Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Maxwell equations

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

		Unità di misura
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$:	Campo elettrico	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)$:	Induzione elettrica	Coulomb/m^2
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$:	Campo magnetico	Ampere/m
$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)$:	Induzione magnetica	Weber/m^2
$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$:	Densità di corrente	Ampere/m^2
$\rho(\vec{\mathbf{r}},t)$:	Densità di carica	Coulomb/m^3

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

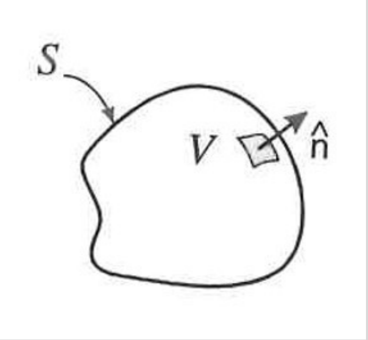
Mathematical tools to be exploited

Mathematics

… memo…

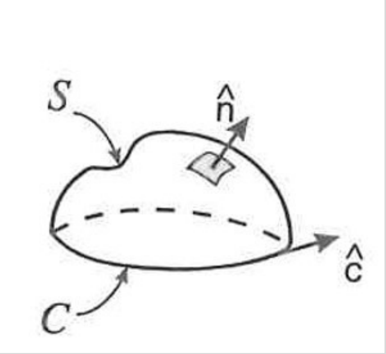
I) Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oint\!\!\!\oint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



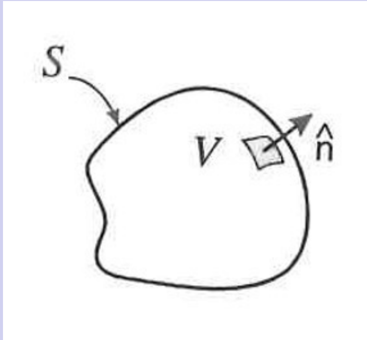
II) Curl

$$\left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \lim_{S \to 0} \frac{1}{S} \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



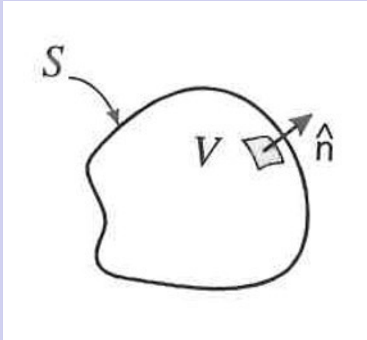
- Scalar quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

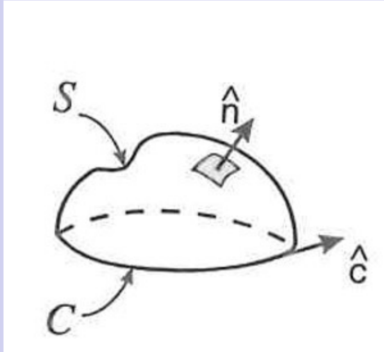


Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

Curl

$$\left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \lim_{S \to 0} \frac{1}{S} \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



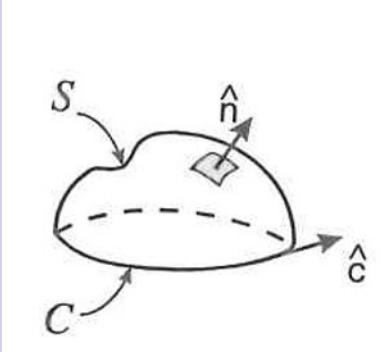
- Vector quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Curl

$$\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}}=\lim_{S\to 0}\frac{1}{S}\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$



Stokes theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$

Cartesian Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = A_x(x,y,z,t)\hat{i}_x + A_y(x,y,z,t)\hat{i}_y + A_z(x,y,z,t)\hat{i}_z$$

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{i}_z$$

Spherical Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t)=A_r(r,\vartheta,\varphi,t)\hat{i}_r + A_\vartheta(r,\vartheta,\varphi,t)\hat{i}_\vartheta + A_\varphi(r,\vartheta,\varphi,t)\hat{i}_\varphi$$

$$\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right)+\frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\vartheta\right)+\frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r\sin\vartheta}\left[\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\varphi\right)-\frac{\partial A_\vartheta}{\partial\varphi}\right]\hat{i}_r+\frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}\left(rA_\varphi\right)\right]\hat{i}_\vartheta+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\vartheta\right)-\frac{\partial A_r}{\partial\vartheta}\right]\hat{i}_\varphi$$

Color legend

New formulas, important considerations,
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Memo

Mathematical tools to be exploited

Mathematics

Maxwell equations

James Clerk Maxwell 1831-1879

Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
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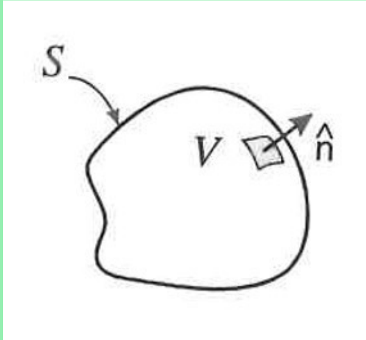
Maxwell equations: integral form

… mathematical tools that we will exploit today…

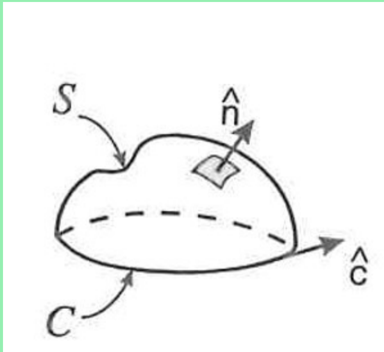
I) Gauss theorem

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}$$



II) Stokes theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}} = \oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$



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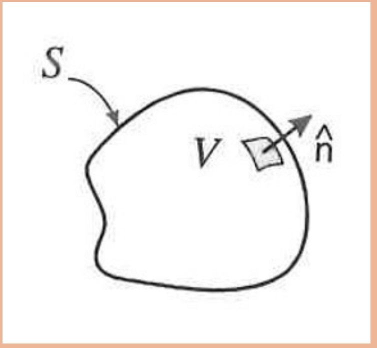

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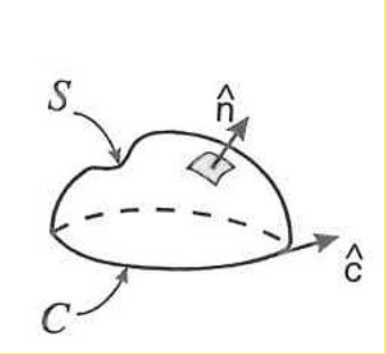
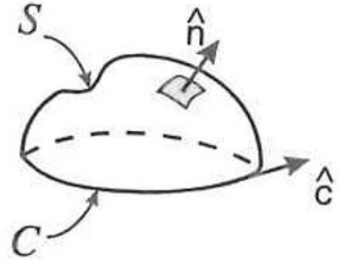


Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

Maxwell equations: integral form



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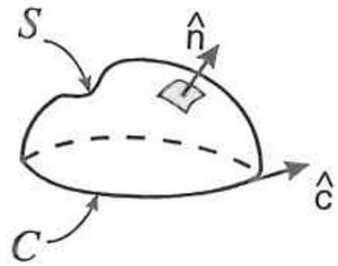
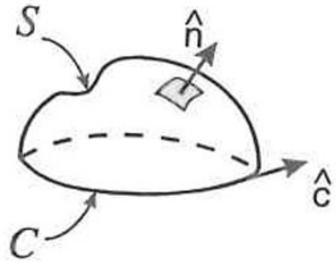


$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad \Rightarrow \quad \iint_S dS \left(\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = -\iint_S dS \frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) \cdot \hat{n} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{c}$$



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Maxwell equations: integral form

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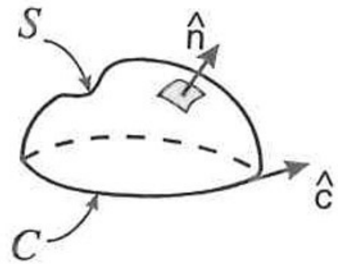
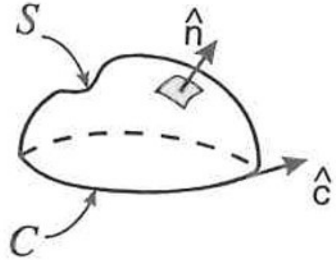


$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \Rightarrow \iint_S dS\left(\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=-\iint_S dS\,\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}$$

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} \quad =$$

Maxwell equations: integral form



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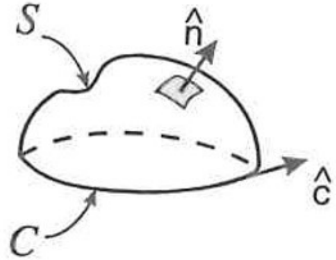
$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form



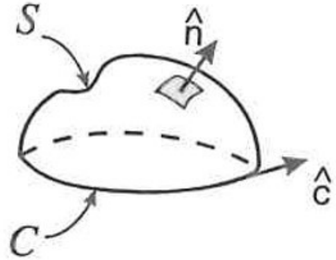
$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

➡

Lenz-Neumann law

$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form



$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\quad\Rightarrow$$

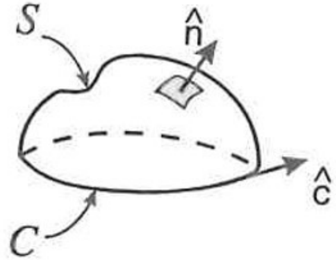
Lenz-Neumann law

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}}=-\frac{d}{dt}\iint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ $\Rightarrow$ $\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}}=0$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad \Rightarrow$$

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$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt} = 0\right) \Rightarrow \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = 0$ **Kirchhoff's second law**

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

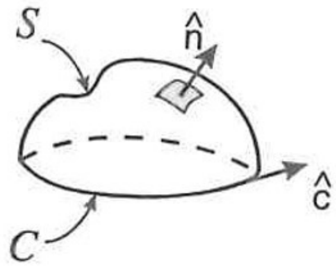
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \end{cases}$$

Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

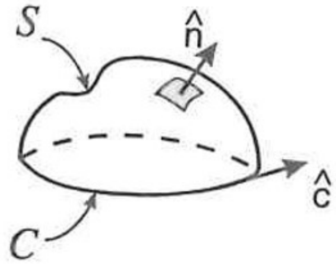


$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS \left(\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = \iint_S dS \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}} + \iint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



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$$\downarrow$$

Maxwell equations: integral form

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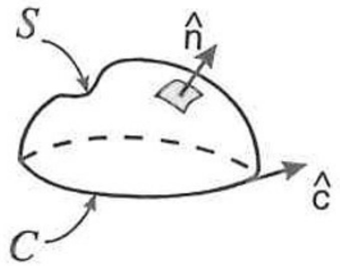

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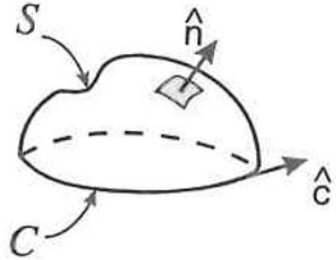
$$\downarrow$$

$$\oint_C dc\,\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}}=$$

Maxwell equations: integral form



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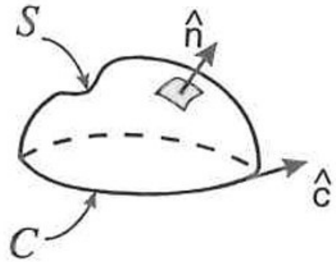
$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form



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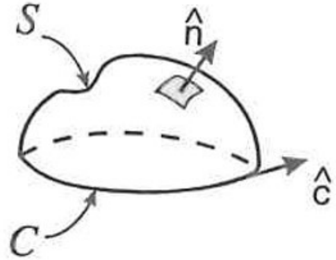
Maxwell equations: integral form



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➡

Ampere-Faraday law

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t)$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \end{cases}$$