# Campi Elettromagnetici

## Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

**a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno**


**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Maxwell equations

**James Clerk Maxwell 1831-1879**

## Differential form

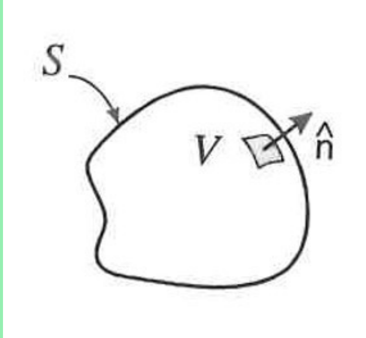
$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

# Maxwell equations: integral form

# … mathematical tools that we will exploit today…

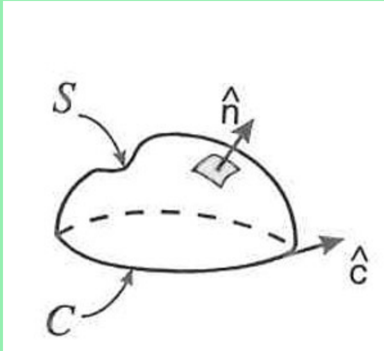
## I) Gauss theorem

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}$$




## II) Stokes theorem

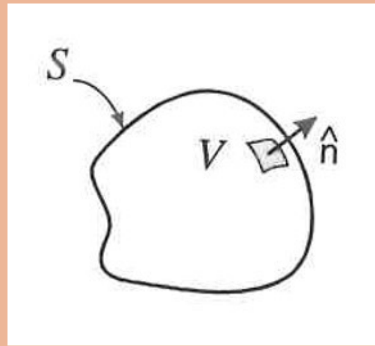
$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}} = \oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$



# … mathematical tools that we will exploit today…

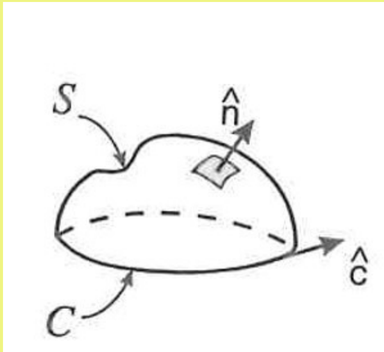
## I) Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

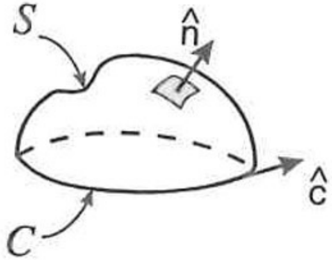



## II) Stokes theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



# Maxwell equations: integral form




$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad \Rightarrow$$

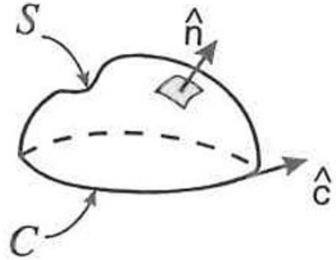
**Lenz-Neumann law**

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\frac{d}{dt}\iint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ $\Rightarrow$ $\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = 0$ **Kirchhoff’s second law**

# Maxwell equations: integral form




$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

**Ampere-Faraday law**

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t)$$

# Maxwell equations

## Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

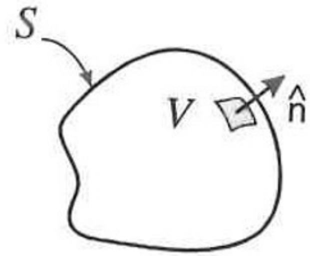
## Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \end{cases}$$

# Maxwell equations: integral form

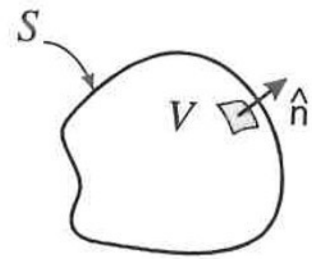
$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

# Maxwell equations: integral form




$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

# Maxwell equations: integral form

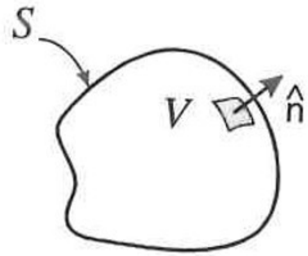



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV \, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV \, \rho(\vec{\mathbf{r}},t)$$

# Maxwell equations: integral form

**Gauss theorem**

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

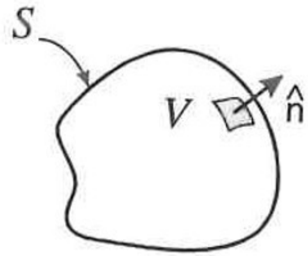



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

# Maxwell equations: integral form

**Gauss theorem**

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

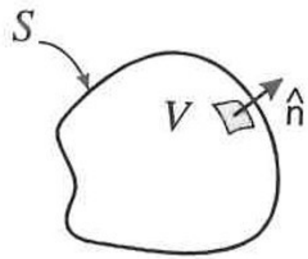



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

$$\downarrow$$

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \;=$$

# Maxwell equations: integral form

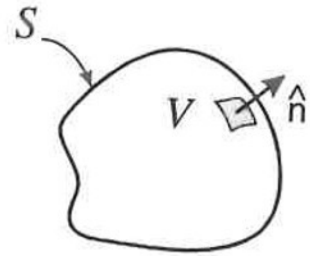



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

$$\downarrow \qquad \downarrow$$

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \;=\; q(t)$$

# Maxwell equations: integral form




$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

➡

**Coulomb law**

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)$$

# Maxwell equations

## Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

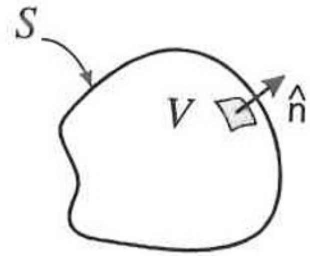
## Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = q(t) \end{cases}$$

# Maxwell equations: integral form

$$\nabla \cdot \vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)=0$$

# Maxwell equations: integral form

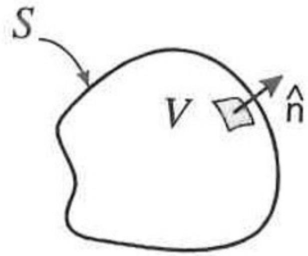



$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV \; \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

# Maxwell equations: integral form

**Gauss theorem**

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

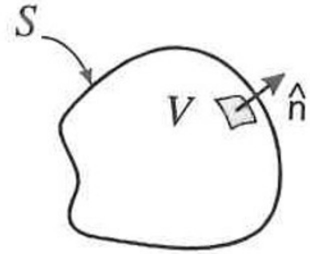



$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

$$\downarrow$$

$$\oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

# Maxwell equations: integral form




$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

# Maxwell equations

## Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

## Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0 \end{cases}$$

# … mathematical tools that we will exploit today…

$$\nabla \cdot \left( \nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) = 0$$

# Maxwell equations

## Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\Rightarrow \quad \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = 0$$

# Maxwell equations

## Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

**Current density equation**

## Integral form

**Current density equation**

# Maxwell equations

## Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

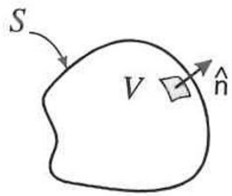
**Current density equation**

## Integral form

**Current density equation**

**Gauss theorem**

$$\iiint_V dV \, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



# Maxwell equations

## Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

**Current density equation**

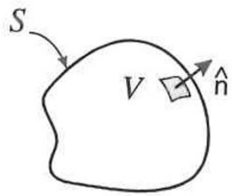
## Integral form

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

**Current density equation**

**Gauss theorem**

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



# Maxwell equations

## Integral form

$$\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

**Current density equation**

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ ➡ $\oiint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0$ **Kirchhoff's first law**

# Maxwell equations

## Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

## Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0 \end{cases}$$

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$