Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations, important formulas, important concepts

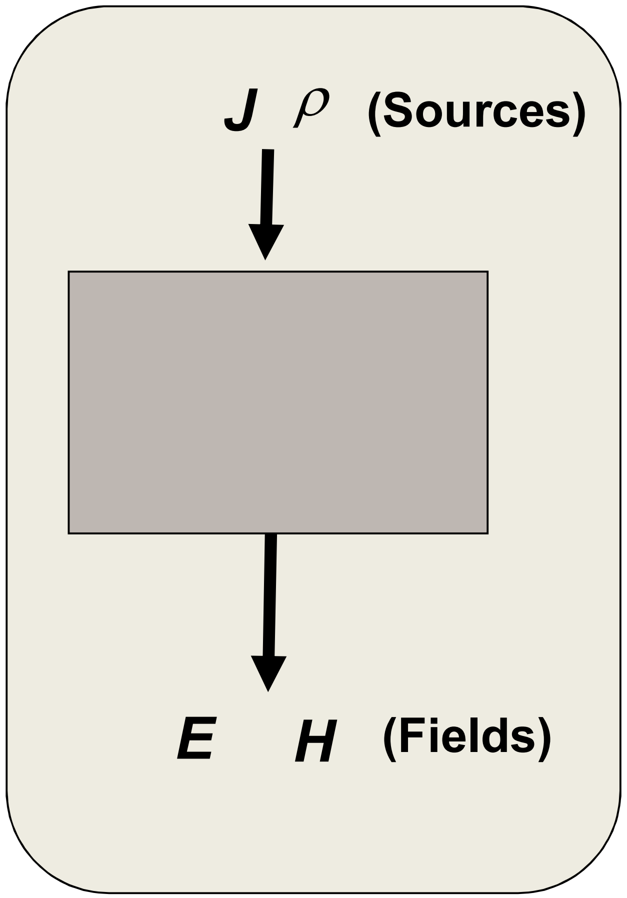
Very important for the discussion

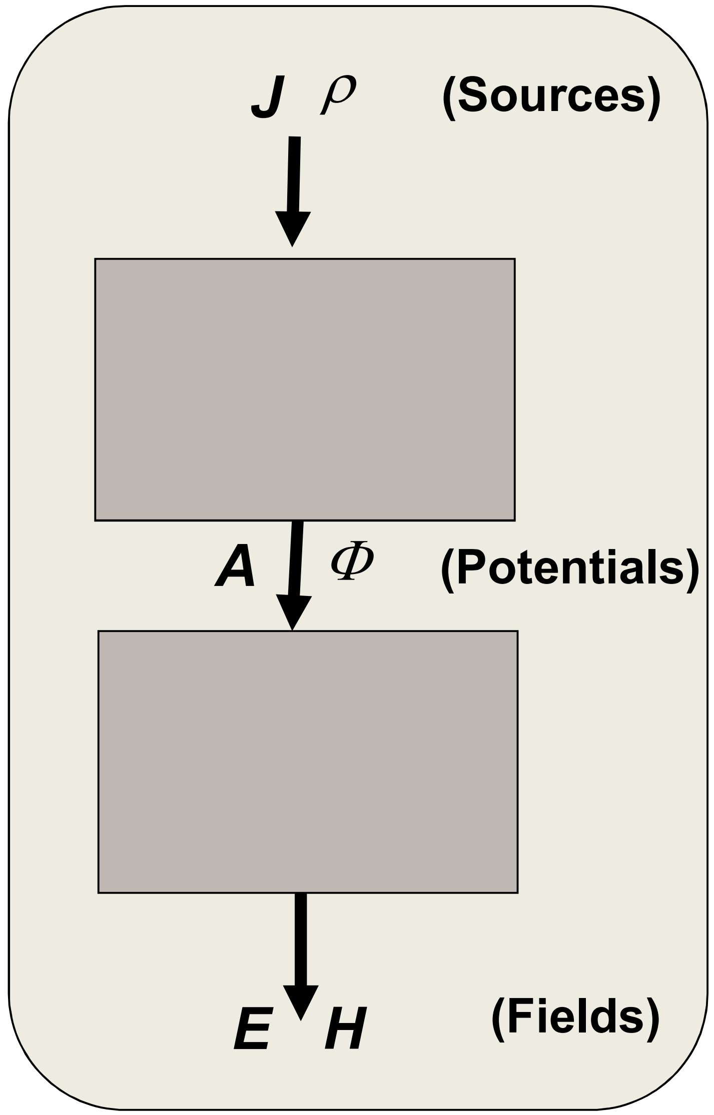
Memo

Mathematical tools to be exploited

Mathematics

Radiation problem





Potentials

$\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

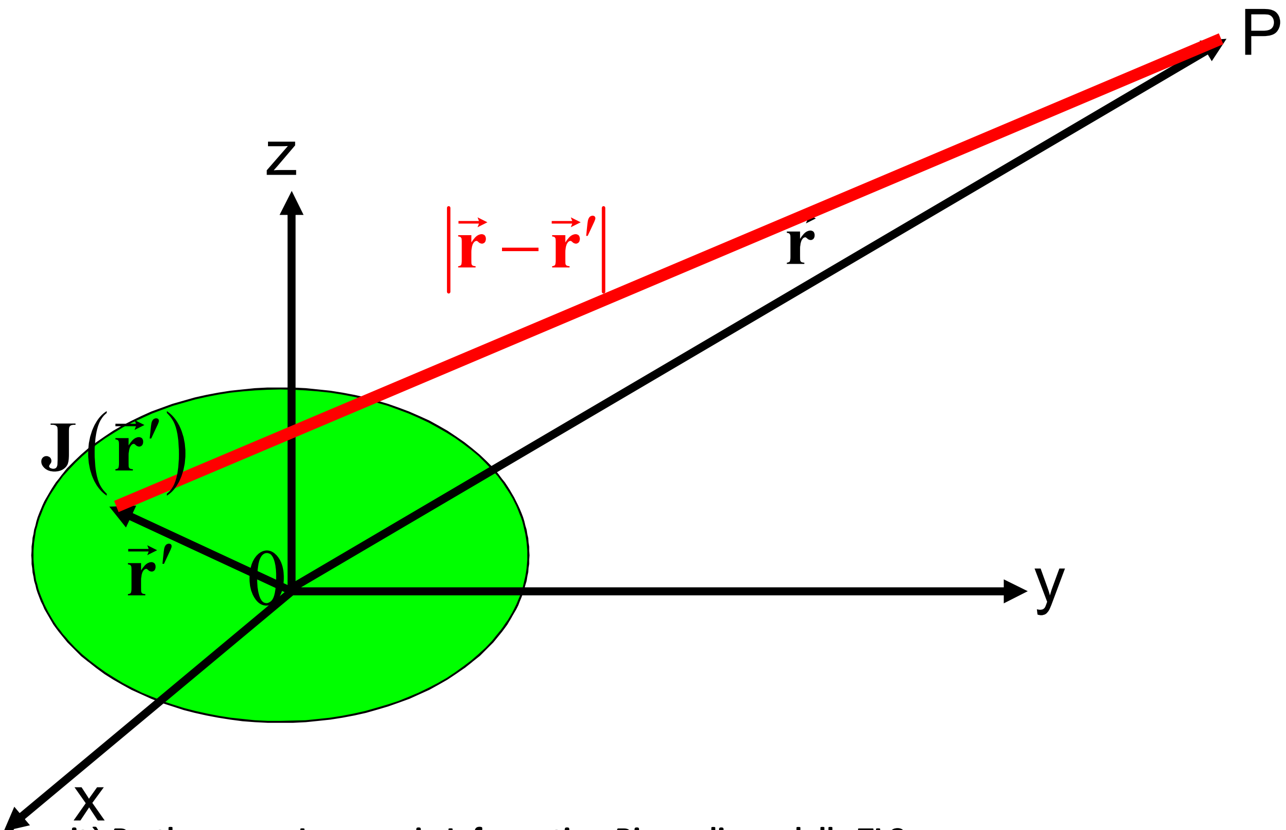
$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \mathbf{J}(\mathbf{r}') \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu} \nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\boldsymbol{E}(\boldsymbol{r})$
$\boldsymbol{H}(\boldsymbol{r})$

Potentials

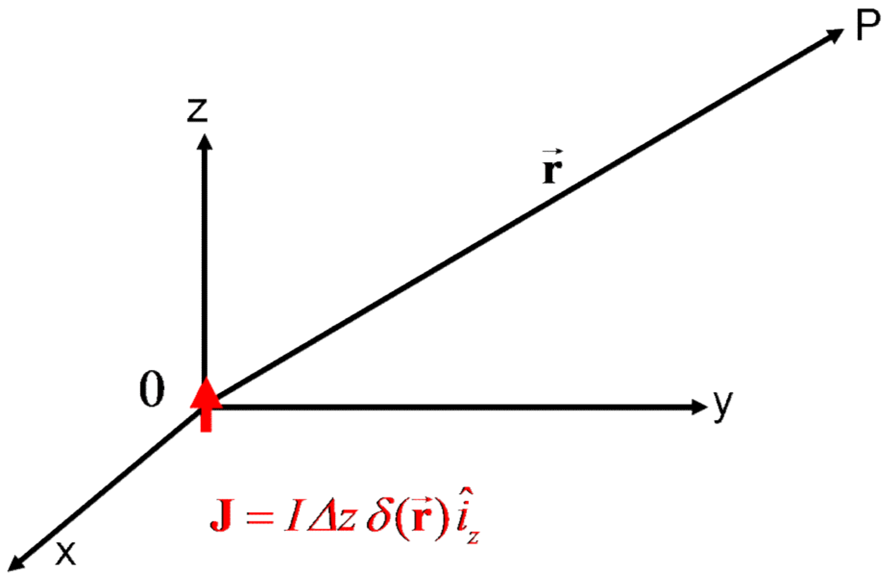
$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$



Elementary electrical dipole

- A δ-source radiating element is also known as elementary electrical dipole.

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$



- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

Elementary electrical dipole

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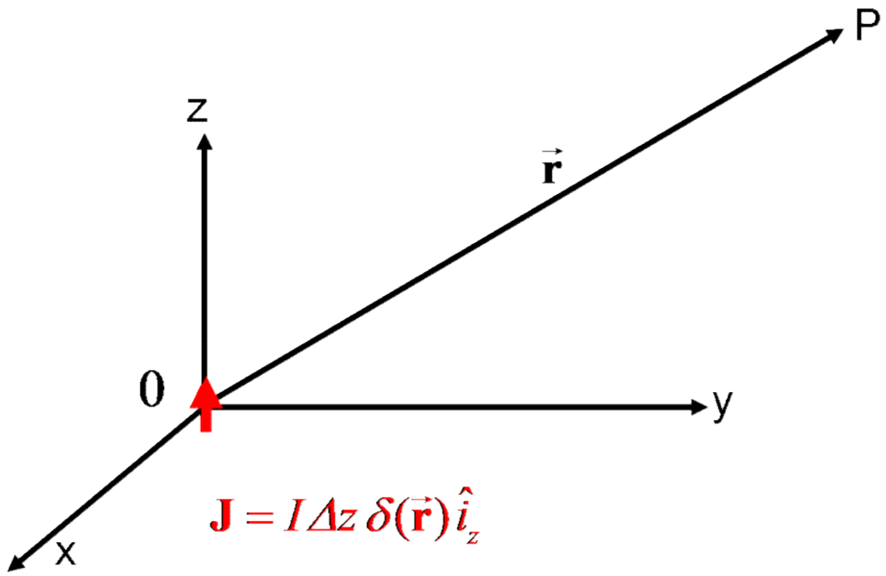

- **Why are we interested in such a radiating element?**
- Why is such a radiating element referred to as elementary electrical dipole?
- How can we physically approximate an elementary electrical dipole?

… memo …

$$\mathbf{A}(\vec{\mathbf{r}}) = \int -\mu \mathbf{J}(\vec{\mathbf{r}}')\, g(\vec{\mathbf{r}} - \vec{\mathbf{r}}')\, d\vec{\mathbf{r}}'$$

$$\nabla^2 g(\vec{\mathbf{r}}) + k^2 g(\vec{\mathbf{r}}) = \delta(\vec{\mathbf{r}})$$

$$\delta(\vec{\mathbf{r}}) \longrightarrow \boxed{} \longrightarrow g(\vec{\mathbf{r}})$$

Mathematically, a δ-source radiating element is related to the radiation of any antenna!

Elementary electrical dipole

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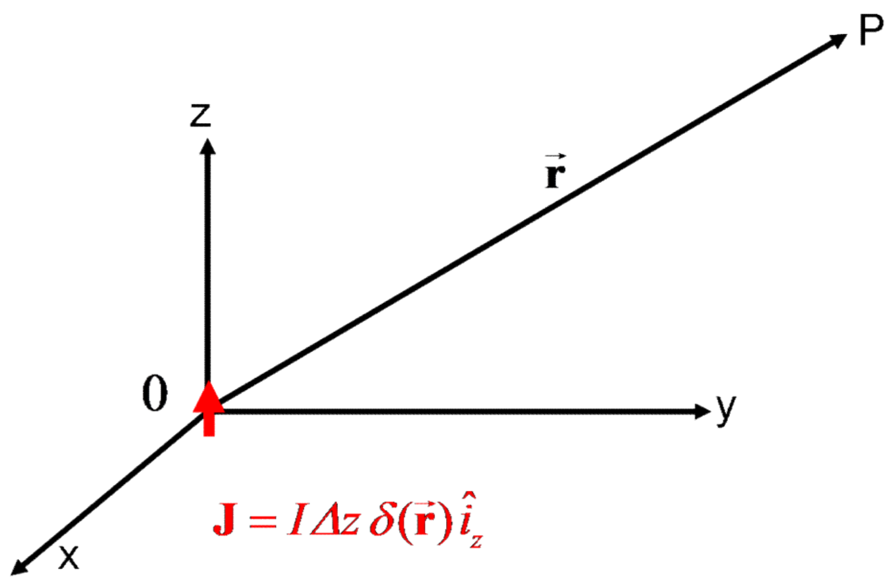

- Why are we interested in such a radiating element?
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Elementary electrical dipole

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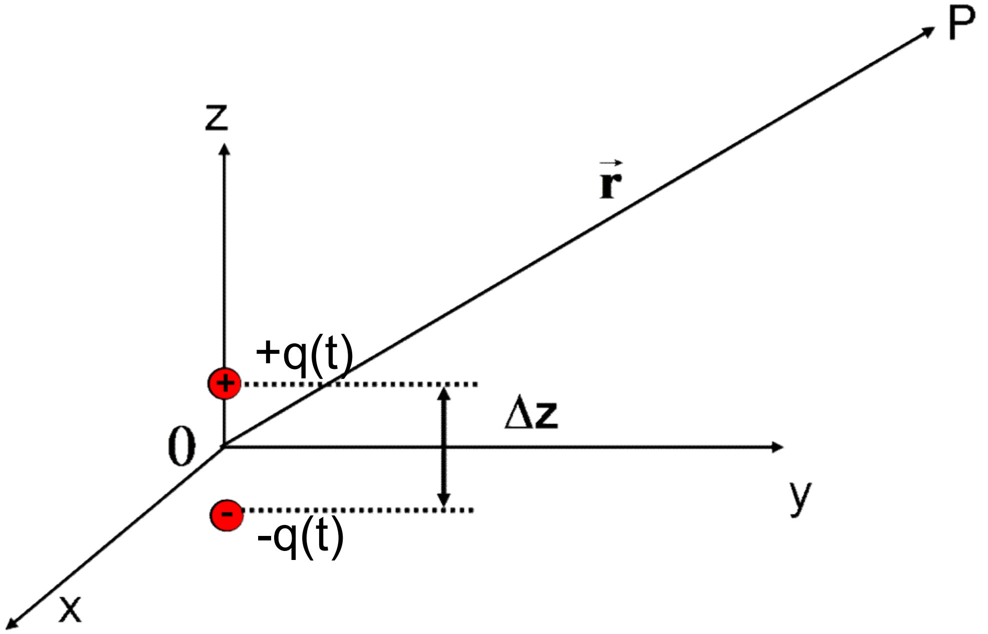

It can be easily shown that this electric current density is the same as that of an electrical dipole such that:

1) the two charges, of opposite sign, have equal time variation;

2) in the spectral domain, the relation between *I* and the time-varying charge *Q* is:

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Elementary electrical dipole

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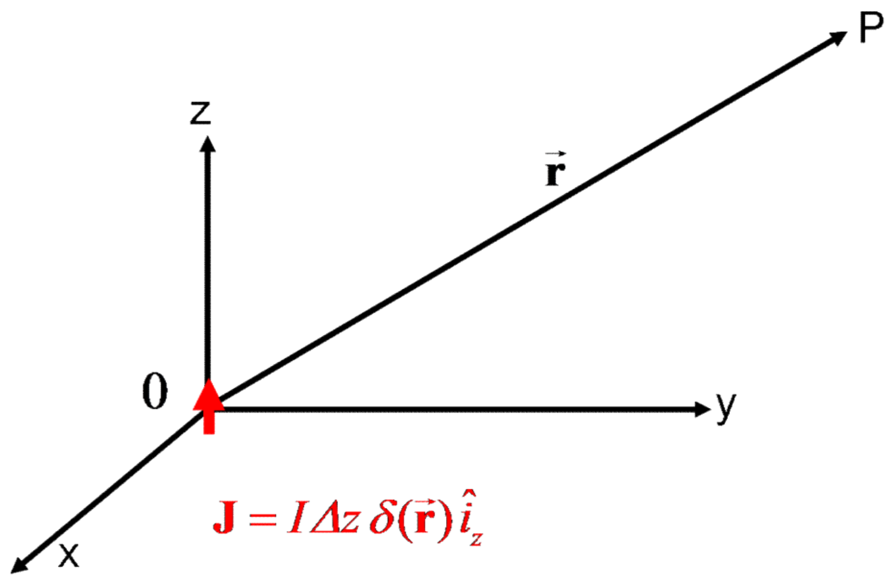


- Why are we interested in such a radiating element?
- Why is such a radiating element referred to as elementary electrical dipole?
- **How can we physically approximate an elementary electrical dipole?**

Hertzian dipole

$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\,\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\,\hat{i}_z$$

• Of course in real life one cannot physically build up a δ-source radiating element but only an approximation.

•An approximation of the elementary dipole was used by Hertz in his experiments, in fact the elementary dipole is often called as Hertzian dipole.

•Note however that an Hertzian dipole is a dipole characterized by:

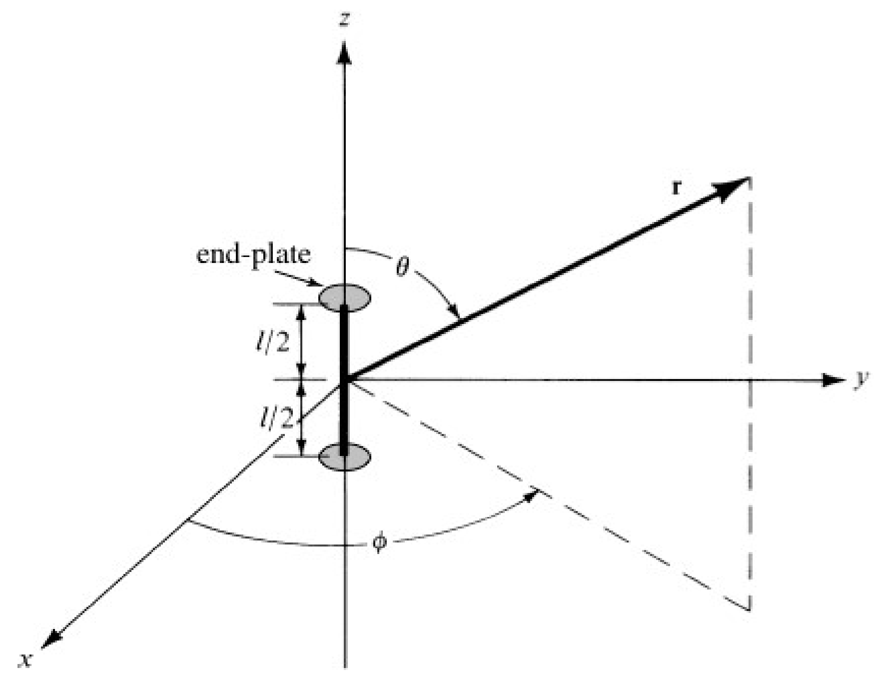
$$\mathbf{J} = I\delta(x)\delta(y)\,\mathrm{rect}\left[\frac{z}{\Delta z}\right]\hat{i}_z$$

$$when\ \ \Delta z \to 0\ \ \ then$$

$$\mathrm{rect}\left[\frac{z}{\Delta z}\right] \to \Delta z\ \delta(z)$$

Hertzian dipole

- The creation of the constant current distribution can be made by two large charge “tanks” at the two edges.
- Note that in practical case this model is meant to be suitable for electrical dipole smaller than λ/50.



Infinitesimal dipole

Elementary electrical dipole

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Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

J

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

A

$$\begin{cases} \nabla^2 A_x + k^2 A_x = -\mu J_x \\ \nabla^2 A_y + k^2 A_y = -\mu J_y \\ \nabla^2 A_z + k^2 A_z = -\mu J_z \end{cases}$$

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

J

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

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$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 \\ \nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I\Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I\Delta z\,\delta(\vec{\mathbf{r}}) \end{cases}$$

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

$\boldsymbol{J}$ $\downarrow$

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

$\downarrow$ $\boldsymbol{A}$

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\, \delta(x)\delta(y)\delta(z) = -\mu I \Delta z\, \delta(\vec{\mathbf{r}}) \end{cases}$$

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$\boldsymbol{J}$

$\downarrow$

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

$\downarrow$

$\boldsymbol{A}$

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \boxed{\nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I \Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I \Delta z\,\delta(\vec{\mathbf{r}})} \end{cases}$$

Elementary electrical dipole

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$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

J ↓

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

↓ ***A***

$$\begin{cases} \nabla^2 A_x + k^2 A_x = 0 & \Rightarrow A_x = 0 \\ \nabla^2 A_y + k^2 A_y = 0 & \Rightarrow A_y = 0 \\ \boxed{\nabla^2 A_z + k^2 A_z = -\mu J_z = -\mu I\Delta z\,\delta(x)\delta(y)\delta(z) = -\mu I\Delta z\,\delta(\vec{\mathbf{r}})} \end{cases}$$

$$\nabla^2 g(\vec{\mathbf{r}}) + k^2 g(\vec{\mathbf{r}}) = -\mu I\Delta z\;\delta(\vec{\mathbf{r}})$$

$$-\mu I\Delta z\;\delta(\vec{\mathbf{r}}) \longrightarrow \boxed{} \longrightarrow g(\vec{\mathbf{r}}) = (-\mu I\Delta z)\,-\frac{1}{4\pi}\frac{e^{-jkr}}{r} = \frac{\mu}{4\pi} I\Delta z \frac{e^{-jkr}}{r}$$

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$\downarrow$ $\boldsymbol{J}$

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ **Vector Helmholtz equation**

$\downarrow$ $\mathbf{A} = \frac{\mu}{4\pi} I\Delta z \frac{e^{-jkr}}{r}\hat{i}_z$

$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A}$ and $\boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla\cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$

$\downarrow$ $\boldsymbol{E}$, $\boldsymbol{H}$

Elementary electrical dipole

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The E.M. field radiated by the elementary electrical dipole

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_r(r,\vartheta)\hat{i}_r + \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2} + \dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r} + \dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

Because of the problem symmetry there is no dependence on the azimuth angle φ.

Elementary electrical dipole

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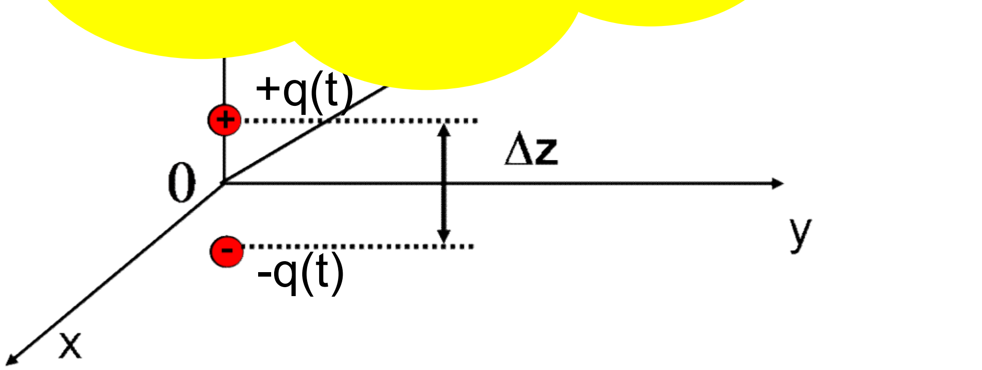
$$\mathbf{J} = I\Delta z\,\delta(\vec{\mathbf{r}})\hat{i}_z = I\Delta z\,\delta(x)\delta(y)\delta(z)\hat{i}_z$$

All the quantities, included the expressions of the fields, can be provided in terms of dipole moment U

$$I\Delta z = j\omega Q\Delta z = j\omega U$$



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The E.M. field radiated by the elementary electrical dipole

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$$j\omega Q = I$$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{j\omega \mathrm{Q} \Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{j\omega \mathrm{Q} \Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{j\omega \mathrm{Q} \Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$j\omega Q = I$$

… for $\omega=0$ simplifies as

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta \boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \end{cases}$$

$\rightarrow 1$

$j\omega Q = I$

... for ω=0 simplifies as

$$\begin{cases} E_r = \dfrac{\zeta}{\sqrt{\mu\varepsilon}} \dfrac{Q\Delta z}{2\pi} \dfrac{1}{r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\zeta}{\sqrt{\mu\varepsilon}} \dfrac{Q\Delta z}{4\pi} \dfrac{1}{r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

$\beta = \omega\sqrt{\mu\varepsilon} \to 0$

$\Rightarrow \omega\beta = \omega^2\sqrt{\mu\varepsilon} \to 0$

$\Rightarrow \omega/\beta = 1/\sqrt{\mu\varepsilon}$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta \boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \end{cases}$$

$\rightarrow 1$

$j\omega Q = I$

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$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \frac{\zeta}{\sqrt{\mu\varepsilon}} = \frac{1}{\varepsilon}$$

Elementary electrical dipole: static case

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{jQ\Delta z}{2\pi}\left(\cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\cos\vartheta \boxed{\exp(-j\beta r)} \\ E_\vartheta = \zeta \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}} + \dfrac{\omega}{j\beta r^3}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \\ H_\varphi = \dfrac{jQ\Delta z}{4\pi}\left(\cancel{\dfrac{j\omega\beta}{r}} + \cancel{\dfrac{\omega}{r^2}}\right)\sin\vartheta \boxed{\exp(-j\beta r)} \end{cases}$$

1

$$j\omega Q = I$$

... for ω=0 simplifies as

$$\begin{cases} E_r = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{2\pi}\dfrac{1}{r^3}\cos\vartheta = \dfrac{Q\Delta z}{2\pi}\dfrac{1}{\varepsilon r^3}\cos\vartheta \\ E_\vartheta = \dfrac{\zeta}{\sqrt{\mu\varepsilon}}\dfrac{Q\Delta z}{4\pi}\dfrac{1}{r^3}\sin\vartheta = \dfrac{Q\Delta z}{4\pi}\dfrac{1}{\varepsilon r^3}\sin\vartheta \\ H_\varphi = 0 \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\Rightarrow \quad \frac{\zeta}{\sqrt{\mu\varepsilon}} = \frac{1}{\varepsilon}$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$
\begin{cases}
E_r = \zeta \dfrac{I\Delta z}{2\pi}\boxed{\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)}\cos\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\dfrac{1}{j\beta r}+\dfrac{1}{[j\beta r]^2}\right)\cos\vartheta\exp(-j\beta r) \\
E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\boxed{\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)}\sin\vartheta\exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}+\dfrac{1}{[j\beta r]^2}\right)\sin\vartheta\exp(-j\beta r) \\
H_\varphi = \dfrac{I\Delta z}{4\pi}\boxed{\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)}\sin\vartheta\exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}\right)\sin\vartheta\exp(-j\beta r)
\end{cases}
$$

$$
\left(\frac{j\beta}{r}+\frac{1}{r^2}+\frac{1}{j\beta r^3}\right)=\frac{j\beta}{r}\left(1+\frac{r}{j\beta}\frac{1}{r^2}+\frac{r}{j\beta}\frac{1}{j\beta r^3}\right) = \frac{j\beta}{r}\left(1+\frac{1}{j\beta r}+\frac{1}{[j\beta r]^2}\right)
$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\dfrac{1}{j\beta r}+\dfrac{1}{\left[j\beta r\right]^2}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}+\dfrac{1}{\left[j\beta r\right]^2}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\dfrac{1}{j\beta r}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) & = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\dfrac{j\beta}{r}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) & = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

… for $\beta r >> 1$ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{r}\boxed{\dfrac{\beta}{4\pi}}\sin\vartheta \exp(-j\beta r) = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{r}\dfrac{\beta}{4\pi}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

$$\frac{\beta}{4\pi} = \frac{2\pi}{\lambda}\frac{1}{4\pi} = \frac{1}{2\lambda}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$2\pi f = \omega$$

$$c = 1/\sqrt{\mu\varepsilon}$$

$$\lambda = c/f$$

Elementary electrical dipole: far field

The E.M. field radiated by the elementary electrical dipole

$$\begin{cases} E_r = \zeta \dfrac{I\Delta z}{2\pi}\left(\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\cos\vartheta \exp(-j\beta r) = \zeta \dfrac{I\Delta z}{2\pi}\dfrac{j\beta}{r}\left(\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\cos\vartheta \exp(-j\beta r) \\ E_\vartheta = \zeta \dfrac{I\Delta z}{4\pi}\left(\boxed{\dfrac{j\beta}{r}}+\dfrac{1}{r^2}+\dfrac{1}{j\beta r^3}\right)\sin\vartheta \exp(-j\beta r) = \zeta \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}+\cancel{\dfrac{1}{[j\beta r]^2}}\right)\sin\vartheta \exp(-j\beta r) \\ H_\varphi = \dfrac{I\Delta z}{4\pi}\left(\boxed{\dfrac{j\beta}{r}}+\dfrac{1}{r^2}\right)\sin\vartheta \exp(-j\beta r) = \dfrac{I\Delta z}{4\pi}\dfrac{j\beta}{r}\left(1+\cancel{\dfrac{1}{j\beta r}}\right)\sin\vartheta \exp(-j\beta r) \end{cases}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

... for $\beta r >> 1$ simplifies as

$$\begin{cases} E_r = 0 \\ E_\vartheta = j\zeta \dfrac{I\Delta z}{r}\dfrac{\beta}{4\pi}\sin\vartheta \exp(-j\beta r) = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_\varphi = j\dfrac{I\Delta z}{r}\dfrac{\beta}{4\pi}\sin\vartheta \exp(-j\beta r) = \dfrac{E_\vartheta}{\zeta} \end{cases}$$

$$\beta r >> 1 \Rightarrow \frac{2\pi}{\lambda} r >> 1 \Rightarrow r >> \lambda$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$2\pi f = \omega$$

$$c = 1/\sqrt{\mu\varepsilon}$$

$$\lambda = c/f$$

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… for r>>λ simplifies as

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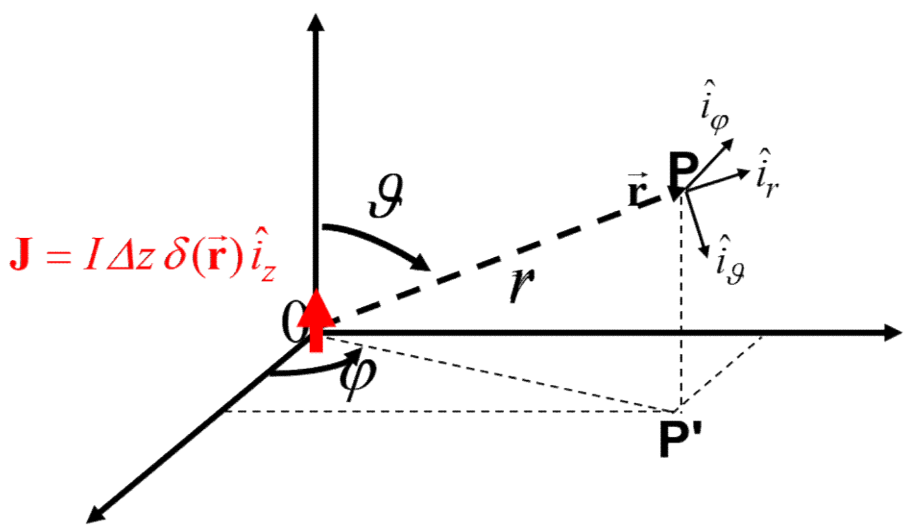
In the far-field case (r>>λ) the elementary electrical dipole behaves as follows

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathrm{E}_{\vartheta}(r,\vartheta)\hat{i}_{\vartheta}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_{\varphi}(r,\vartheta)\hat{i}_{\varphi}$$

$$\begin{cases} E_{\vartheta} = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ H_{\varphi} = \dfrac{E_{\vartheta}}{\zeta} \end{cases}$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

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$$\zeta\vec{\mathbf{H}} = \zeta \mathrm{H}_\varphi \hat{i}_\varphi = E_\vartheta \hat{i}_\varphi$$

$$\hat{i}_r \times \vec{\mathbf{E}} = \hat{i}_r \times E_\vartheta \hat{i}_\vartheta = E_\vartheta \hat{i}_\varphi$$

$$\Rightarrow \quad \zeta\mathbf{H} = \hat{i}_r \times \mathbf{E}$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$

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$$\begin{cases} E_\vartheta = j\zeta \dfrac{I\Delta z}{2\lambda r}\sin\vartheta \exp(-j\beta r) \\ \boxed{H_\varphi = \dfrac{E_\vartheta}{\zeta}} \end{cases}$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{1}{2}E_\vartheta\hat{i}_\vartheta \times \left(H_\varphi\hat{i}_\varphi\right)^* = \boxed{\frac{1}{2}E_\vartheta H_\varphi^*\hat{i}_r}$$

$$\hat{i}_\varphi = \hat{i}_r \times \hat{i}_\vartheta$$
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$$\frac{1}{2}E_\vartheta H_\varphi^*\hat{i}_r = \frac{1}{2\zeta}E_\vartheta E_\vartheta^*\hat{i}_r = \frac{1}{2\zeta}\left|E_\vartheta\right|^2\hat{i}_r = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r$$

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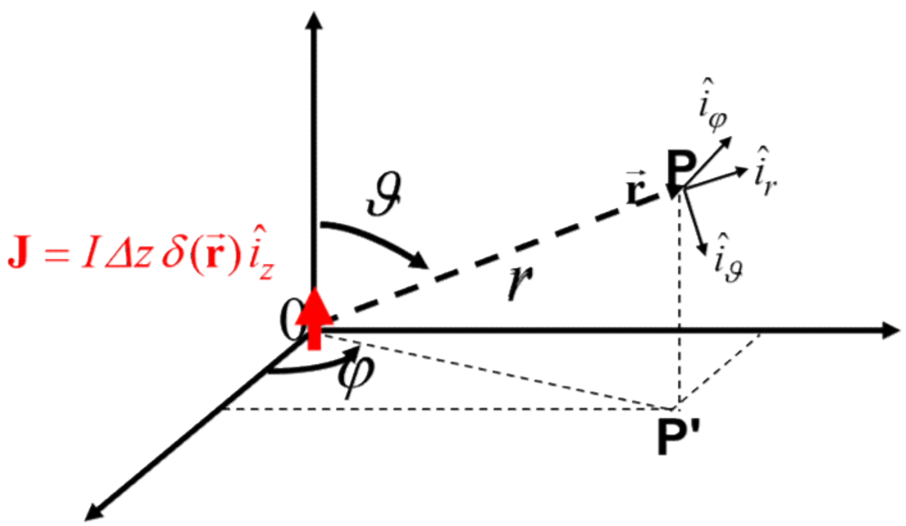
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \mathrm{H}_\varphi(r,\vartheta)\hat{i}_\varphi$$



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