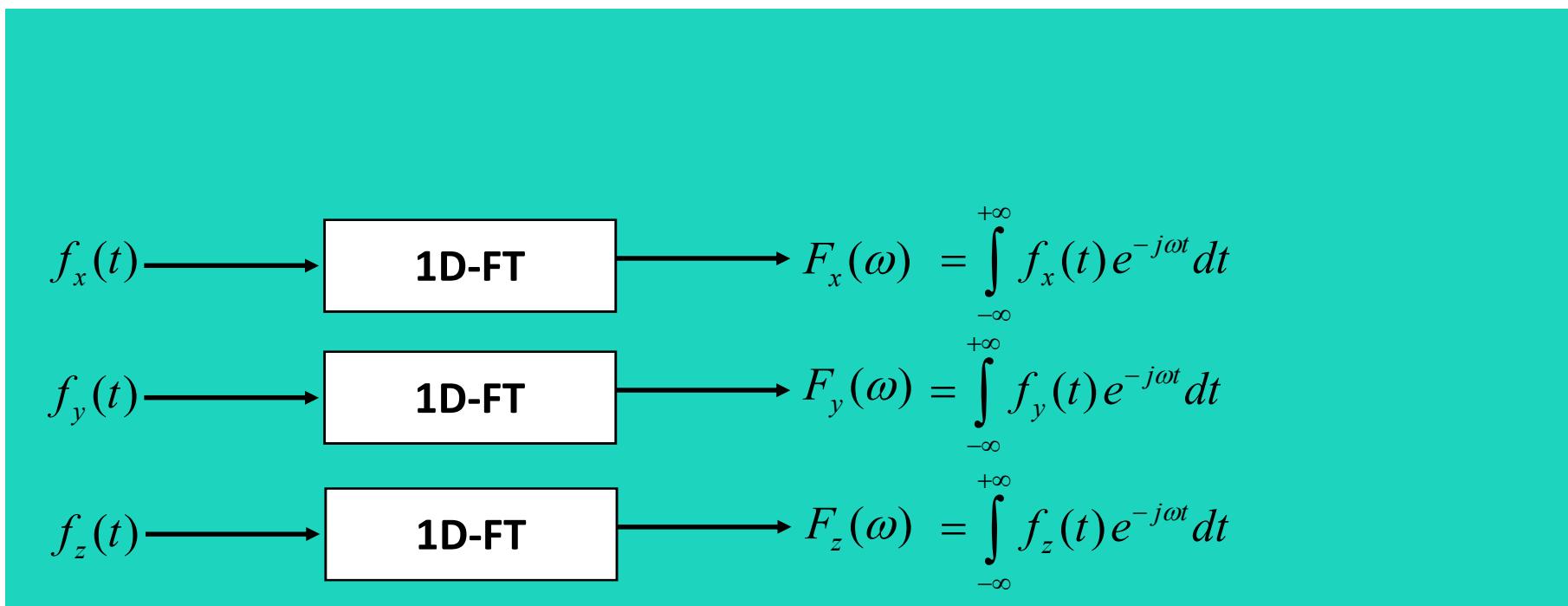
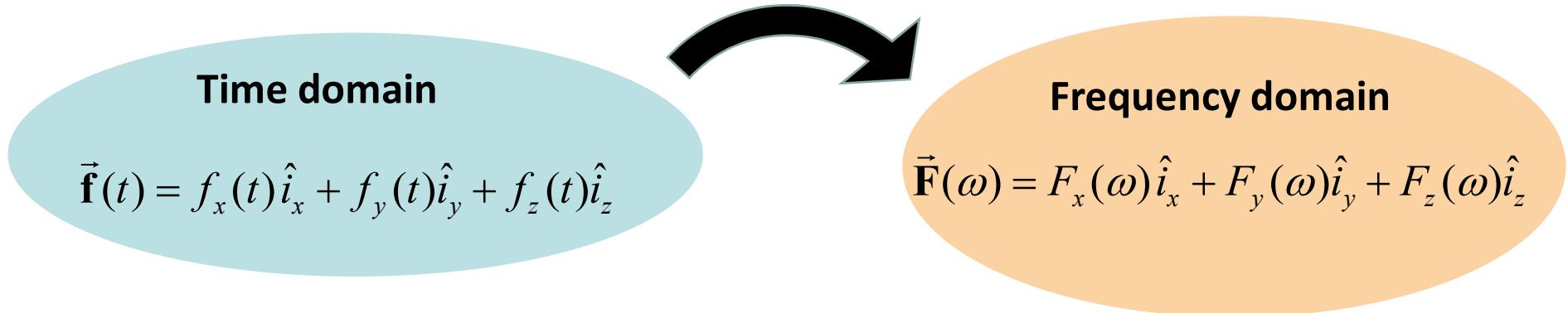
Fourier Transform and vector functions



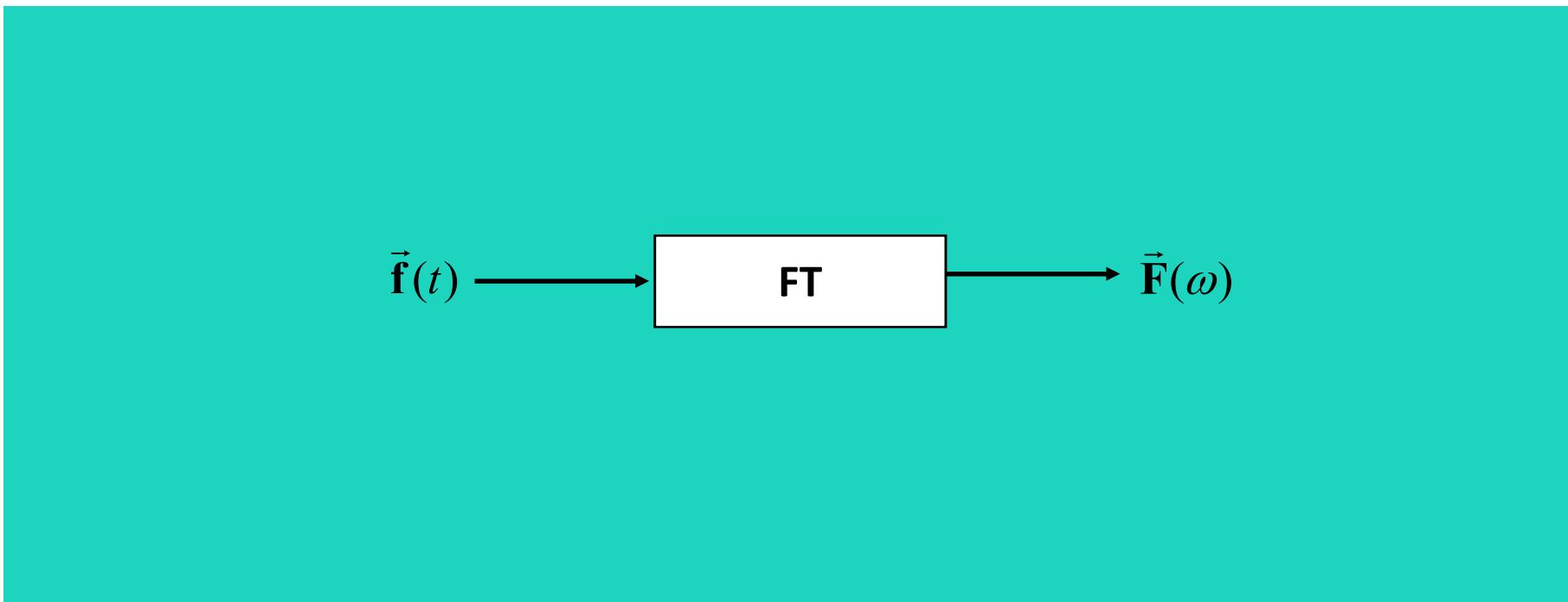
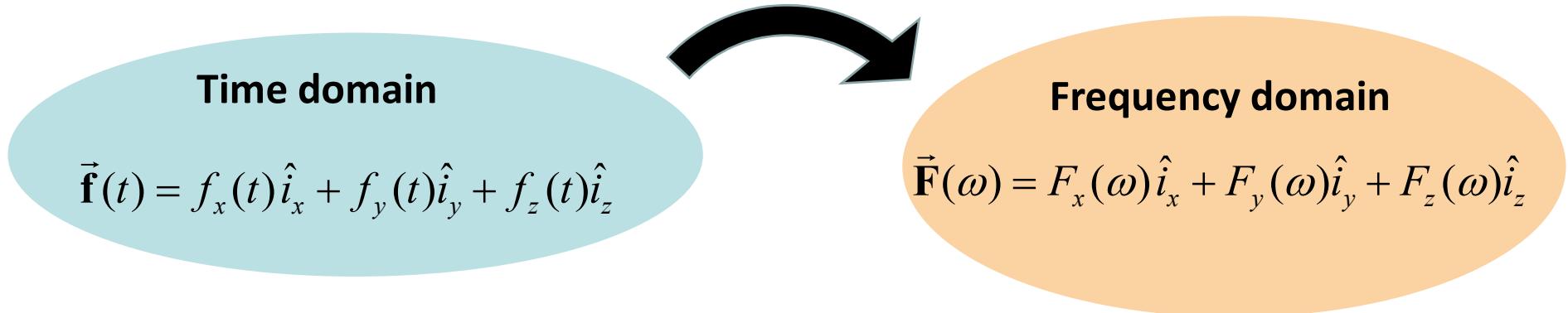
Fourier Transform and vector functions



Fourier Transform and vector functions



Fourier Transform and vector functions



Fourier Transform and vector functions

Time domain

$$\vec{f}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z$$

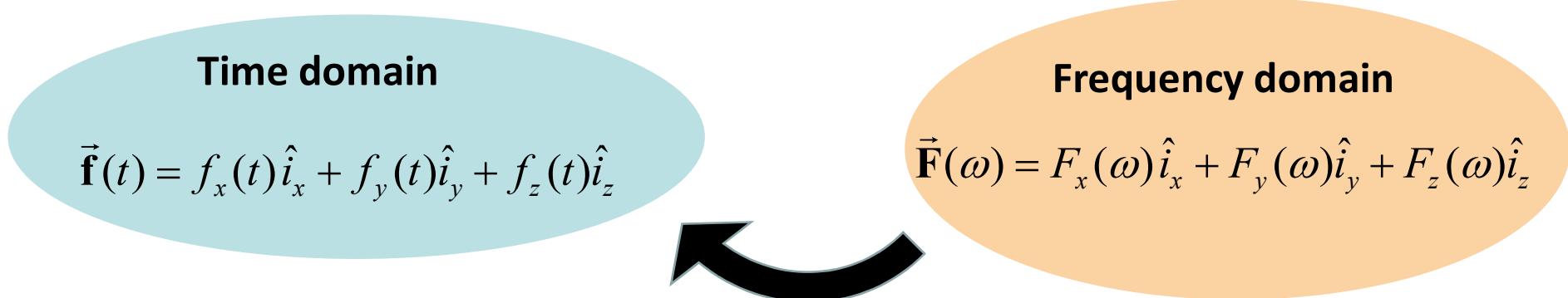
Frequency domain

$$\vec{F}(\omega) = F_x(\omega)\hat{i}_x + F_y(\omega)\hat{i}_y + F_z(\omega)\hat{i}_z$$

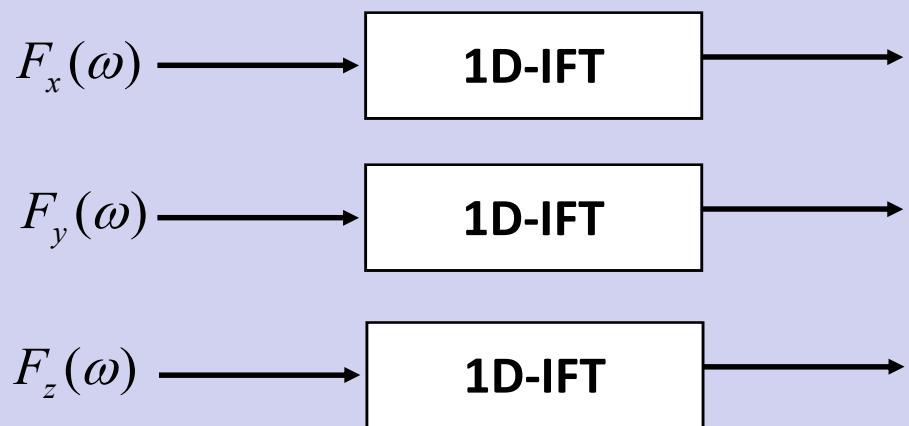


1) How to jump back from the Spectral domain to the Time domain

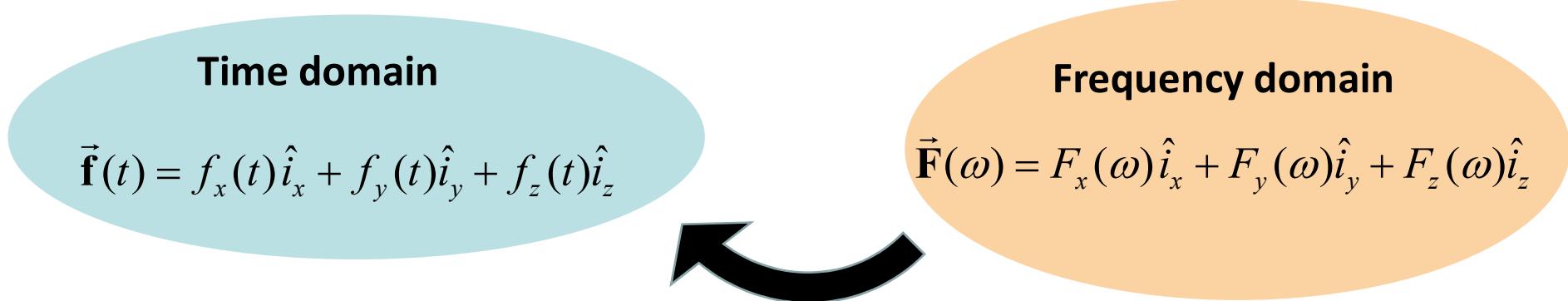
Fourier Transform and vector functions



1) How to jump back from the Spectral domain to the Time domain



Fourier Transform and vector functions

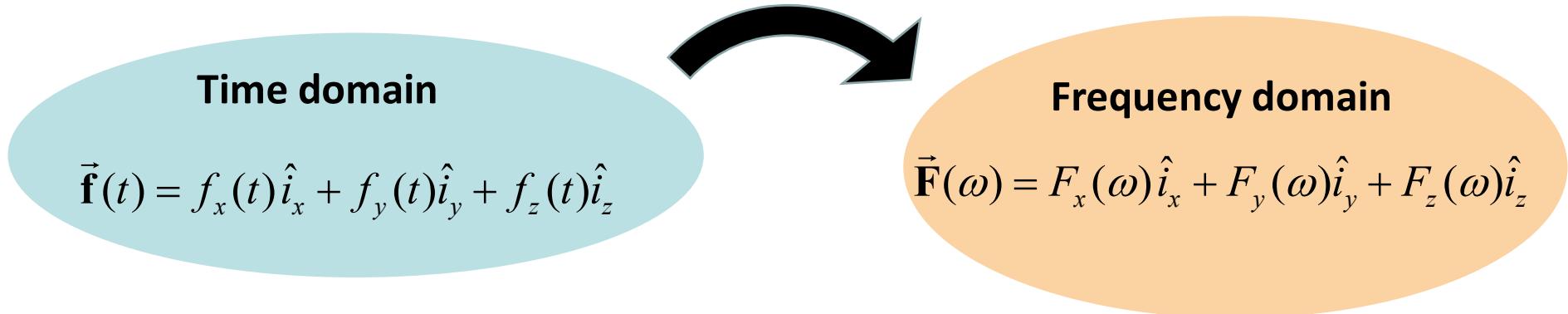


1) How to jump back from the Spectral domain to the Time domain

The diagram shows three parallel processes for jumping back from the Spectral domain to the Time domain. Each process consists of an input signal, a "1D-IFT" block, and a corresponding output equation:

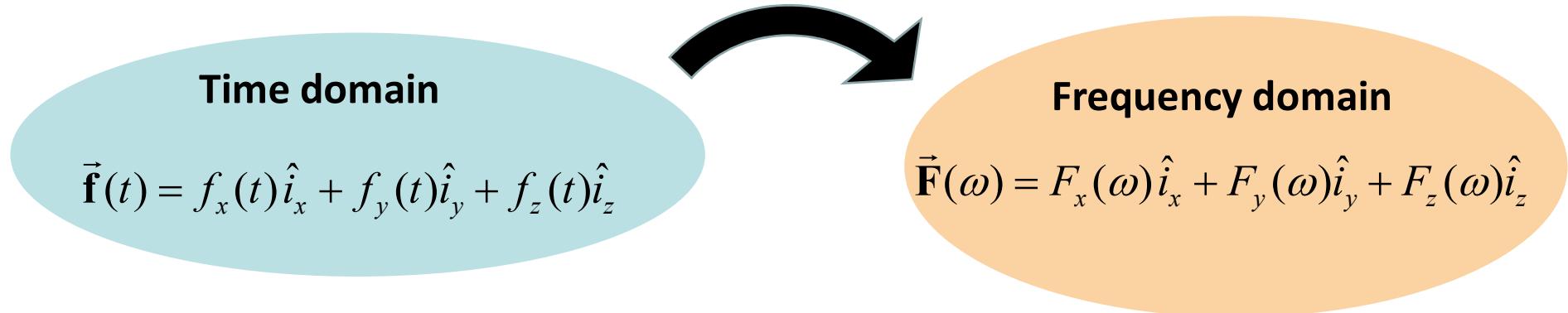
- Input: $F_x(\omega)$ → 1D-IFT → Output: $f_x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_x(\omega) e^{j\omega t} d\omega$
- Input: $F_y(\omega)$ → 1D-IFT → Output: $f_y(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_y(\omega) e^{j\omega t} d\omega$
- Input: $F_z(\omega)$ → 1D-IFT → Output: $f_z(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_z(\omega) e^{j\omega t} d\omega$

Fourier Transform and vector functions

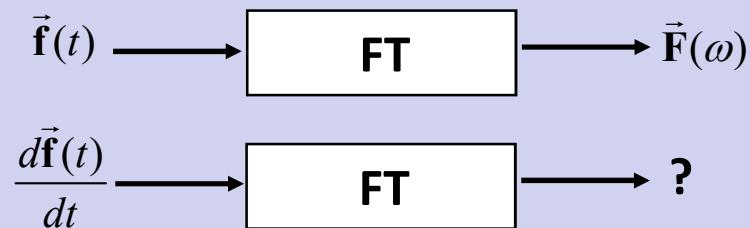


2) Time domain derivative and Fourier Transform

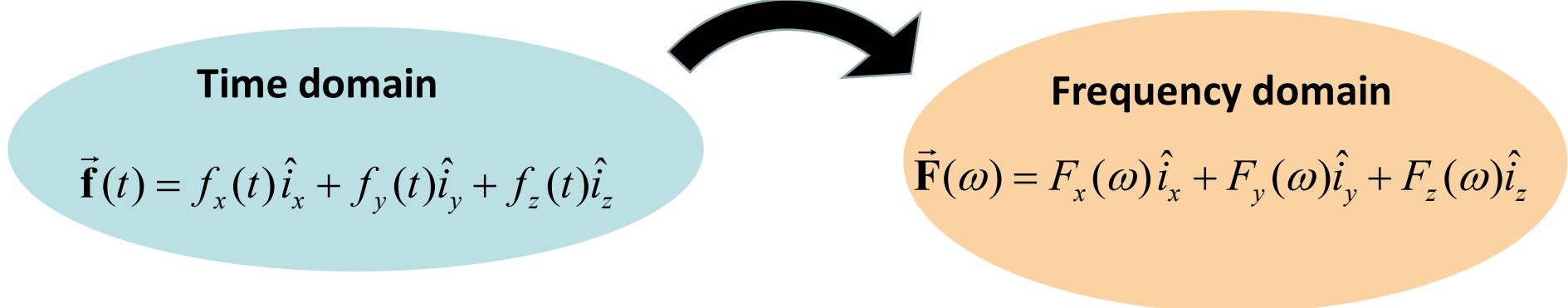
Fourier Transform and vector functions



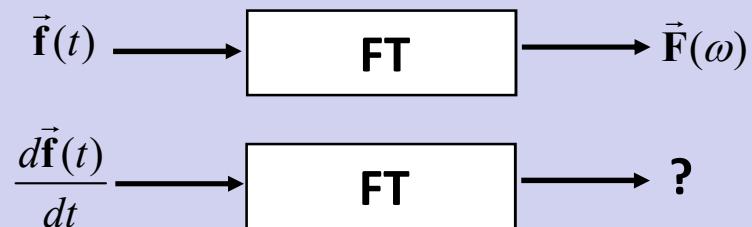
2) Time domain derivative and Fourier Transform



Fourier Transform and vector functions

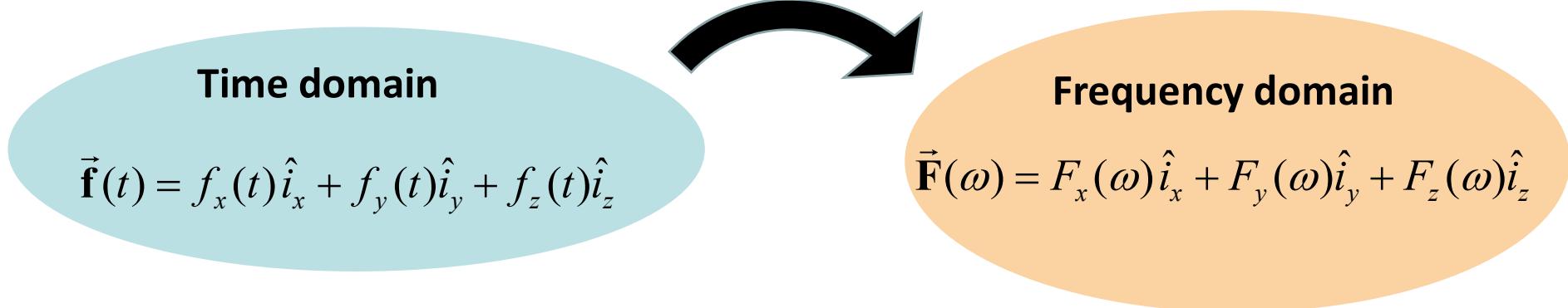


2) Time domain derivative and Fourier Transform

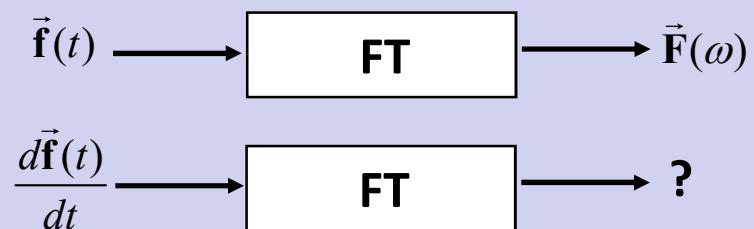


$$\frac{d\vec{f}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

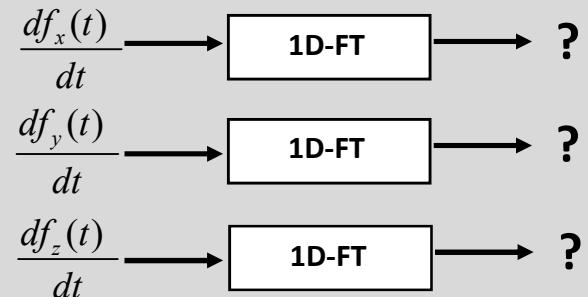
Fourier Transform and vector functions



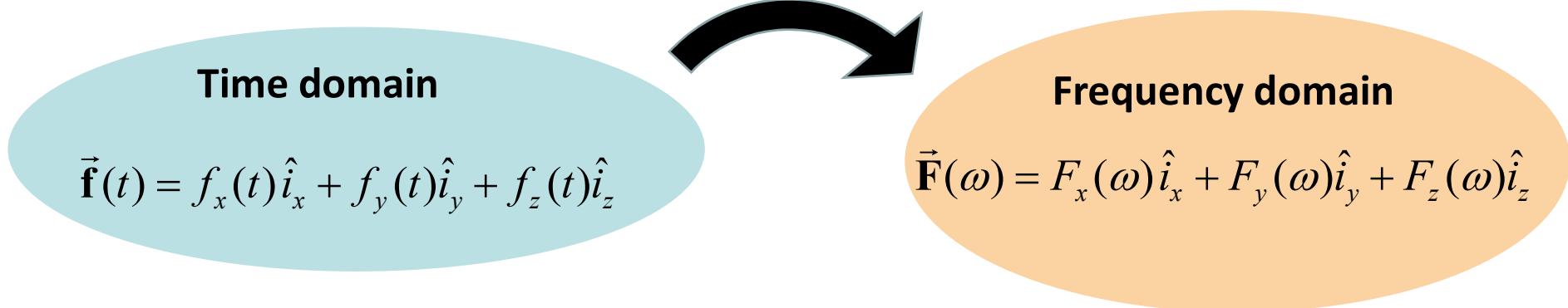
2) Time domain derivative and Fourier Transform



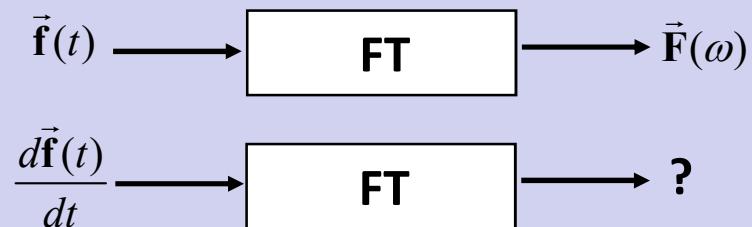
$$\frac{d\vec{f}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$



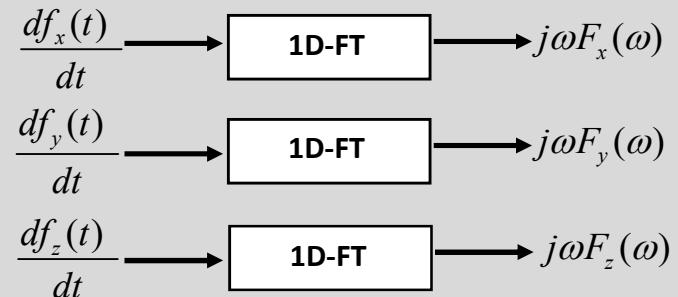
Fourier Transform and vector functions



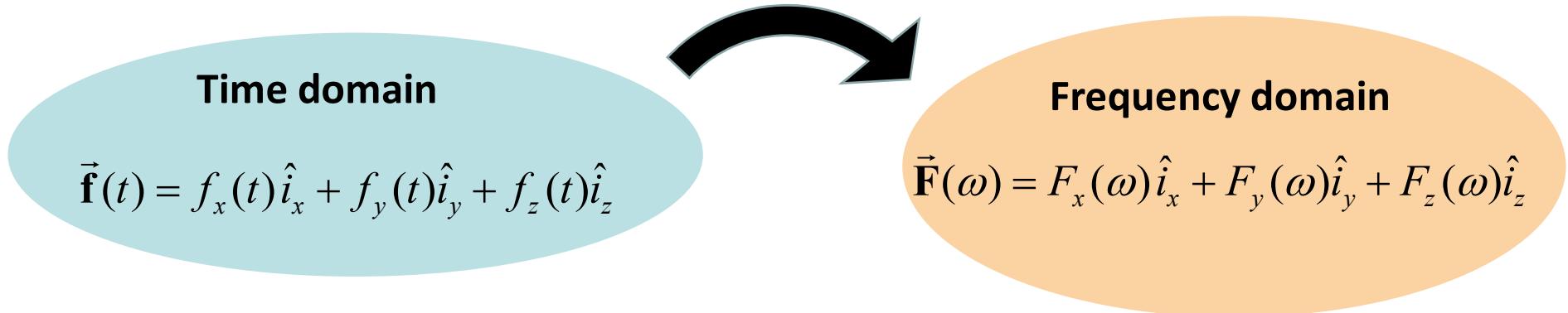
2) Time domain derivative and Fourier Transform



$$\frac{d\vec{f}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$



Fourier Transform and vector functions



2) Time domain derivative and Fourier Transform

$$\vec{f}(t) \xrightarrow{\text{FT}} \vec{F}(\omega)$$

$$\frac{d\vec{f}(t)}{dt} \xrightarrow{\text{FT}}$$

$$j\omega F_x(\omega)\hat{i}_x + j\omega F_y(\omega)\hat{i}_y + j\omega F_z(\omega)\hat{i}_z$$

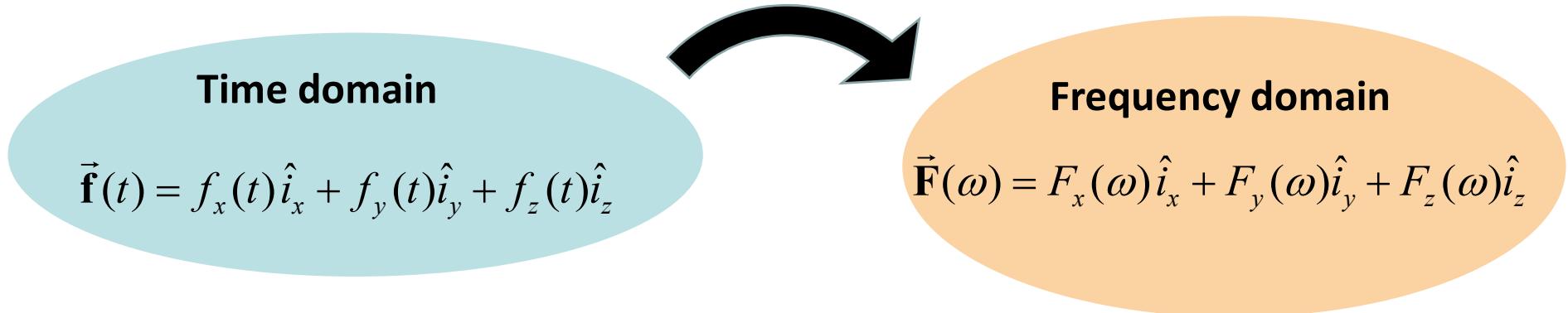
$$\frac{d\vec{f}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \xrightarrow{1\text{-D-FT}} j\omega F_x(\omega)$$

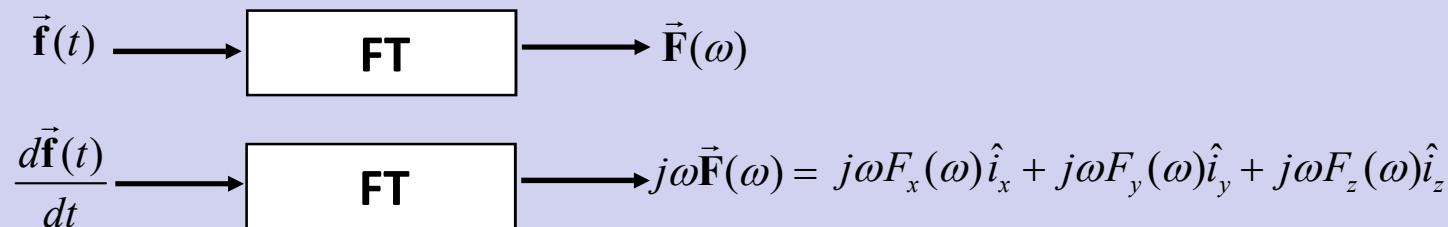
$$\frac{df_y(t)}{dt} \xrightarrow{1\text{-D-FT}} j\omega F_y(\omega)$$

$$\frac{df_z(t)}{dt} \xrightarrow{1\text{-D-FT}} j\omega F_z(\omega)$$

Fourier Transform and vector functions



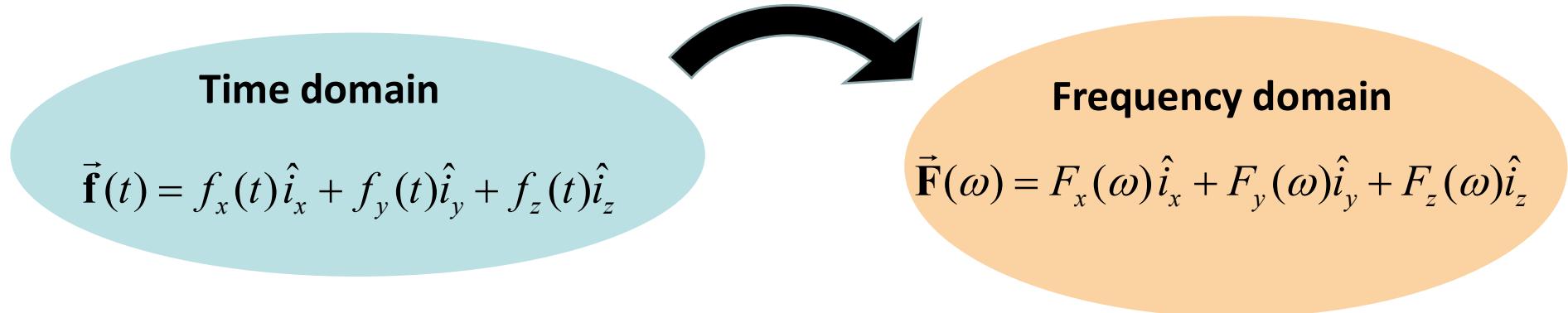
2) Time domain derivative and Fourier Transform



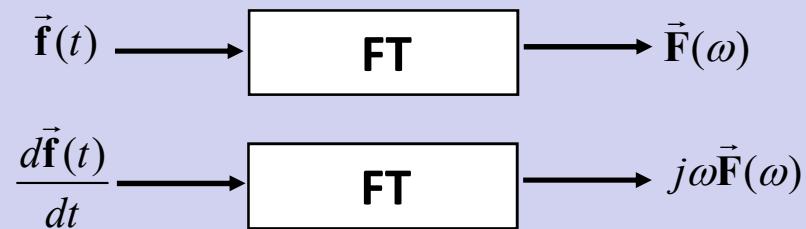
$$\frac{d\vec{f}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$



Fourier Transform and vector functions



2) Time domain derivative and Fourier Transform



Frequency domain

- Fourier Transform and functions of n variables
- Fourier Transform and vector functions
- Fourier Transform and vector functions of n variables**

1) How to jump back from the Frequency domain to the Time domain

2) Time domain derivative and Fourier Transform