Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

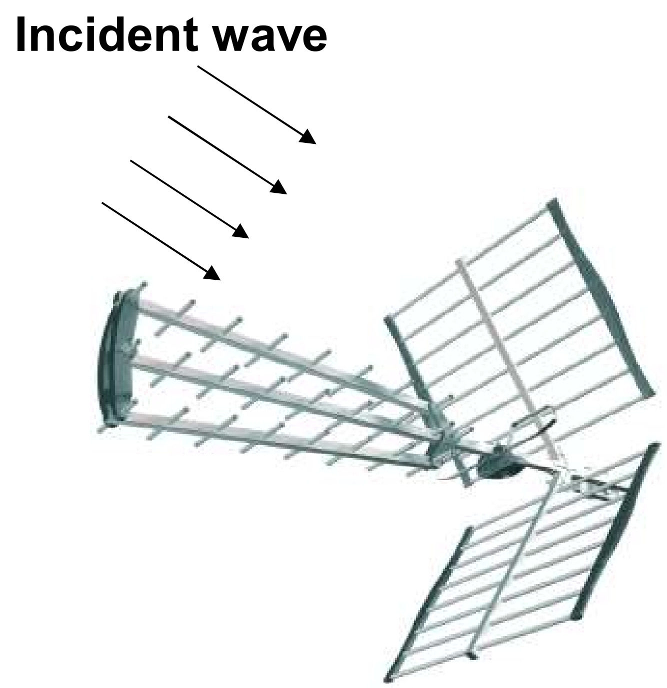
Receiving mode

When an antenna is operating as a receiving antenna, it extracts a certain amount of power from an incident electromagnetic wave.

Since an incident wave comes from a far distance may be thought of as a uniform (local) plane wave being intercepted by the antenna.

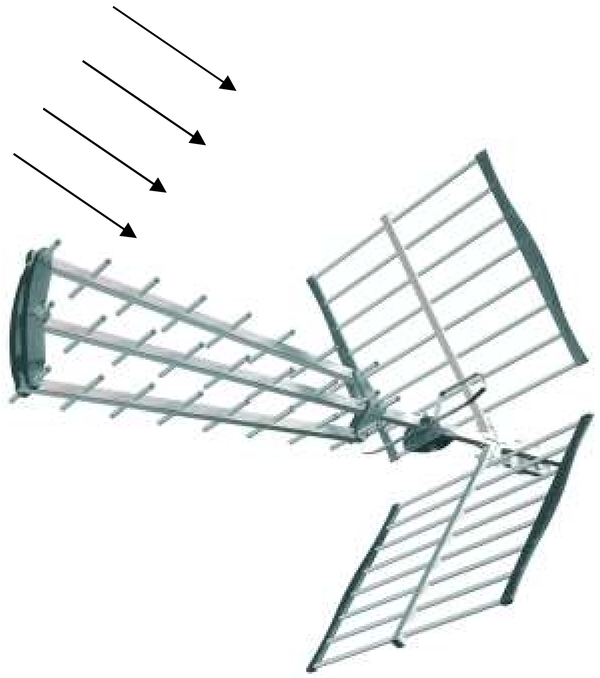
The use of the antenna in the receiving mode is shown in Figure.

The incident wave impinges upon the antenna, and it induces a voltage V_0 at the input terminals .



Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



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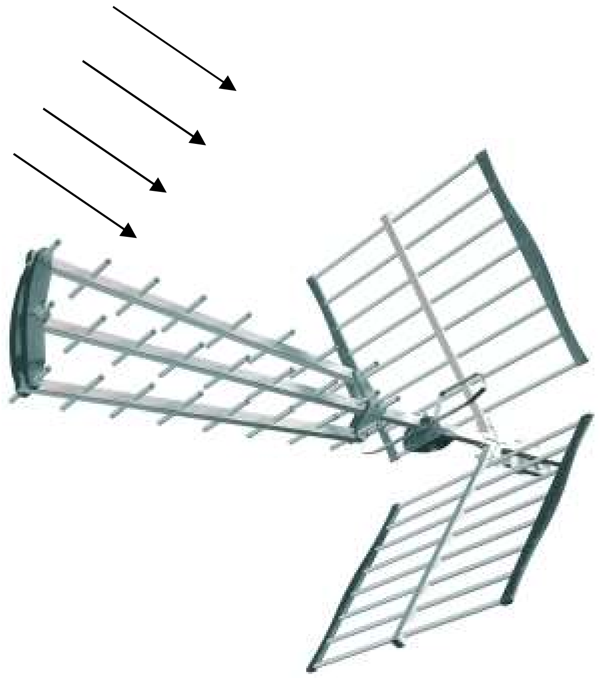

Rx effective length

Tx effective length $\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$

Fraunhofer region $\mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$

Elementary electrical dipole $\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$

Small loop antenna $\mathbf{l}(\vartheta,\varphi) = -j\beta\Delta S \sin\vartheta\, \hat{i}_\varphi$

It can be shown that **for an elementary electrical dipole or for a small loop antenna**, the following property is valid:

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

$\mathbf{l}$ is the tx antenna effective length

$\boldsymbol{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

Rx effective length

Elementary electrical dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

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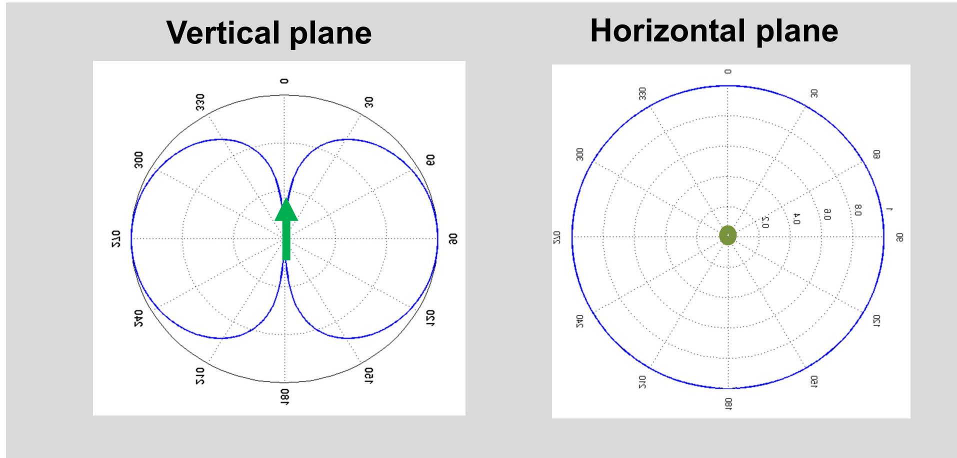
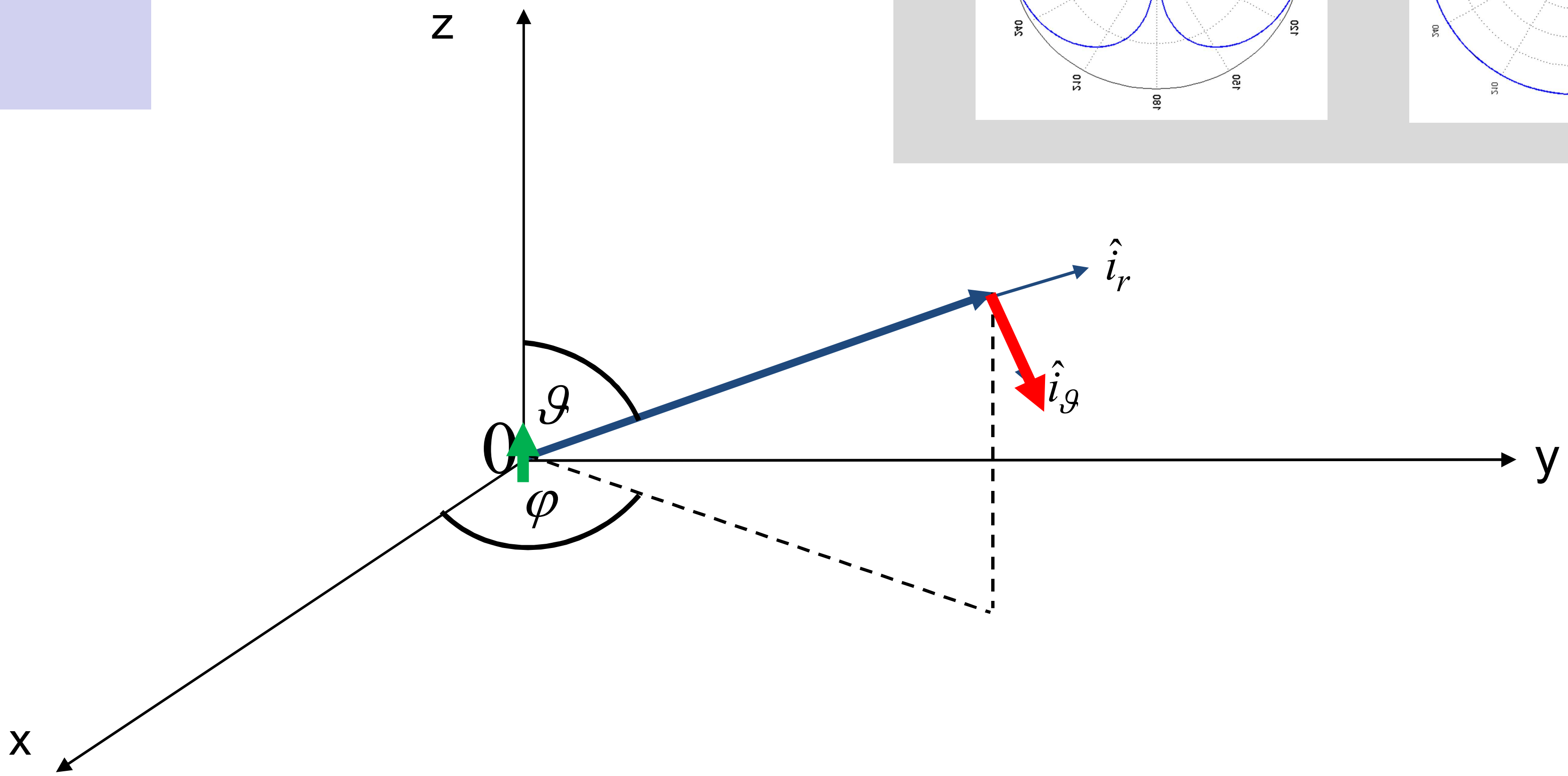




Rx effective length

Elementary electrical dipole

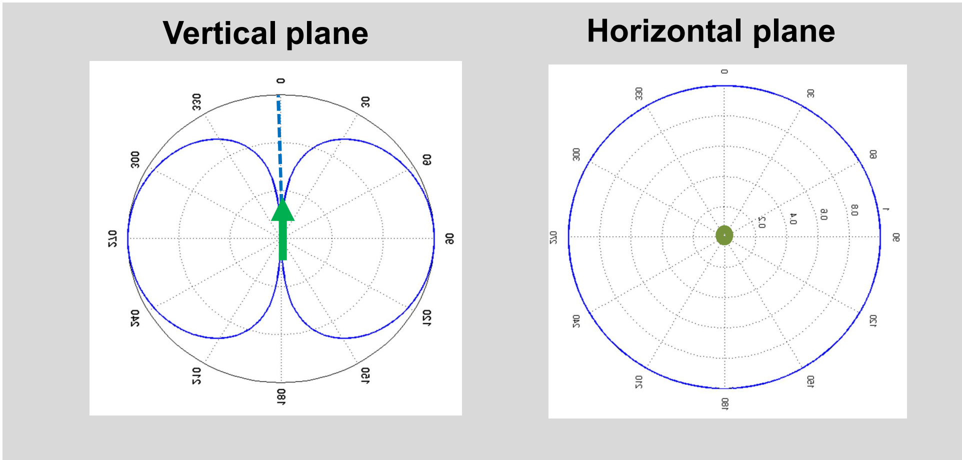
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\,\hat{i}_\vartheta$$

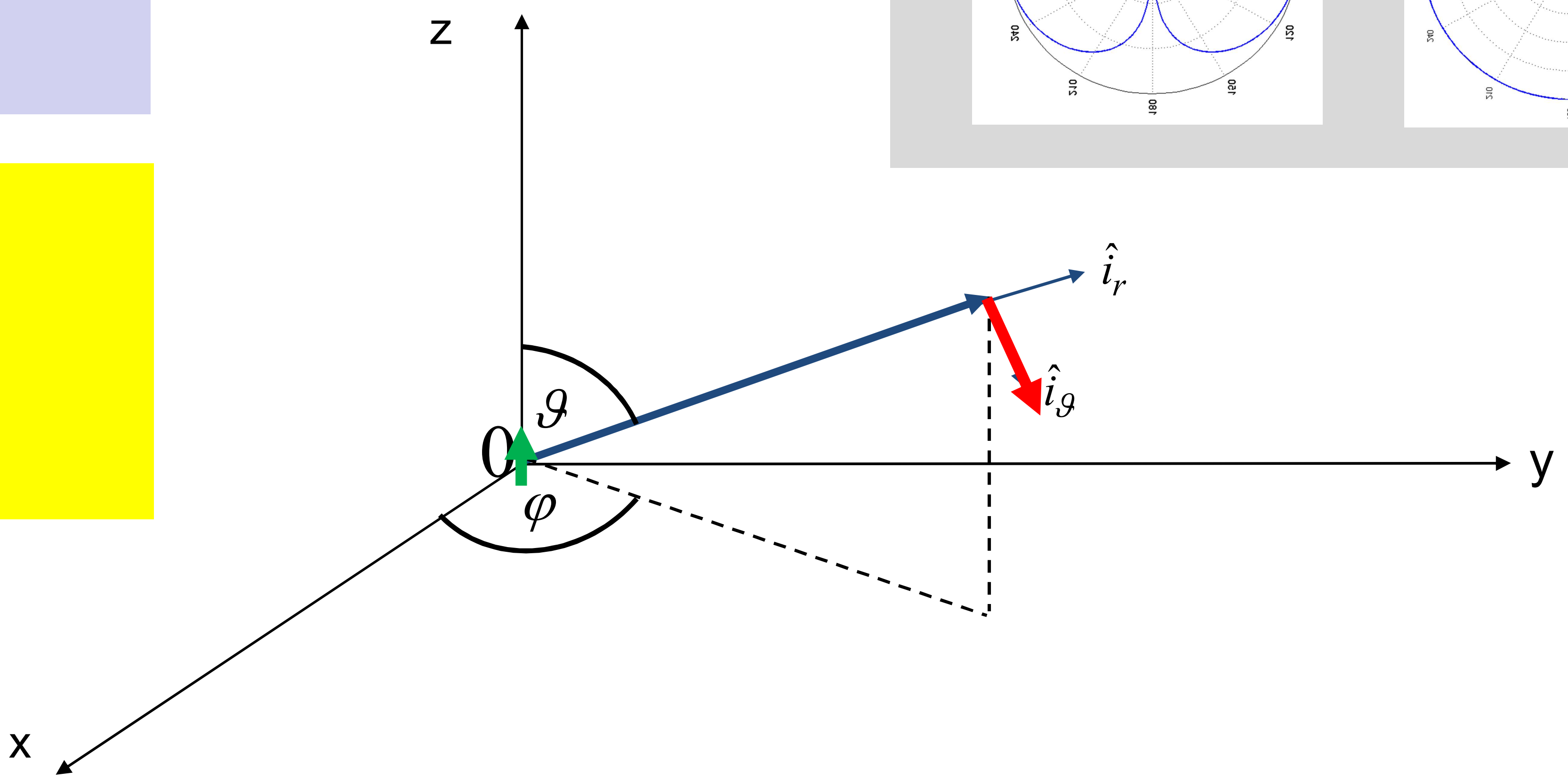
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

First example

$$\vartheta = 0$$

$$|V_0| = 0$$





Rx effective length

Elementary electrical dipole

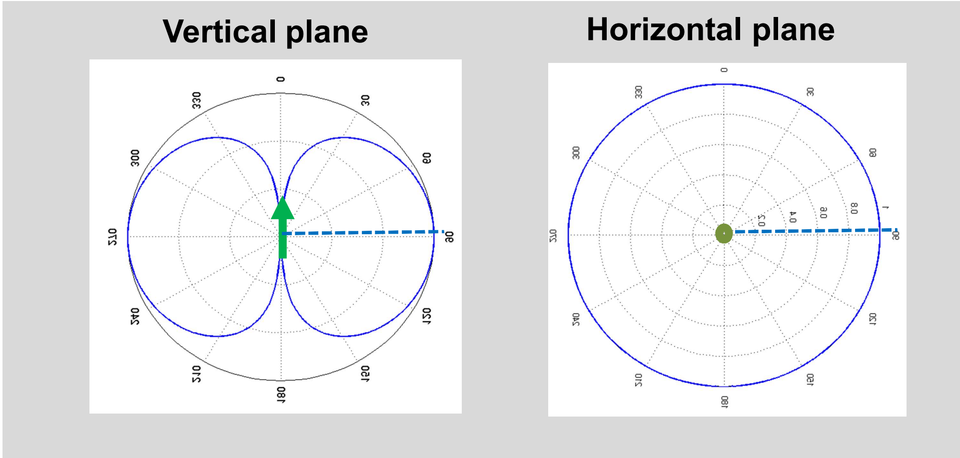
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta \, \hat{i}_\vartheta$$

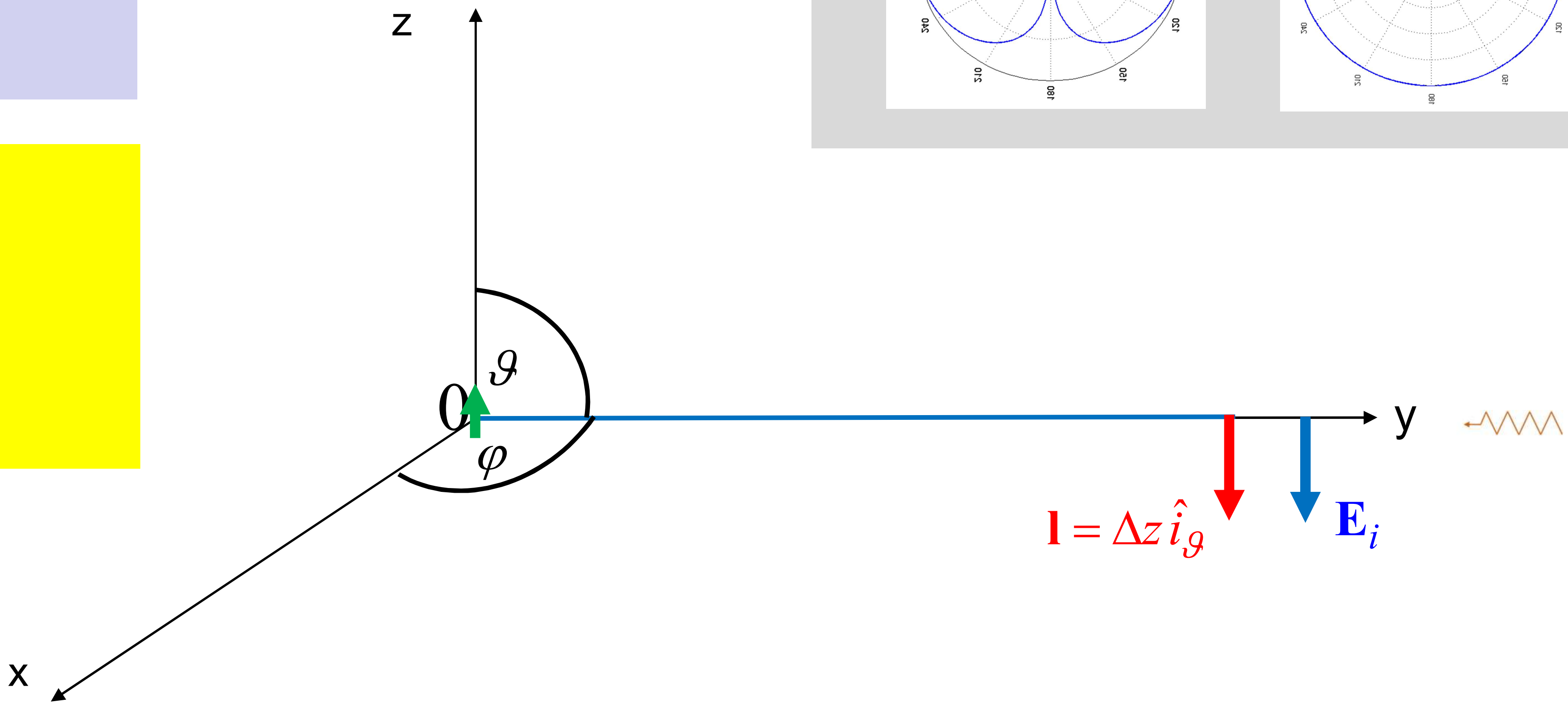
$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

Second example

$$\vartheta = \frac{\pi}{2}; \quad \varphi = \frac{\pi}{2}$$

$$|V_0| = |\Delta z \mathbf{E_i}|$$





Rx effective length

Elementary electrical dipole

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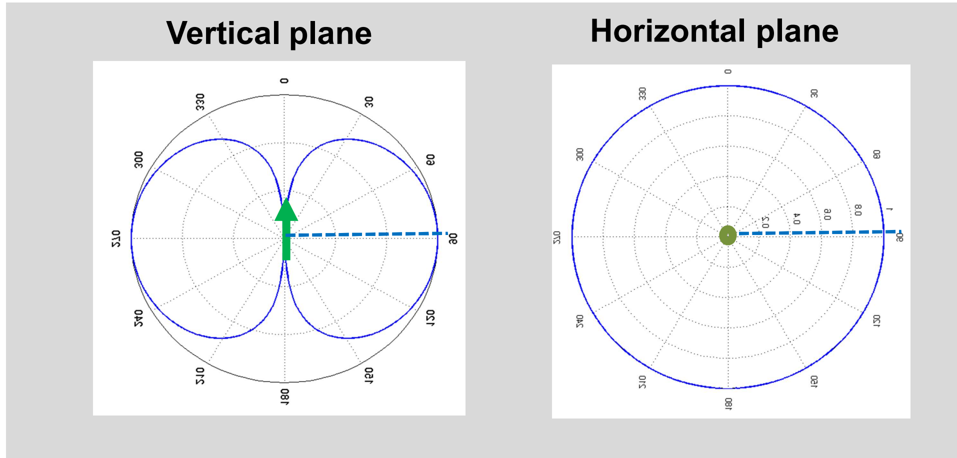
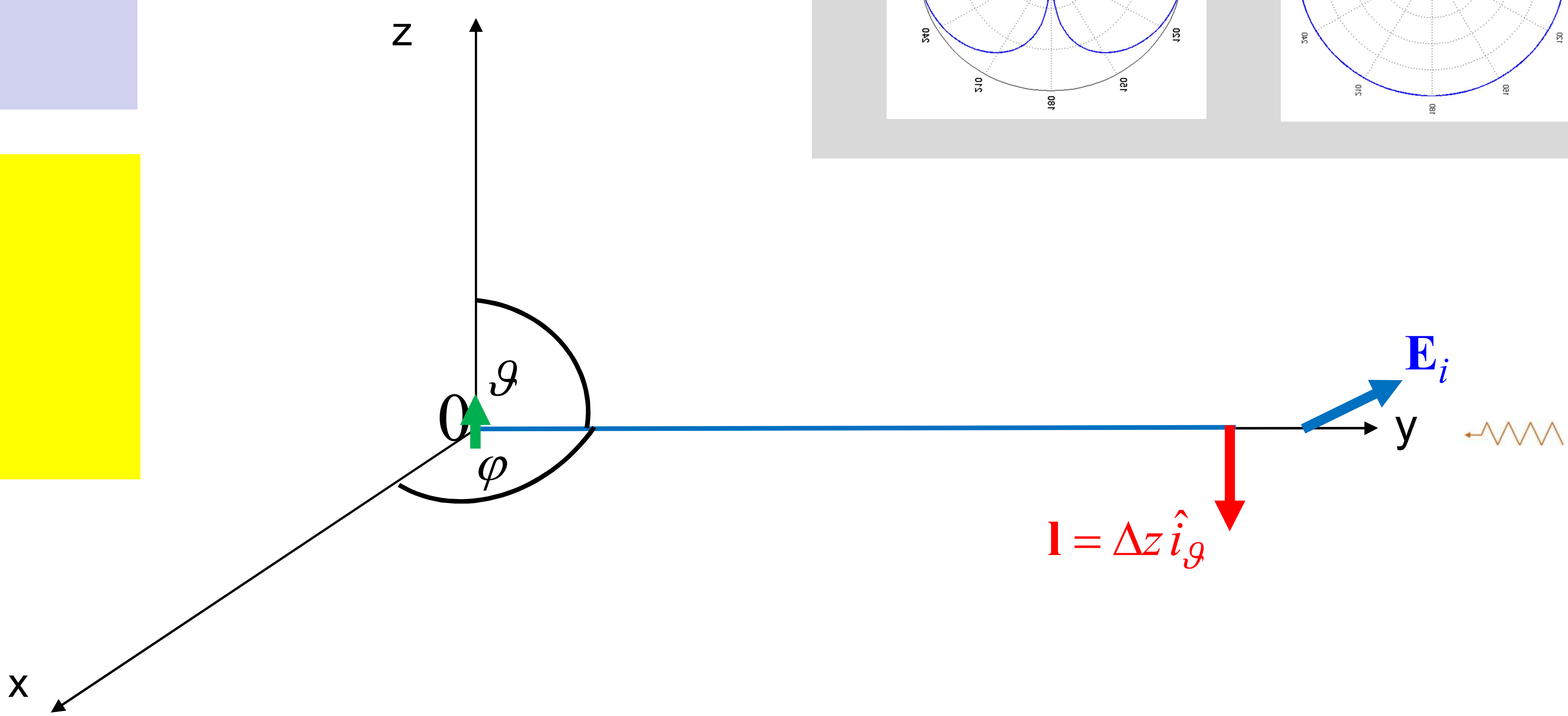
$$\vartheta = \frac{\pi}{2}; \quad \varphi = \frac{\pi}{2}$$

$$|V_0| = 0$$





Rx effective length

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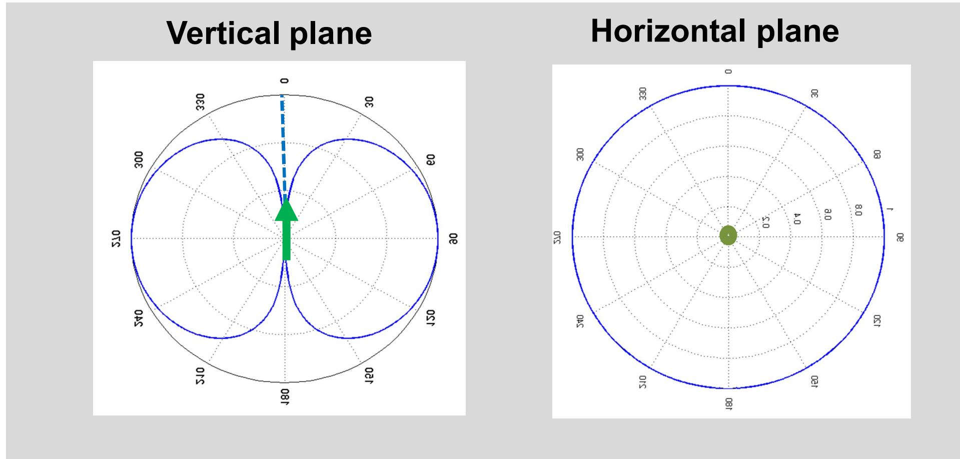

First example

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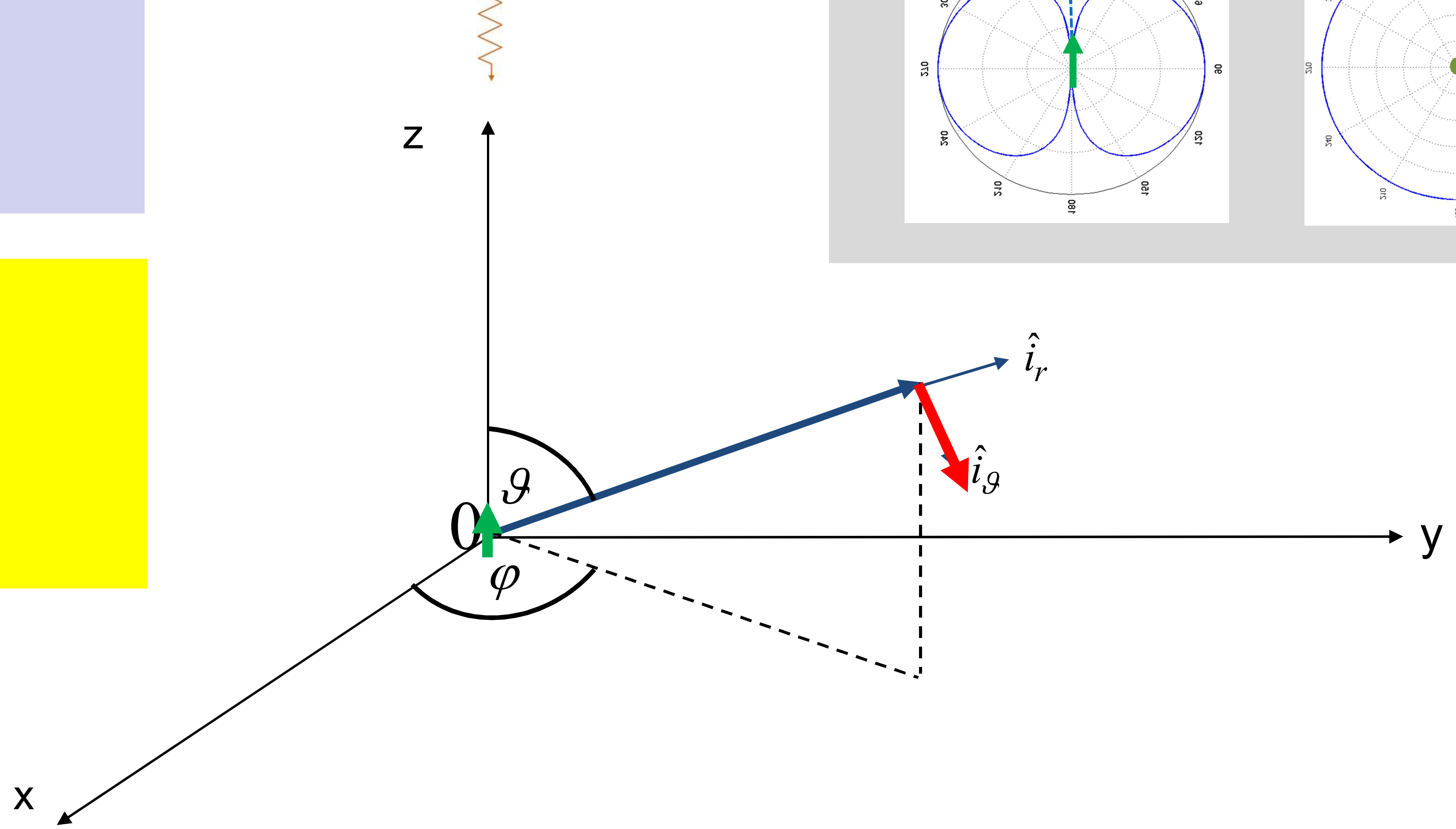


Rx effective length

Interestingly, this result can be extended to **ALL** the antennas by applying the **RECIPROCITY THEOREM**

It can be shown that **for an elementary electrical dipole or for a small loop antenna**, the following property is valid:

$$|V_0| = |\mathbf{E_i} \cdot \mathbf{l}|$$

$\mathbf{l}$ is the tx antenna effective length

$\boldsymbol{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

Rx effective length

$$\left|V_0\right| = \left|\mathbf{E_i} \cdot \mathbf{l}\right|$$

Where

$\boldsymbol{E_i}$ is the incident, locally plane, field

V_0 is the voltage induced at the antenna terminals, which are assumed open-circuited

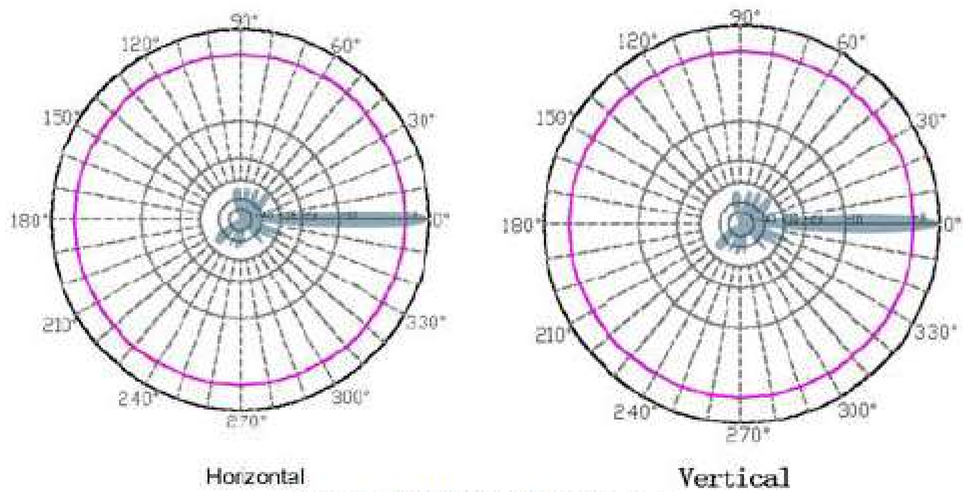
$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$ can referred to as **receiving effective length** of the antenna (and not only transmitting effective length)

Note that this means that the behavior of an antenna when transmitting and when receiving are related.

Rx effective length

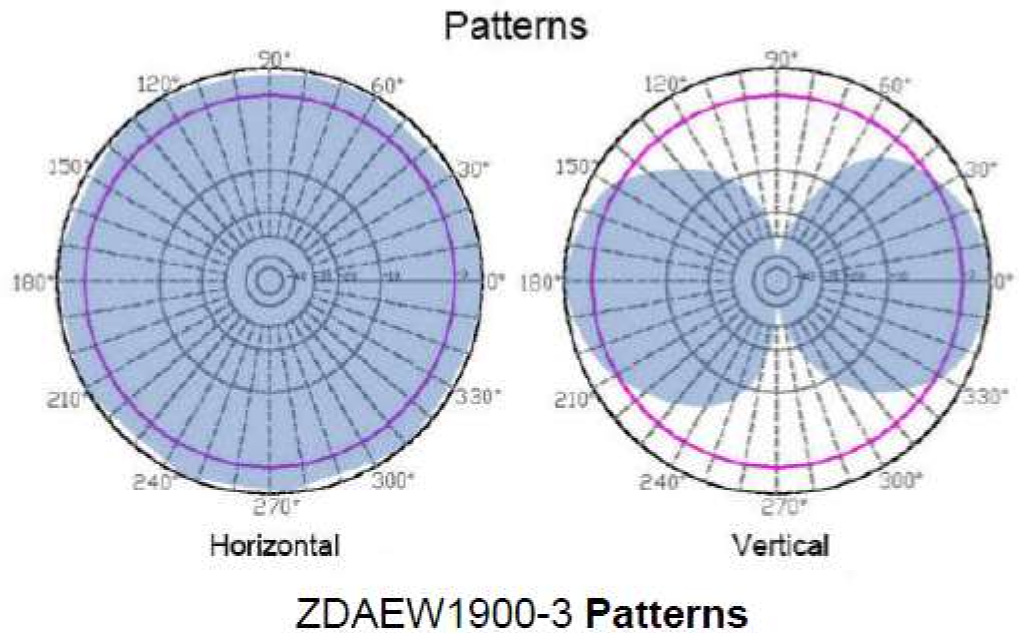
three examples from the real life

Rx effective length

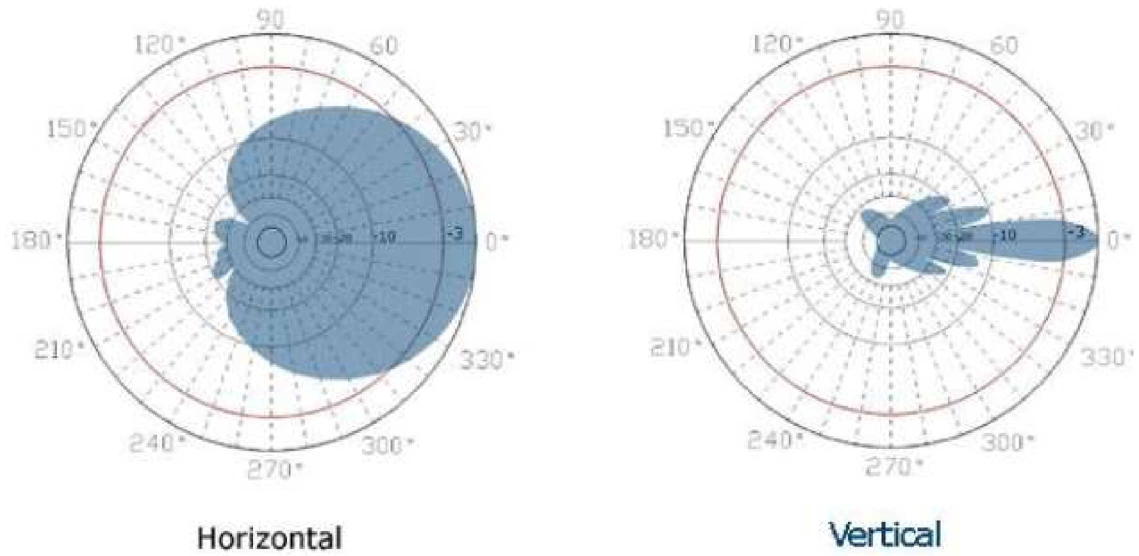


ZDASP5400-29-6 **Patterns**

Rx effective length



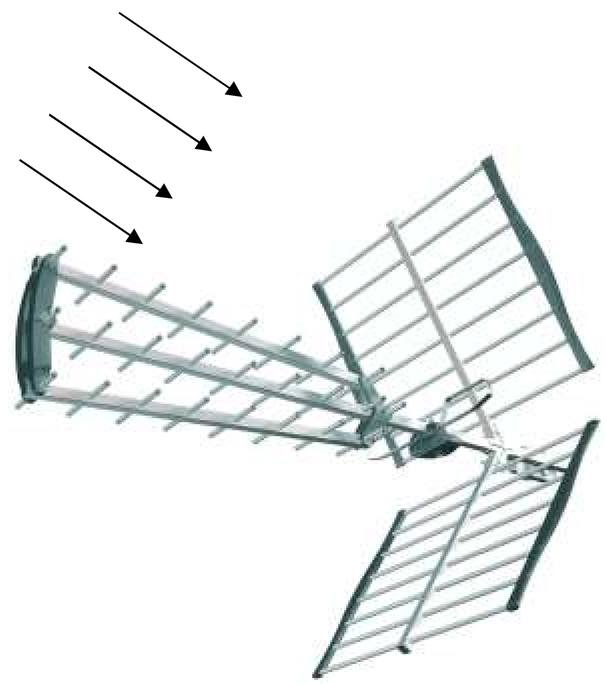
Rx effective length



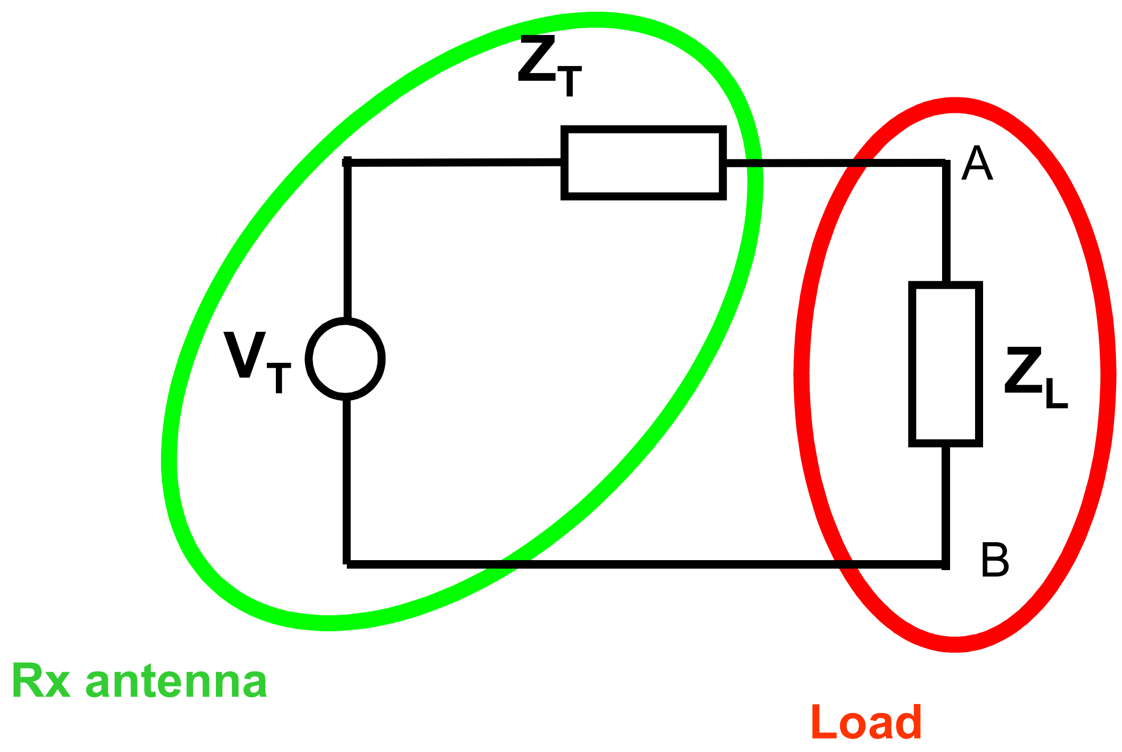
ZDADJ800-13-90 Patterns

Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



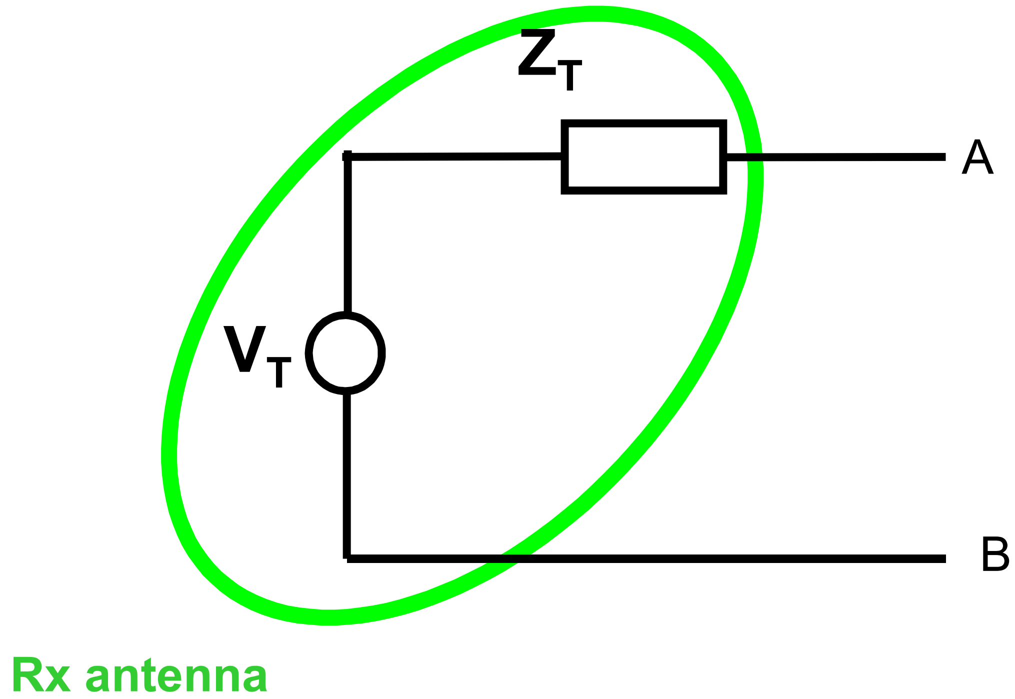
Equivalent circuit of the Rx antenna



$$Z_L = R_L + jX_L$$

The incident electric field sets up currents on the antenna. Such currents may be represented by a Thevenin-equivalent generator, which delivers power to any connected receiving load impedance.

Equivalent circuit of the Rx antenna



$$|V_T| = |V_0| = |\boldsymbol{E_i} \cdot \mathbf{l}|$$

$$Z_L = R_L + jX_L$$

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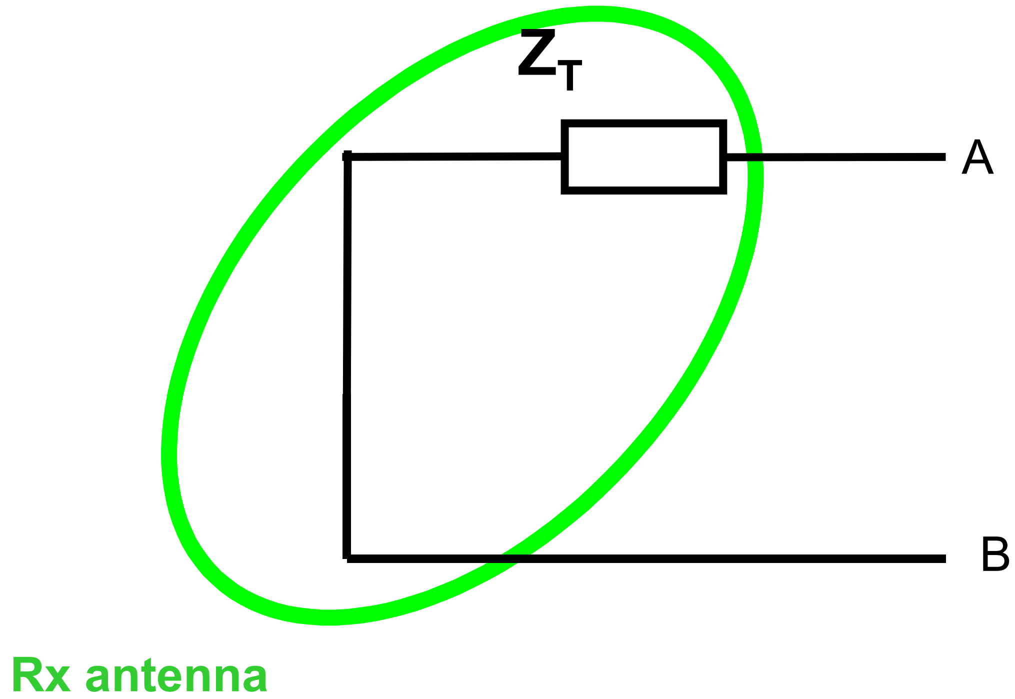
Equivalent circuit of the Rx antenna



$$|V_T| = |V_0| = |\boldsymbol{E_i} \cdot \mathbf{l}|$$

$$Z_T = Z_{in} = R_{in} + jX_{in}$$

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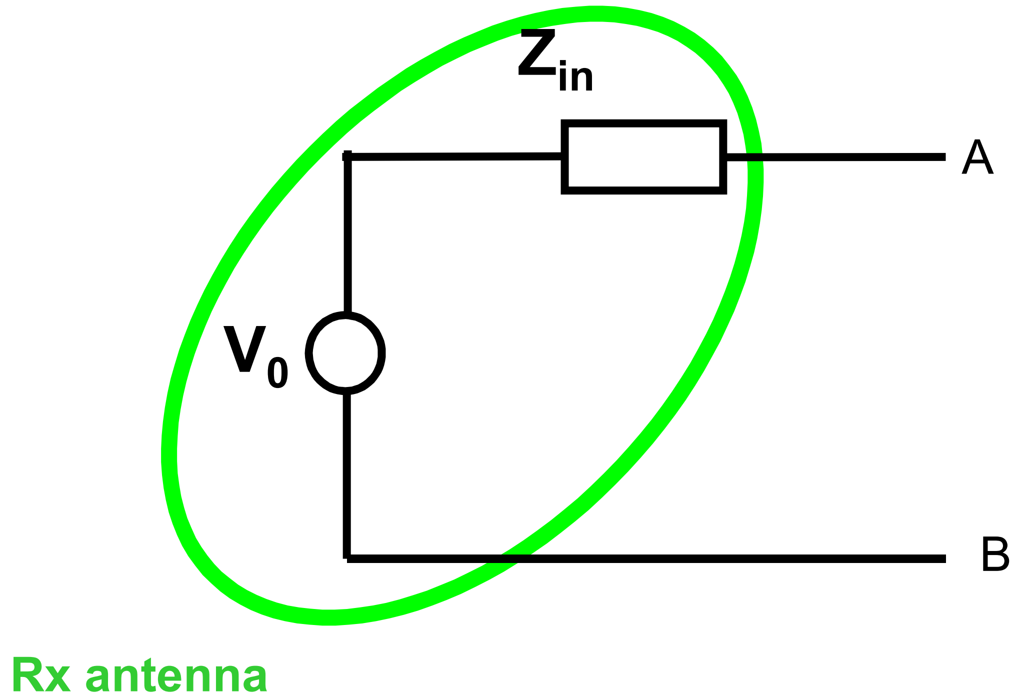

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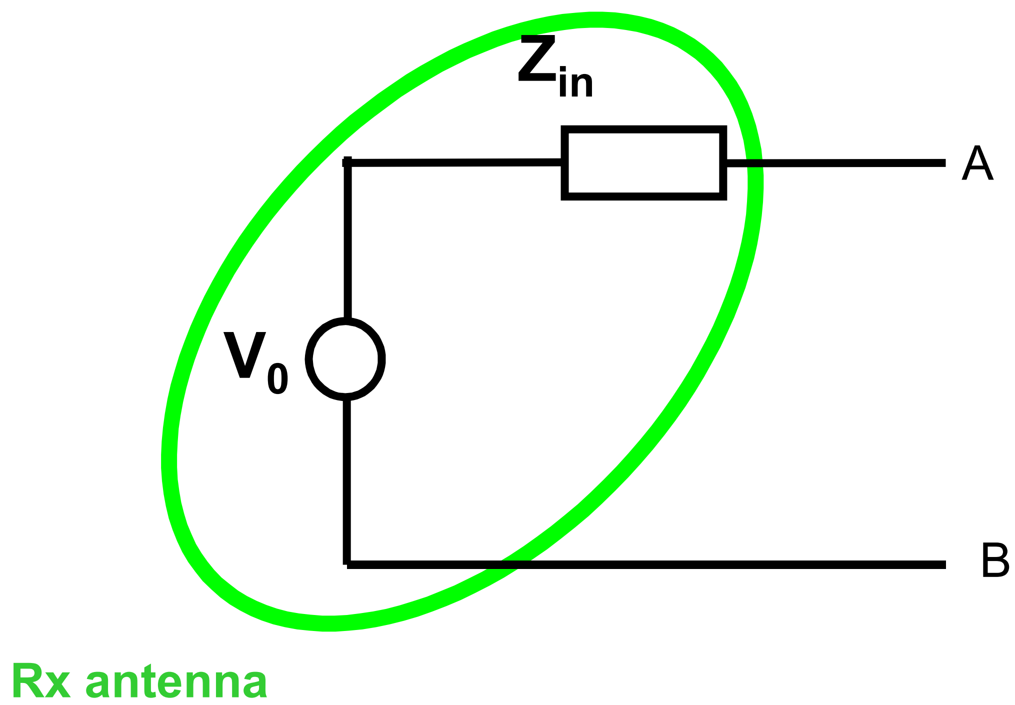

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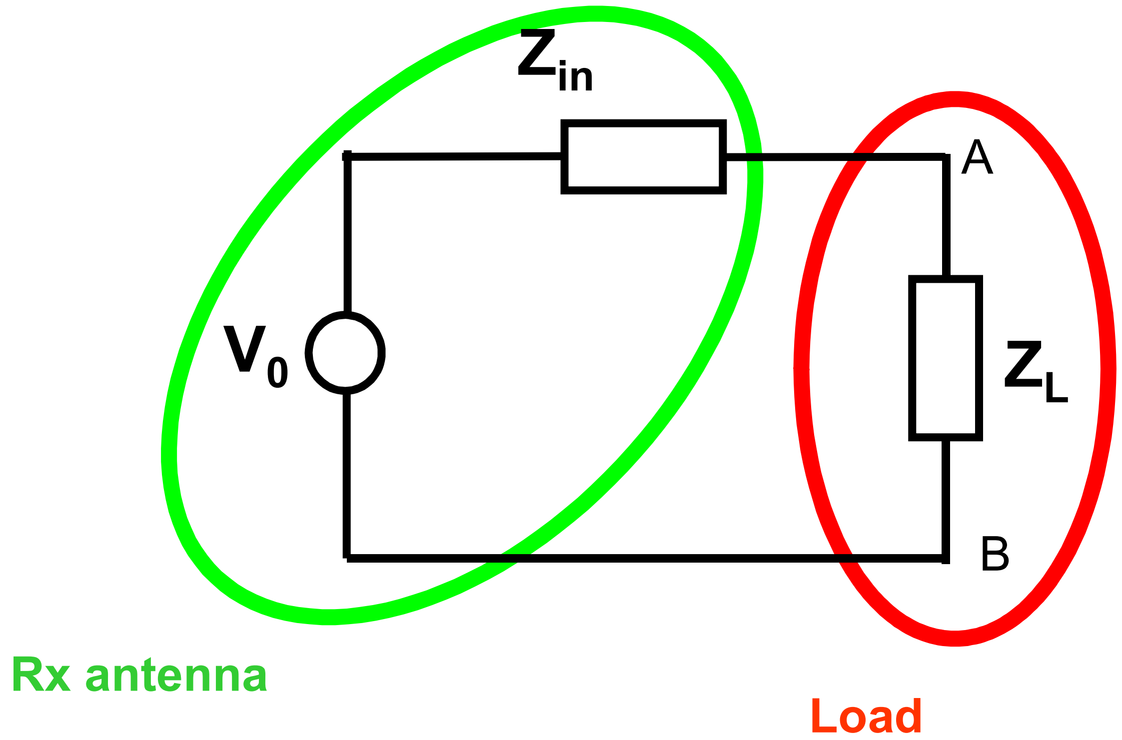

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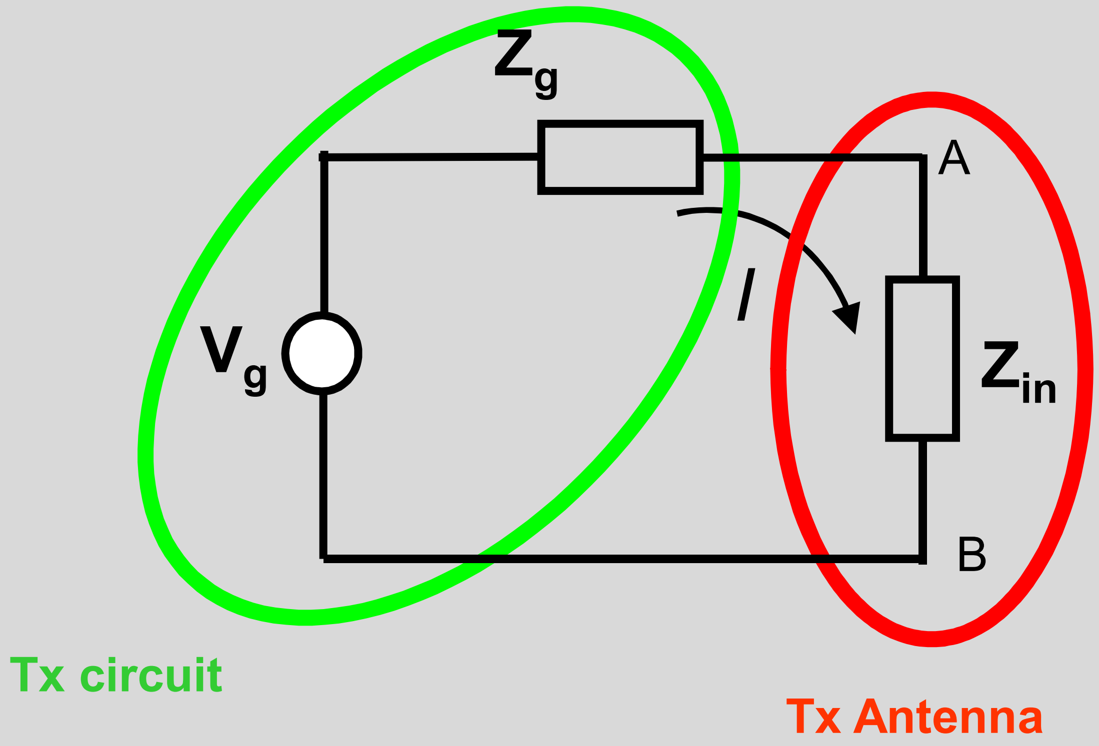
….MEMO..

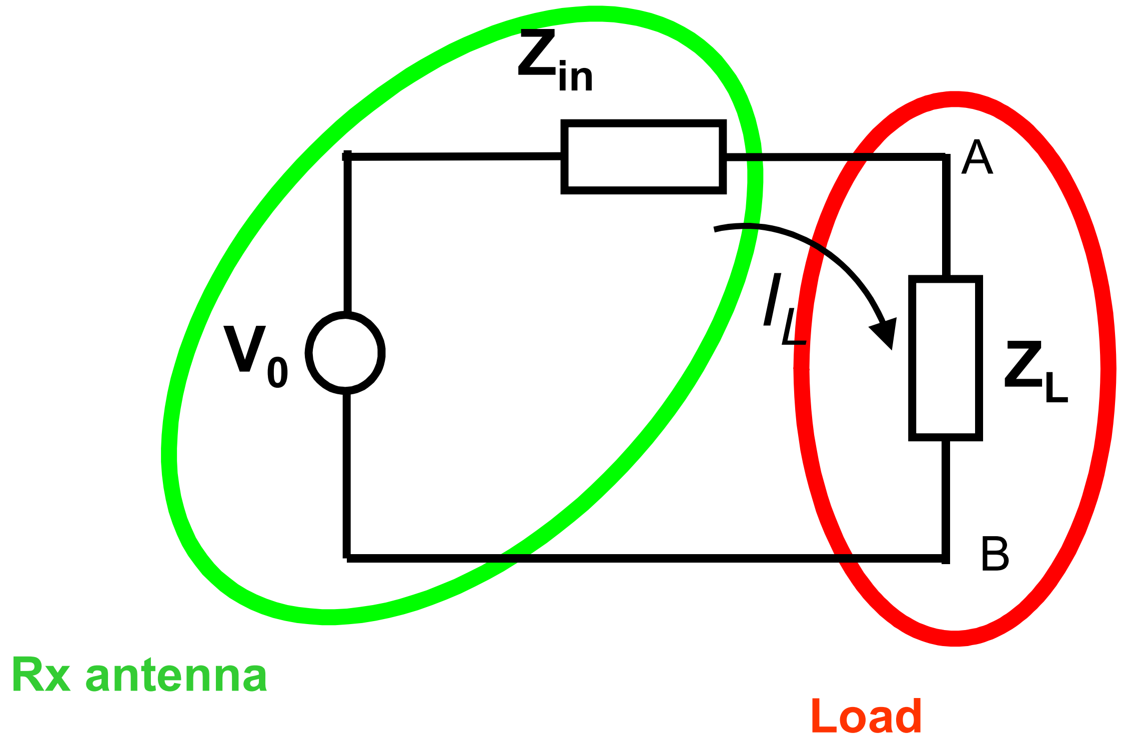
Equivalent circuit of the Tx antenna



$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2$$

Equivalent circuit of the Rx antenna



$$|V_0| = |\boldsymbol{E}_i \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

$$V_0 = (Z_{in} + Z_L) I_L \quad \Rightarrow |I_L|^2 = \frac{|V_0|^2}{|Z_{in} + Z_L|^2}$$

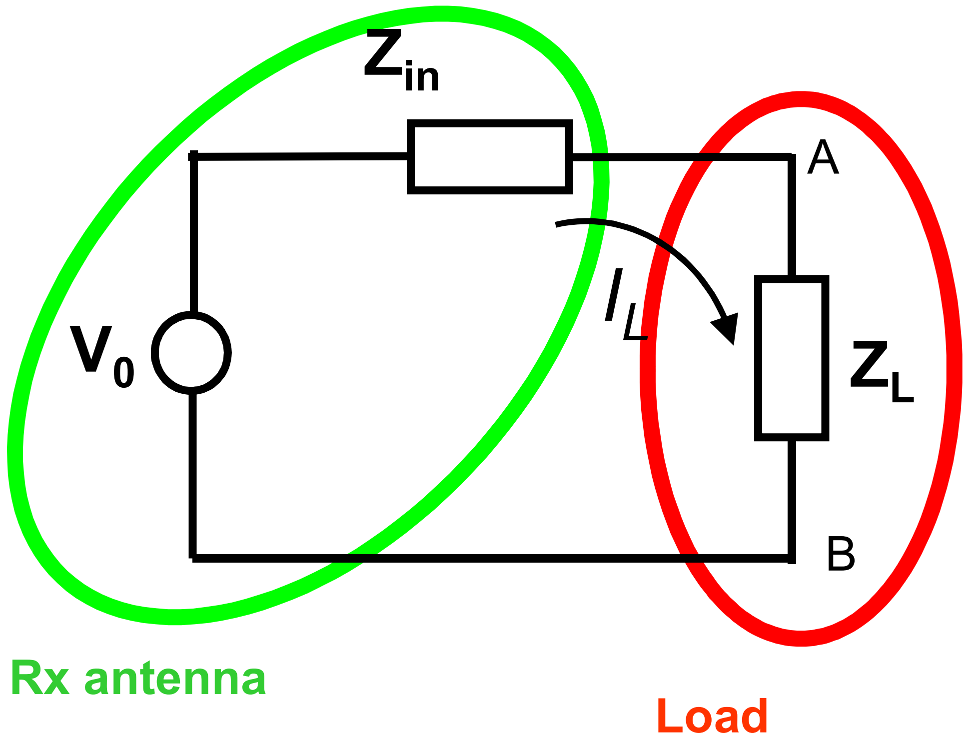
$$P_L = \frac{1}{2} R_L |I_L|^2 = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |V_0|^2 = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\boldsymbol{E}_i \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

$$|V_0| = |\boldsymbol{E}_i \cdot \mathbf{l}|$$

$$Z_{in} = R_{in} + jX_{in}$$

$$Z_L = R_L + jX_L$$

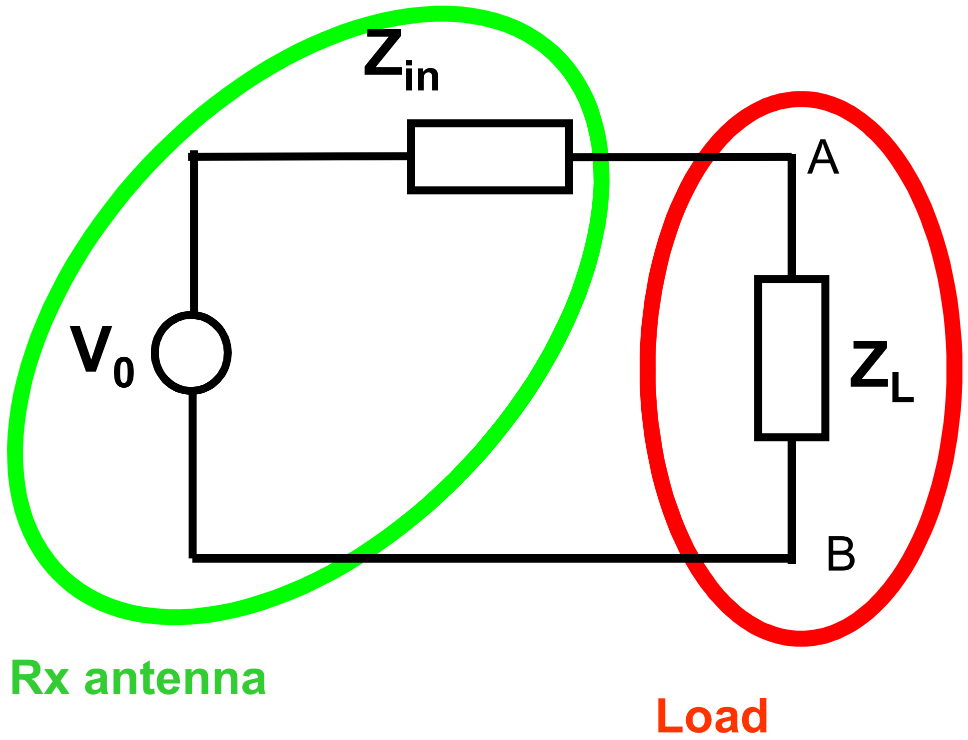


$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in} + Z_L|^2}|\boldsymbol{E}_i \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$|\boldsymbol{E}_i \cdot \mathbf{l}| = |\boldsymbol{E}_i||\mathbf{l}|$$



$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in} + Z_L|^2}|\boldsymbol{E}_i \cdot \mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

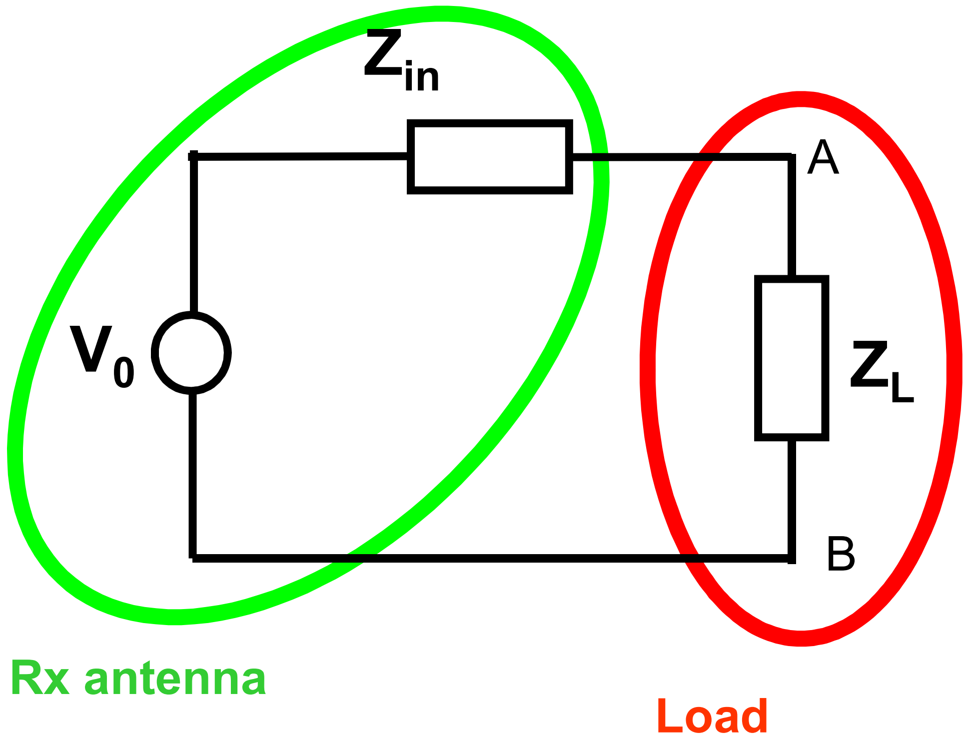
$$|\boldsymbol{E}_i \cdot \mathbf{l}| = |\boldsymbol{E}_i||\mathbf{l}|$$

2) Power matching

$$Z_L = Z_{in}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$

$$\Rightarrow \frac{R_L}{|Z_{in} + Z_L|^2} = \frac{R_{in}}{(2R_{in})^2} = \frac{1}{4R_{in}}$$

$$P_L = \frac{1}{2} \frac{R_L}{|Z_{in} + Z_L|^2} |\boldsymbol{E}_i \cdot \mathbf{l}|^2$$



Equivalent circuit of the Rx antenna

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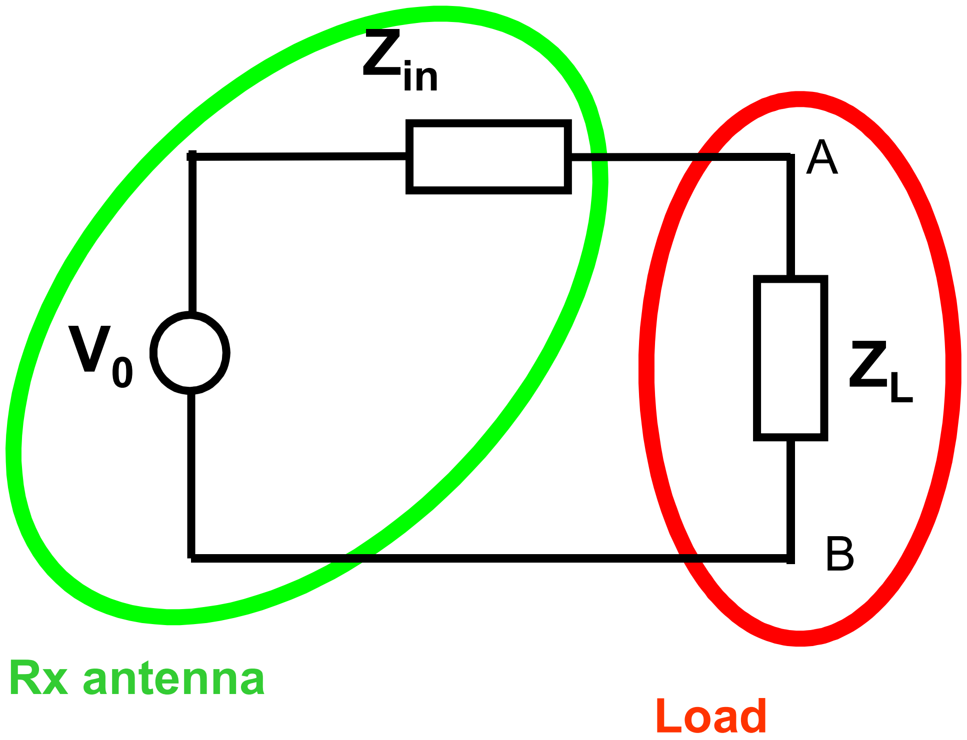


$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in} + Z_L|^2}|\mathbf{E}_i \cdot \mathbf{l}|^2$$

$$\Rightarrow P_{Lmax} = \frac{1}{2}\frac{1}{4R_{in}}|\mathbf{E}_i|^2|\mathbf{l}|^2 = \frac{\zeta}{\zeta}\frac{1}{2}\frac{1}{4R_{in}}|\mathbf{E}_i|^2|\mathbf{l}|^2$$

Equivalent circuit of the Rx antenna

1) Polarization matching

$$|\boldsymbol{E}_i \cdot \mathbf{l}| = |\boldsymbol{E}_i||\mathbf{l}|$$

2) Power matching

$$Z_L = Z_{in}^* \Rightarrow \begin{cases} Z_{in} = R_{in} + jX_{in} \\ Z_L = R_{in} - jX_{in} \end{cases}$$

$$\Rightarrow \frac{R_L}{|Z_{in} + Z_L|^2} = \frac{R_{in}}{(2R_{in})^2} = \frac{1}{4R_{in}}$$

$$P_{Lmax} = \frac{1}{2}\frac{|\boldsymbol{E}_i|^2}{\zeta}\left[\frac{\zeta\,|\mathbf{l}(\vartheta,\varphi)|^2}{4R_{in}}\right]$$

Effective Area

$$A_{eff}(\vartheta,\varphi)$$

$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in} + Z_L|^2}|\boldsymbol{E}_i \cdot \mathbf{l}|^2$$

$$\Rightarrow P_{Lmax} = \frac{1}{2}\frac{1}{4R_{in}}|\boldsymbol{E}_i|^2|\mathbf{l}|^2 = \frac{\zeta}{\zeta}\frac{1}{2}\frac{1}{4R_{in}}|\boldsymbol{E}_i|^2|\mathbf{l}|^2$$

Effective area

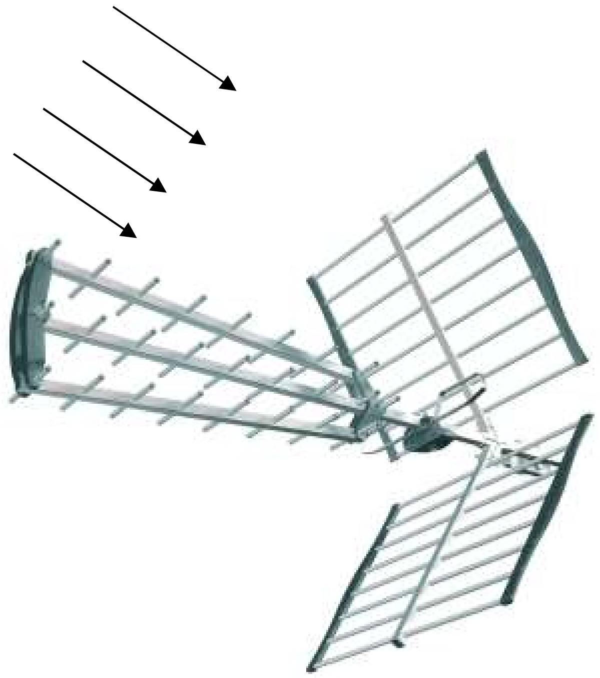
In general, the aperture of an antenna is not directly related to its physical size.

However some types of antennas, for example parabolic dishes and horns, have a physical aperture (opening) which collects the radio waves. In these aperture antennas, the physical aperture A_{phys}
Provides a good approximation of the effective area A.

In any case, the effective aperture A is always less than the area of the antenna's physical aperture A_{phys}.

Parameters of the Rx Antenna

- Rx effective length
- Equivalent circuit of the rx antenna
- Effective Area



Gain and Effective Area

Gain

$$G(\vartheta,\varphi)=\frac{\frac{1}{2}\frac{|\boldsymbol{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

$$P_{in}=\frac{1}{2}R_{in}|I|^2$$

$$\boldsymbol{E}(r,\vartheta,\varphi)=\frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

Effective Area

$$A_{eff}(\vartheta,\varphi)=\frac{\zeta|\mathbf{l}(\vartheta,\varphi)|^2}{4R_{in}}$$

$$G(\vartheta,\phi)=\frac{4\pi r^2}{2\zeta}\frac{|\boldsymbol{E}|^2}{P_{in}}=\frac{4\pi r^2}{2\zeta}\left(\frac{\zeta^2|I|^2}{4\lambda^2}\frac{|\mathbf{l}(\vartheta,\varphi)|^2}{r^2}\right)\frac{2}{R_{in}|I|^2}$$

Gain and Effective Area

Gain

$$G(\vartheta,\varphi)=\frac{\frac{1}{2}\frac{|\boldsymbol{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

$$P_{in}=\frac{1}{2}R_{in}|I|^2$$

$$\boldsymbol{E}(r,\vartheta,\varphi)=\frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

Effective Area

$$A_{eff}(\vartheta,\varphi)=\frac{\zeta\left|\mathbf{l}(\vartheta,\varphi)\right|^2}{4R_{in}}$$

$$G(\vartheta,\phi)=\frac{4\pi r^2}{2\zeta}\frac{|\boldsymbol{E}|^2}{P_{in}}=\frac{4\pi r^2}{2\zeta}\left(\frac{\zeta^2|I|^2}{4\lambda^2}\frac{\left|\mathbf{l}(\vartheta,\varphi)\right|^2}{r^2}\right)\frac{2}{R_{in}|I|^2}=\frac{4\pi}{\lambda^2}\left[\frac{\zeta\left|\mathbf{l}(\vartheta,\varphi)\right|^2}{4R_{in}}\right]$$

Gain and Effective Area

Gain

$$G(\vartheta,\varphi) = \frac{\frac{1}{2}\frac{|\boldsymbol{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

$$P_{in} = \frac{1}{2}R_{in}|I|^2$$

$$\boldsymbol{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

Effective Area

$$A_{eff}(\vartheta,\varphi) = \frac{\zeta|\mathbf{l}(\vartheta,\varphi)|^2}{4R_{in}}$$

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$$G(\vartheta,\varphi) = \frac{4\pi}{\lambda^2}A_{eff}(\vartheta,\varphi)$$

Effective Area

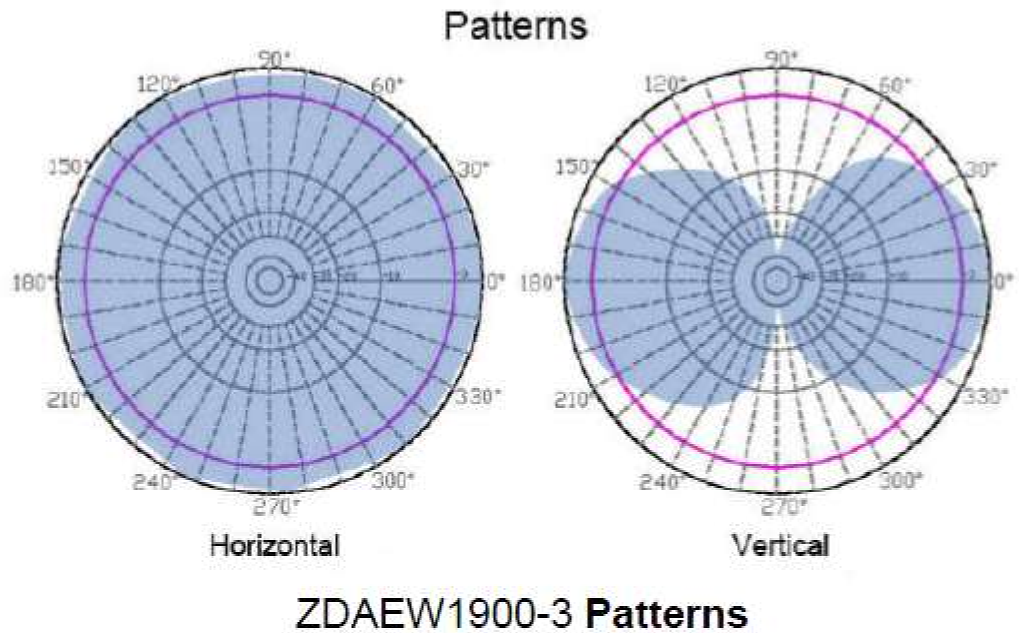
three examples from the real life

Effective Area

three examples from the real life

(maximum) Gain = 3 dB

$$G(\vartheta,\varphi)=\frac{4\pi}{\lambda^2}A_{eff}(\vartheta,\varphi)$$

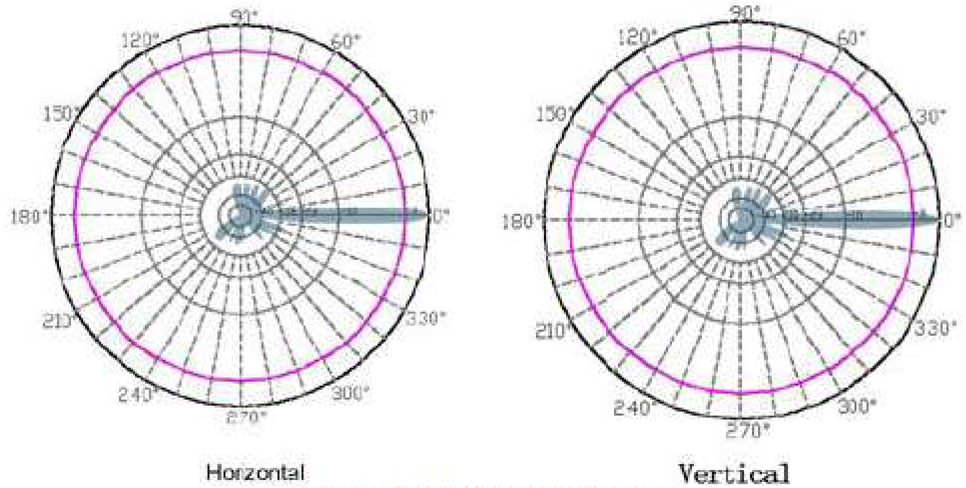


Effective Area

three examples from the real life

(maximum) Gain = 29 dB

$$G(\vartheta,\varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta,\varphi)$$



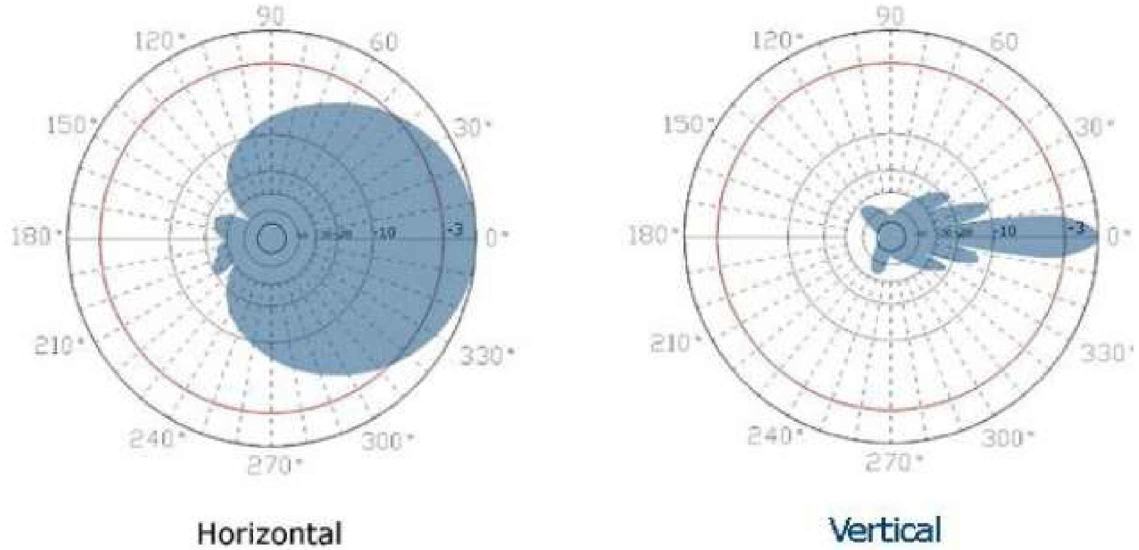
ZDASP5400-29-6 **Patterns**

Effective Area

three examples from the real life

(maximum) Gain = 13 dB

$$G(\vartheta,\varphi) = \frac{4\pi}{\lambda^2} A_{eff}(\vartheta,\varphi)$$



ZDADJ800-13-90 Patterns

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2} \frac{R_L}{\left|Z_{in} + Z_L\right|^2} \left|\mathbf{E}_i \cdot \mathbf{l}\right|^2$$

$$P_{L\,max} = \frac{1}{2} \frac{\left|\mathbf{E}_i\right|^2 \left|\mathbf{l}\right|^2}{4R_{in}}$$

Maximum power transfer to the load is achieved when the two following conditions are simultaneously verified:

1) Polarization matching

$$\Rightarrow \left|\mathbf{E}_i \cdot \mathbf{l}\right| = \left|\mathbf{E}_i\right| \left|\mathbf{l}\right|$$

2) Power matching

$$\Rightarrow \frac{R_L}{\left|Z_{in} + Z_L\right|^2} = \frac{1}{4R_{in}}$$

$$P_{L\,max} = \frac{1}{2} \frac{\left|\mathbf{E}_i\right|^2}{\zeta} A_{eff}\left(\vartheta, \varphi\right)$$

Equivalent circuit of the Rx antenna

$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in}+Z_L|^2}|\boldsymbol{E_i}\cdot\mathbf{l}|^2$$

$$P_{L\,max} = \frac{1}{2}\frac{|\boldsymbol{E_i}|^2|\mathbf{l}|^2}{4R_{in}}$$

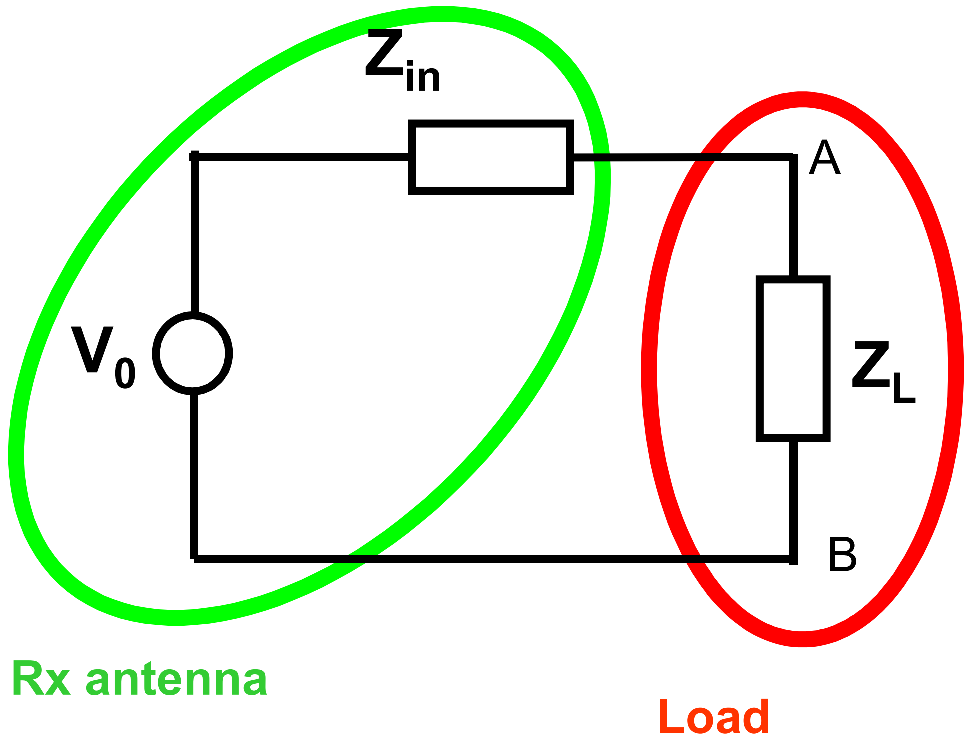
$$P_L = \frac{1}{2}\frac{R_L}{|Z_{in}+Z_L|^2}|\boldsymbol{E_i}\cdot\mathbf{l}|^2 = \underbrace{\frac{1}{2}\frac{|\boldsymbol{E_i}|^2|\mathbf{l}|^2}{4R_{in}}}_{P_{L\,max}}\underbrace{\frac{|\boldsymbol{E_i}\cdot\mathbf{l}|^2}{|\boldsymbol{E_i}|^2|\mathbf{l}|^2}}_{\eta_A}\underbrace{\frac{4R_{in}R_L}{|Z_{in}+Z_L|^2}}_{\eta_B}$$

$$P_L = \eta_A\,\eta_B\,P_{L\,max}$$

Equivalent circuit of the Rx antenna

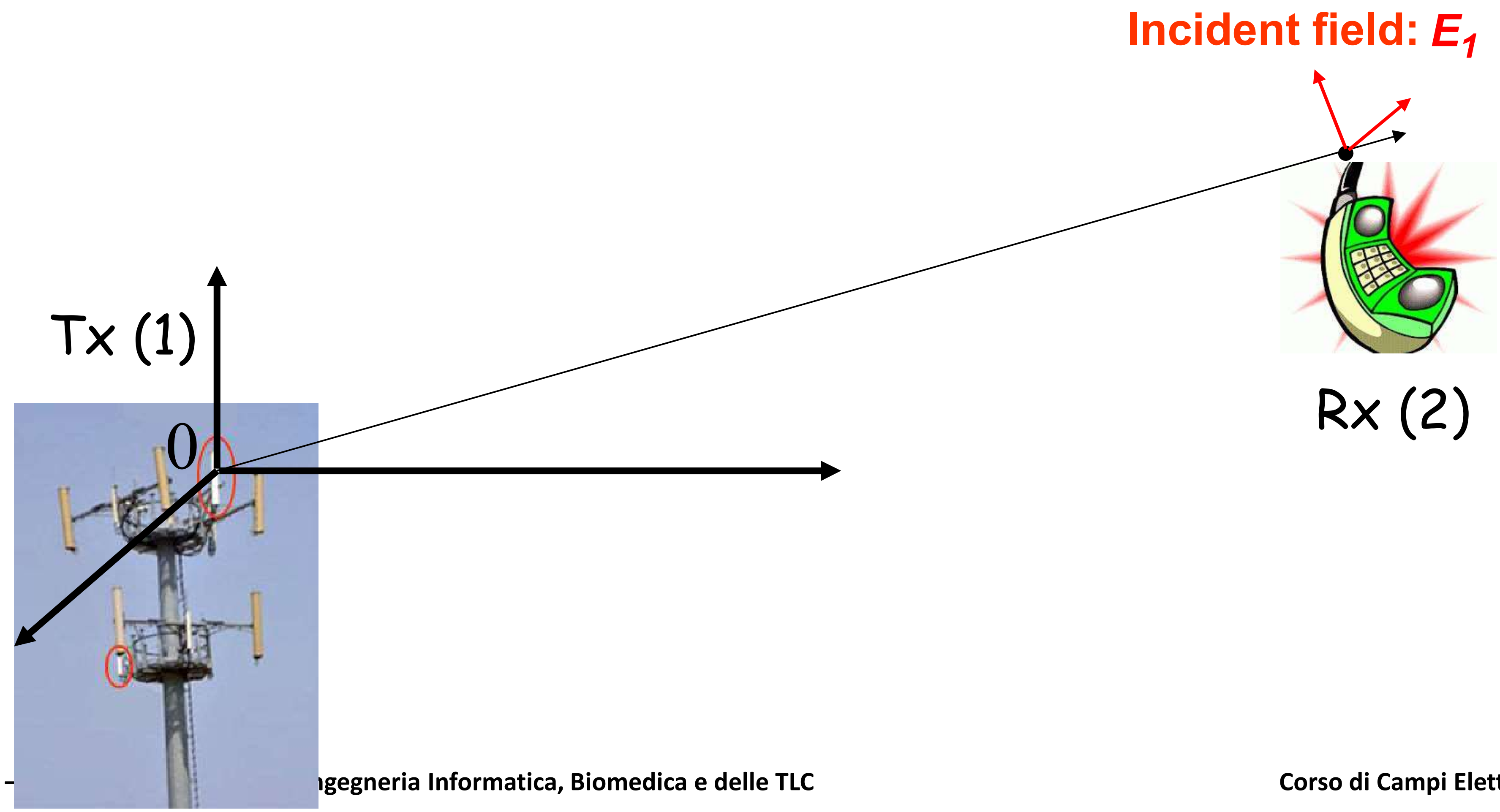
$$P_L = \eta_A \eta_B P_{Lmax}$$

$$P_{Lmax} = \frac{1}{2}\frac{|\boldsymbol{E_i}|^2}{\zeta} A_{eff}(\vartheta,\varphi)$$



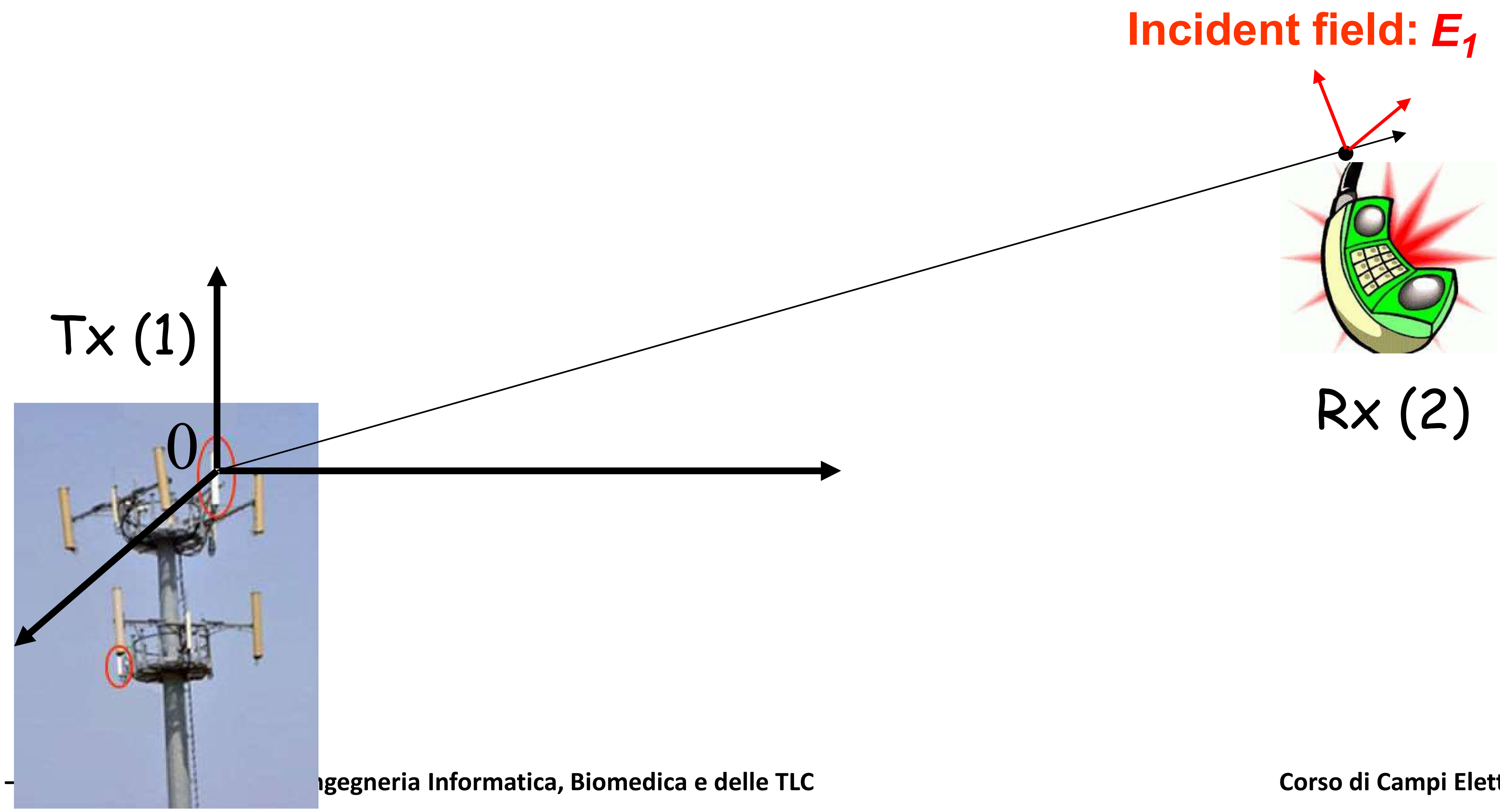
$$P_L = \frac{1}{2}\frac{|\boldsymbol{E_i}|^2}{\zeta} A_{eff}(\vartheta,\varphi)\eta_A \eta_B$$

Radio link equation



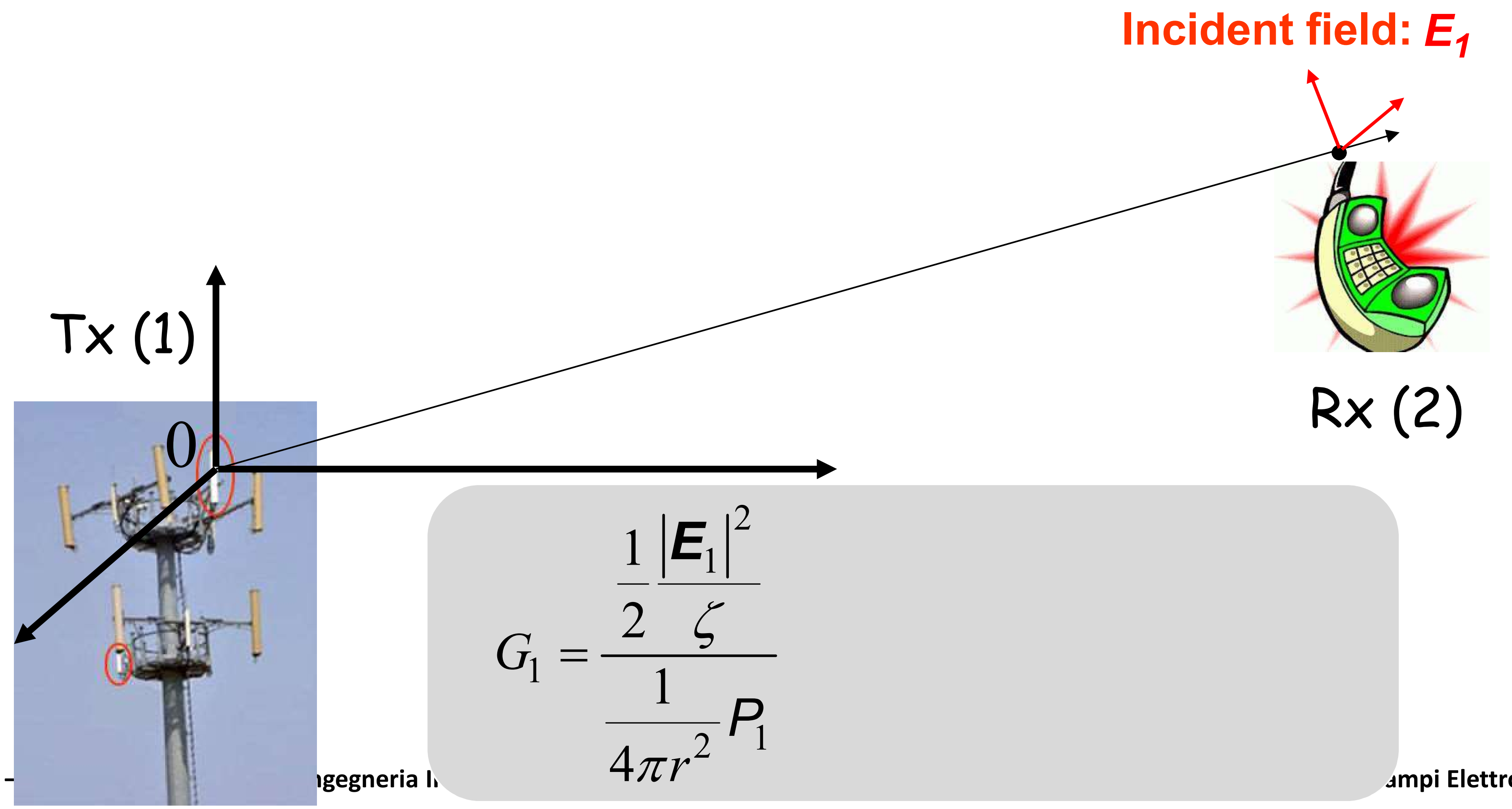
Radio link equation

$$P_2 = \frac{1}{2}\frac{|\mathbf{E}_1|^2}{\zeta} A_{eff\,2}\,\eta_A\,\eta_B$$



Radio link equation

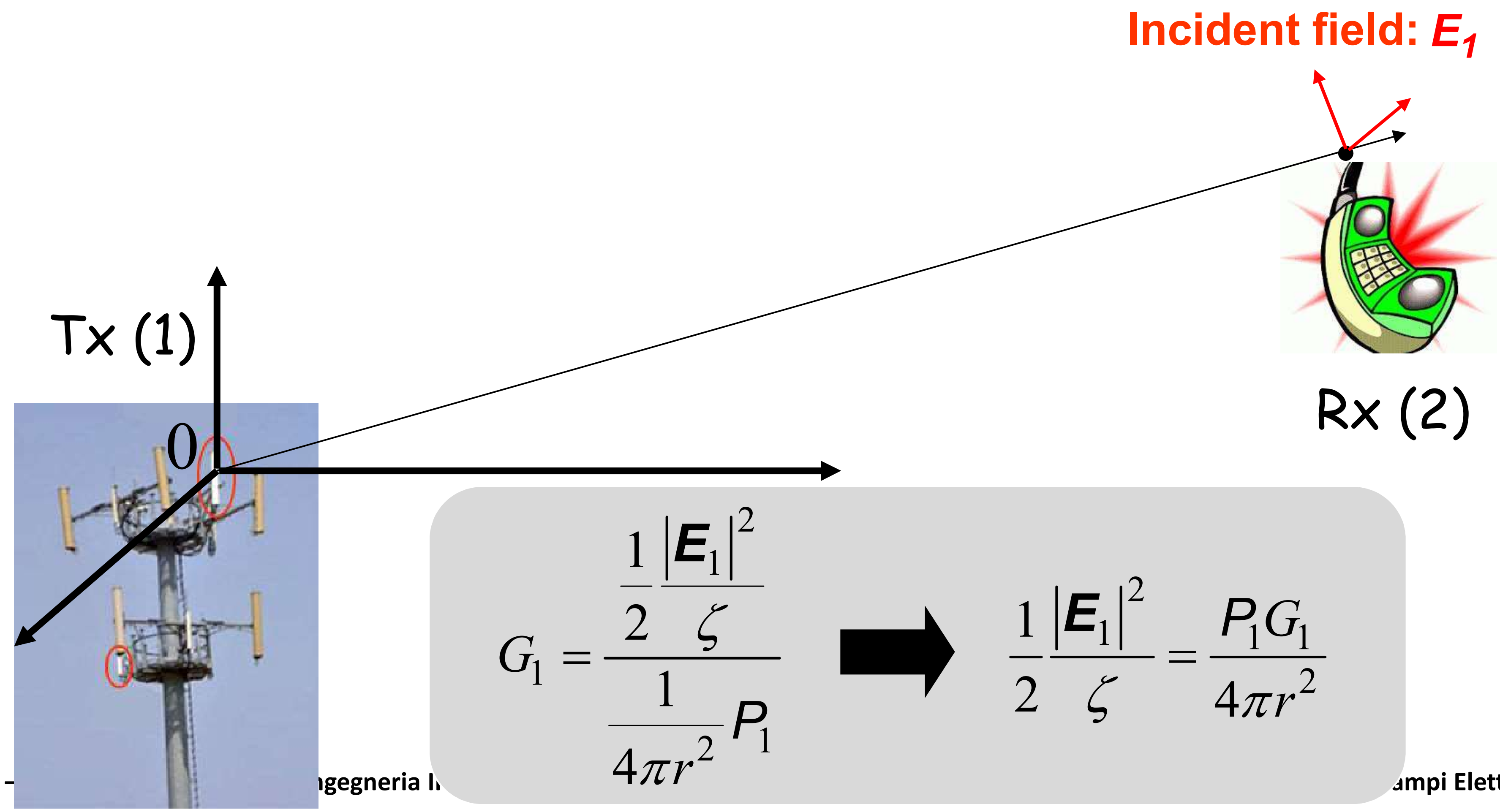
$$P_2 = \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2}\,\eta_A\,\eta_B$$



$$G_1 = \frac{\frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2}P_1}$$

Radio link equation

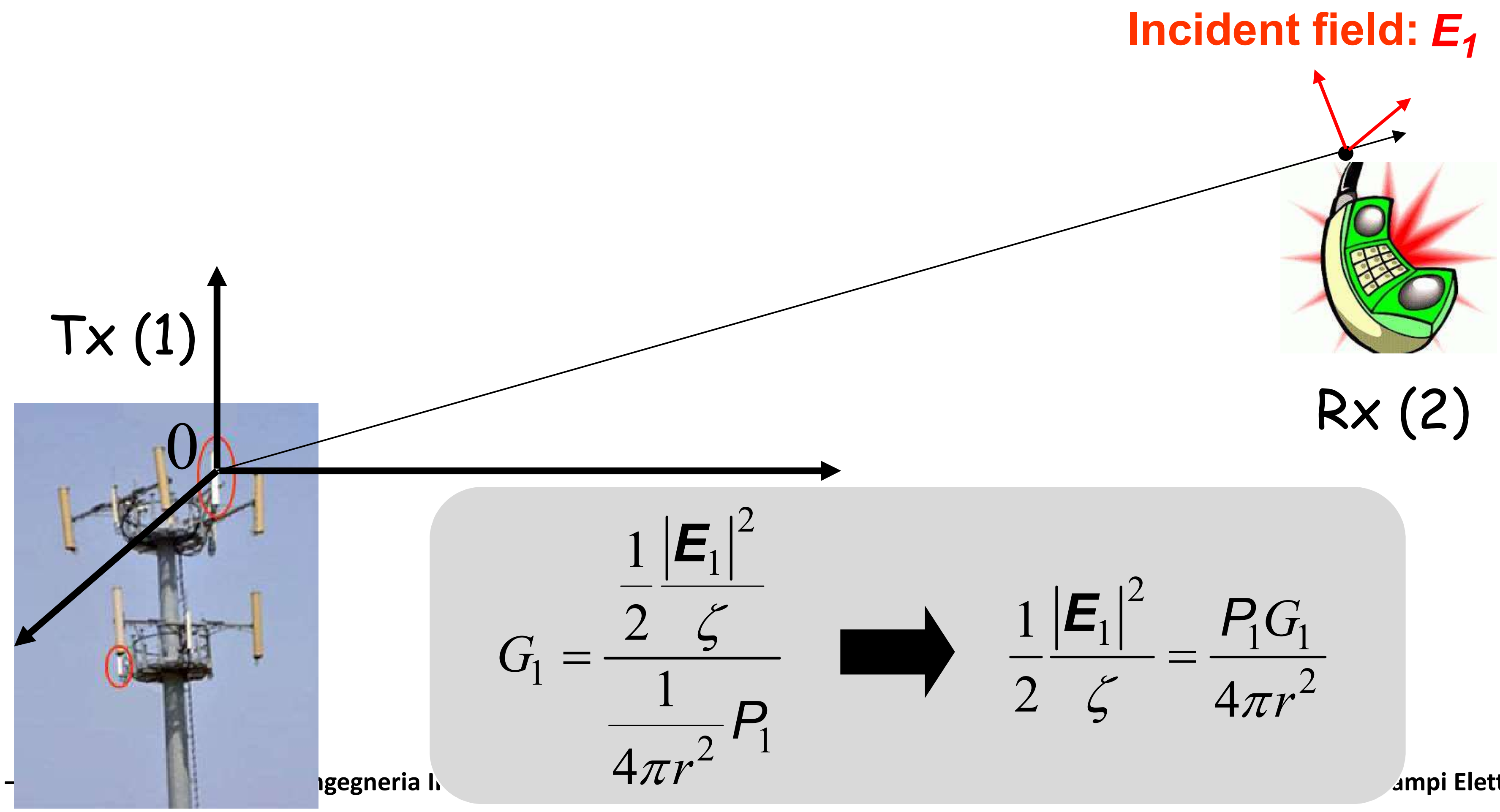
$$P_2 = \frac{1}{2}\frac{|\mathbf{E}_1|^2}{\zeta} A_{eff\,2}\,\eta_A\,\eta_B$$



$$G_1 = \frac{\frac{1}{2}\frac{|\mathbf{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2}P_1} \quad \Rightarrow \quad \frac{1}{2}\frac{|\mathbf{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

Radio link equation

$$P_2 = \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2}\eta_A\,\eta_B = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

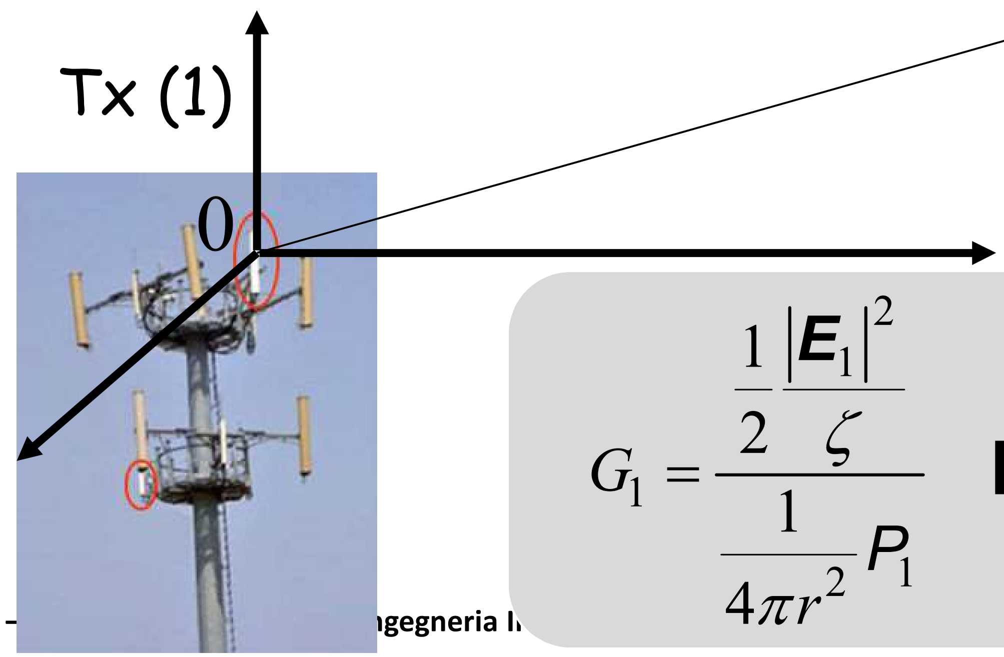


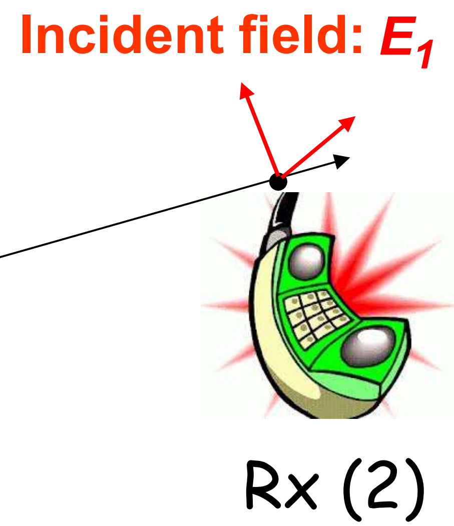
$$G_1 = \frac{\frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2}P_1} \Rightarrow \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

Radio link equation

$$P_2 = \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} A_{eff\,2}\,\eta_A\,\eta_B = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

$$P_2 = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$

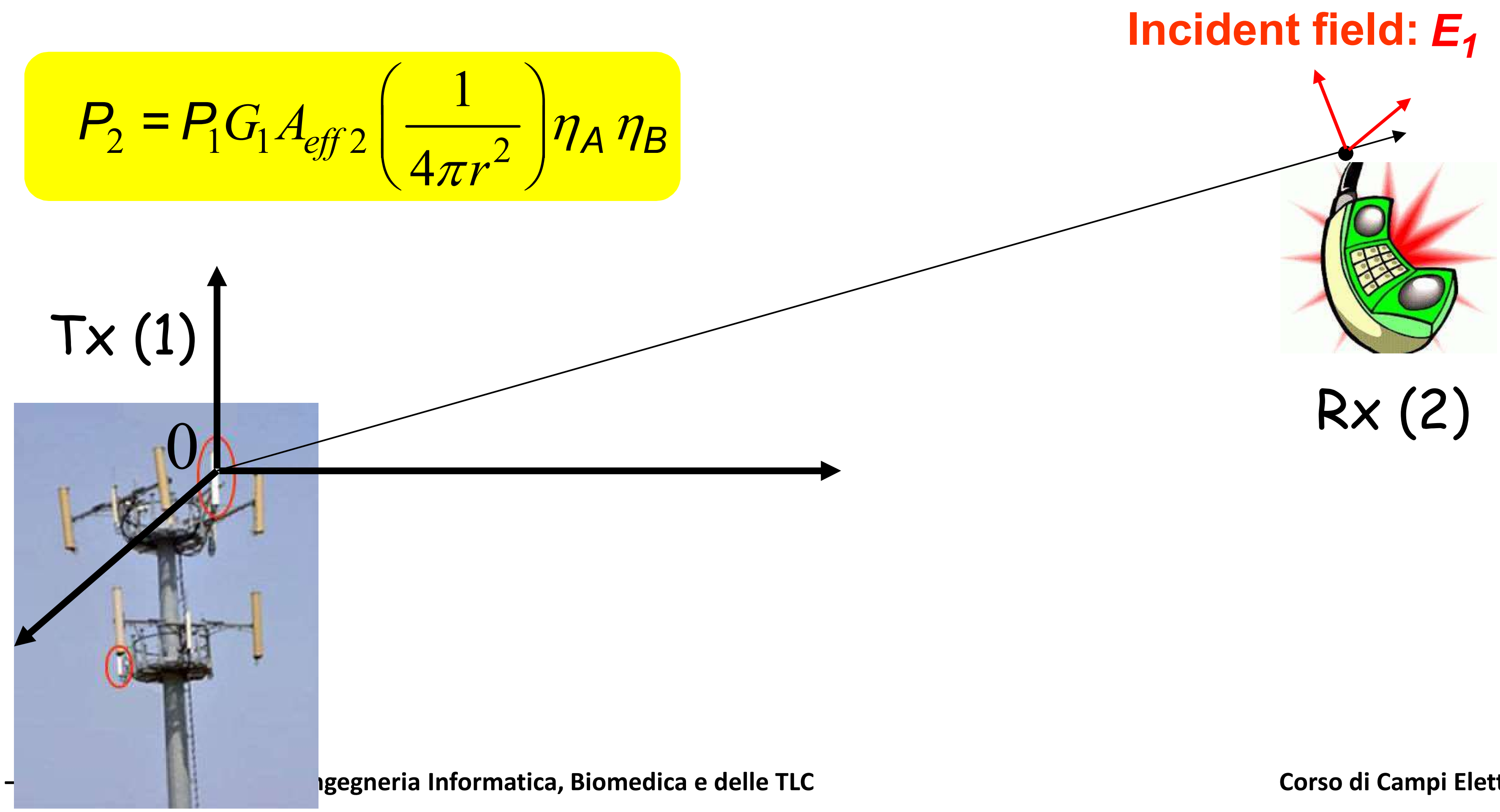




$$G_1 = \frac{\frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta}}{\frac{1}{4\pi r^2}P_1} \quad \Rightarrow \quad \frac{1}{2}\frac{|\boldsymbol{E}_1|^2}{\zeta} = \frac{P_1 G_1}{4\pi r^2}$$

Radio link equation
(Friis equation)

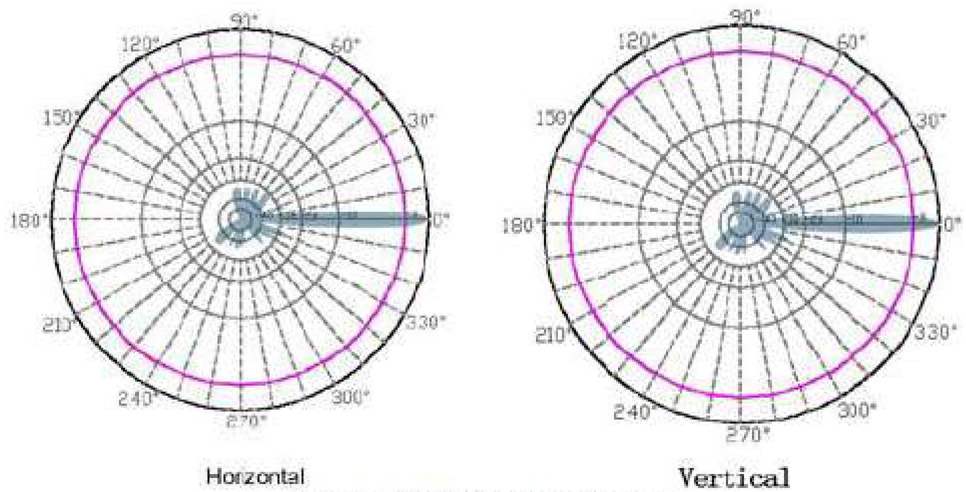
$$P_2 = P_1 G_1 A_{eff\,2}\left(\frac{1}{4\pi r^2}\right)\eta_A\,\eta_B$$



Radio link equation

(maximum) Gain = 29 dB

$$G(\vartheta,\varphi)=\frac{4\pi}{\lambda^2}A_{eff}(\vartheta,\varphi)$$



ZDASP5400-29-6 **Patterns**