

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

# Incidence: Limit and Brewster angles

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

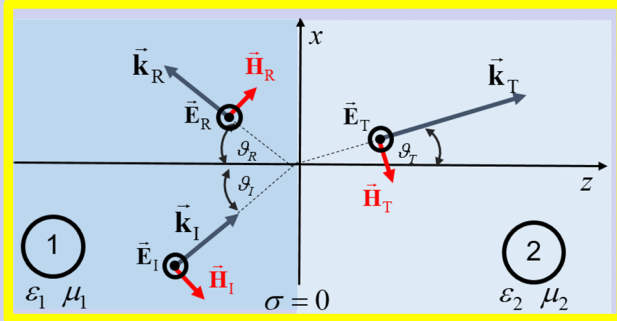
$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$$[H_x, E_y, H_z]$$

$$Z_1 = \frac{\omega \mu_1}{k_{1z}}$$

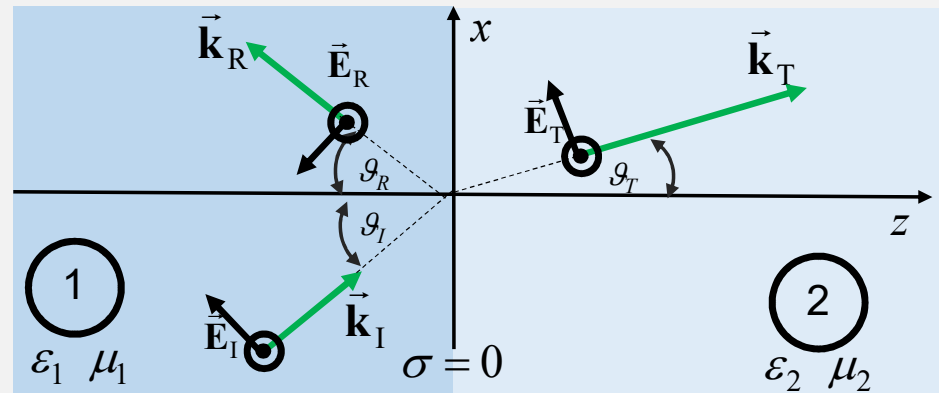
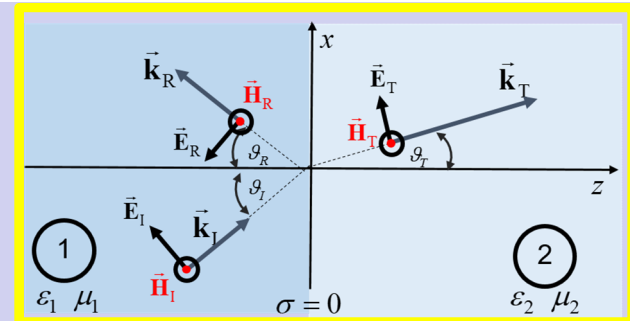
$$Z_2 = \frac{\omega \mu_2}{k_{2z}}$$



$$[E_x, H_y, E_z]$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$



# Incidence: Limit angle

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

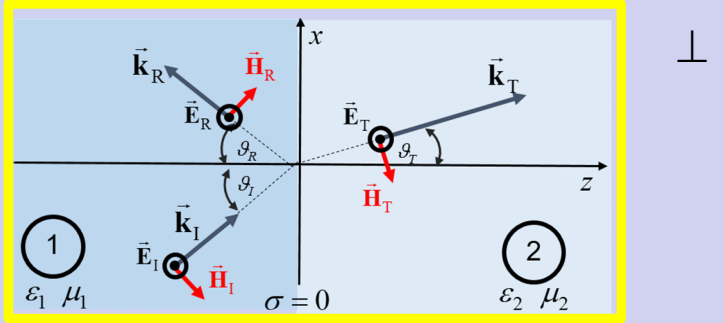
$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$$[H_x, E_y, H_z]$$

$$Z_1 = \frac{\omega \mu_1}{k_{1z}}$$

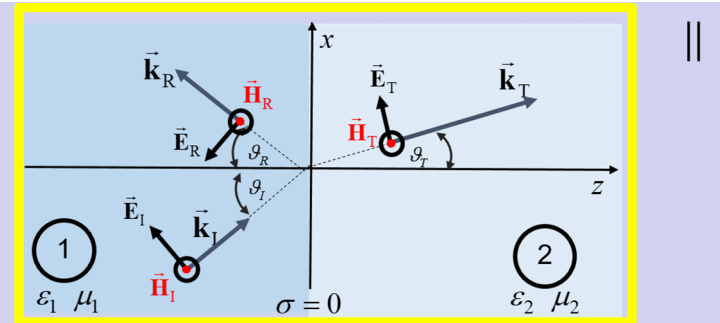
$$Z_2 = \frac{\omega \mu_2}{k_{2z}}$$



$$[E_x, H_y, E_z]$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$$

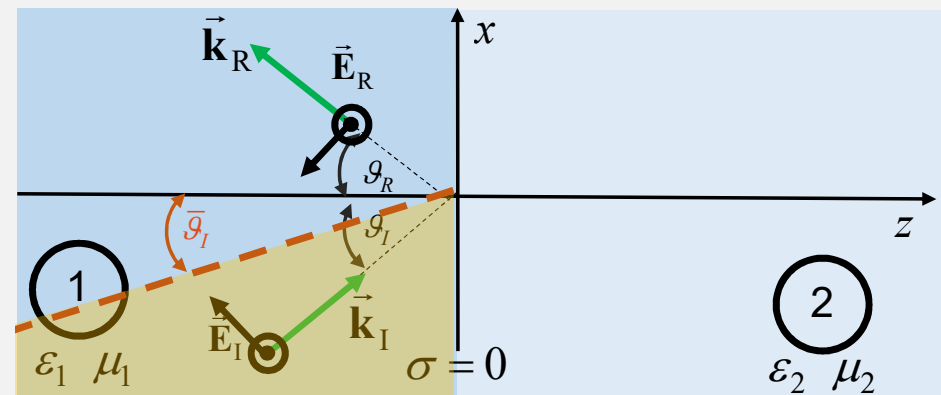
$$Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$



if  $\frac{k_2}{k_1} < 1$

An angle  $\bar{\vartheta}_1$  exists, referred to as **limit angle**, such that for  $\vartheta_1 \geq \bar{\vartheta}_1$  no propagation occurs in the second half-space

$$\sin \bar{\vartheta}_1 = \frac{k_2}{k_1}$$



# Incidence: Brewster angle

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

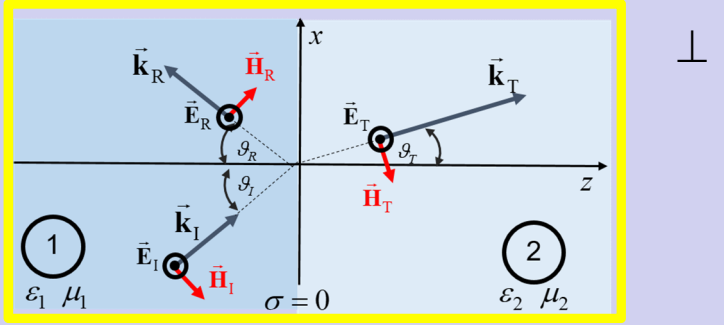
$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$$[H_x, E_y, H_z]$$

$$Z_1 = \frac{\omega \mu_1}{k_{1z}}$$

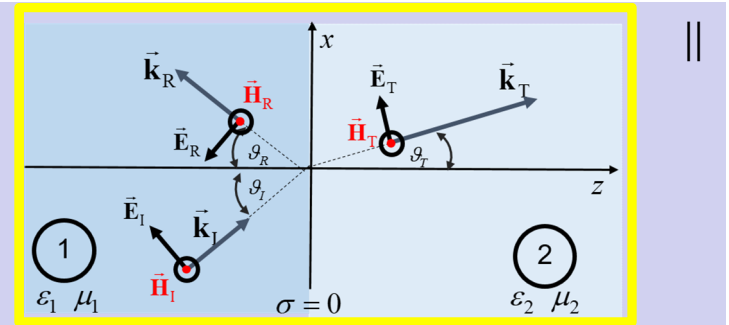
$$Z_2 = \frac{\omega \mu_2}{k_{2z}}$$



$$[E_x, H_y, E_z]$$

$$Z_1 = \frac{k_{1z}}{\omega \varepsilon_1}$$

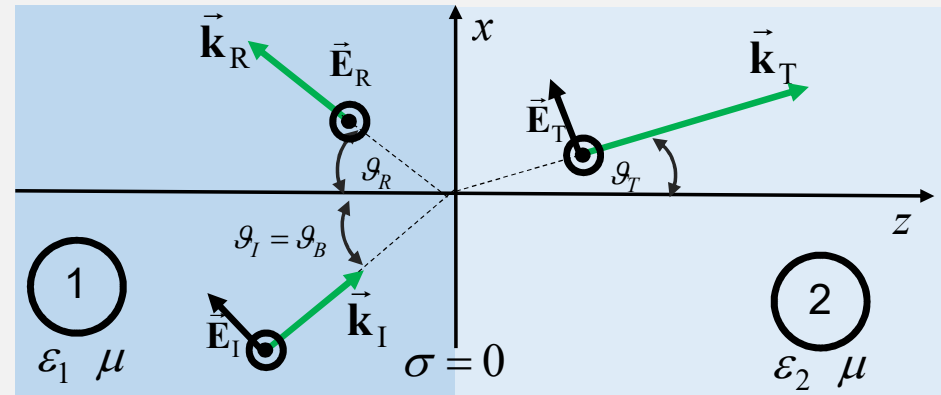
$$Z_2 = \frac{k_{2z}}{\omega \varepsilon_2}$$



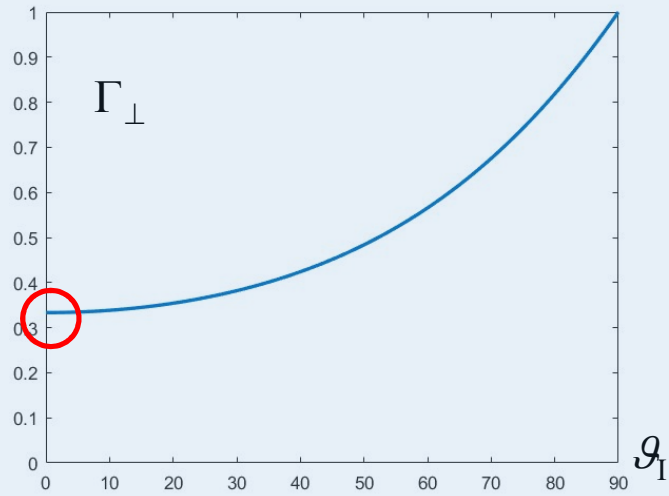
if  $\mu_1 = \mu_2$  and  $\varepsilon_1 \neq \varepsilon_2$

An angle  $\vartheta_B$  exists, referred to as **Brewster angle**, such that an unpolarized plane wave incident at angle  $\vartheta_I = \vartheta_B$  is reflected with perpendicular polarization

$$\sin^2 \vartheta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$



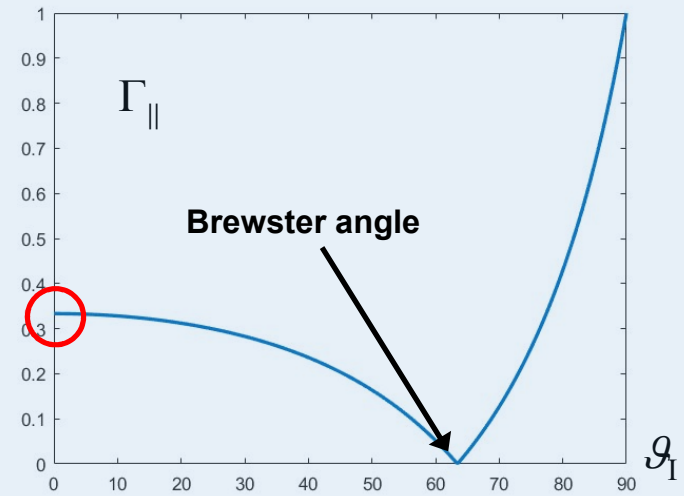
# Fresnel coefficients



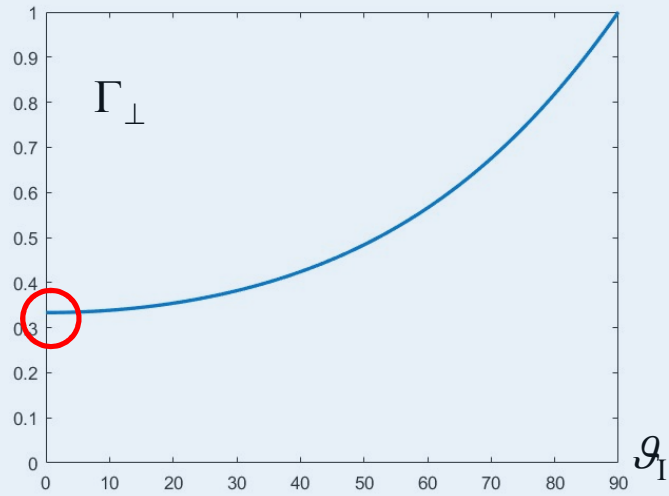
$$\mu_1 = \mu_2$$

$$\varepsilon_2 = 4\varepsilon_1$$

$$\frac{k_2}{k_1} > 1$$



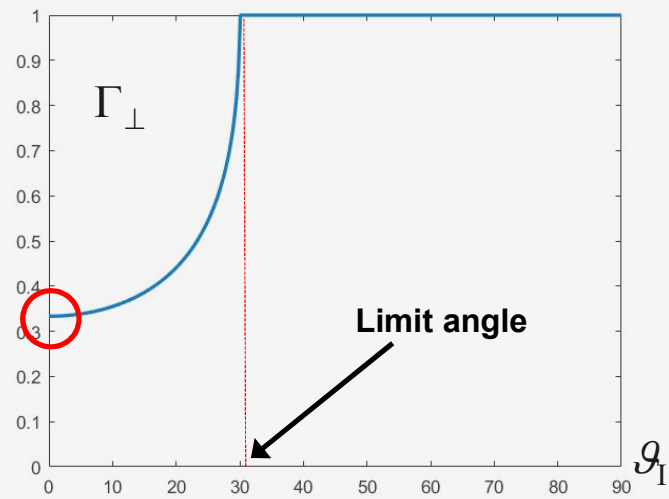
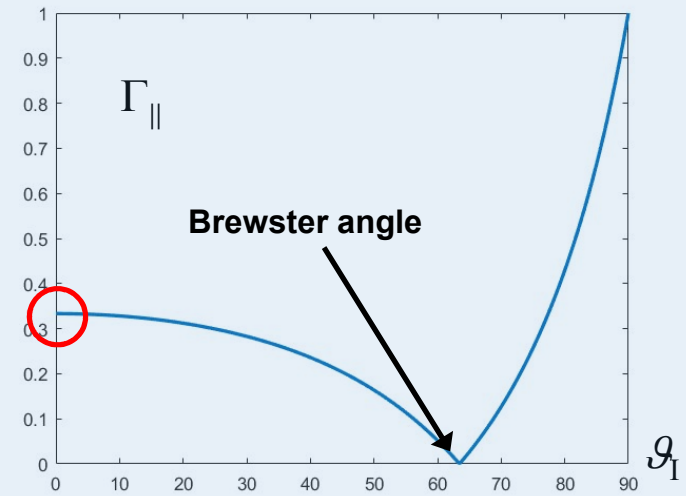
# Fresnel coefficients



$$\mu_1 = \mu_2$$

$$\varepsilon_2 = 4\varepsilon_1$$

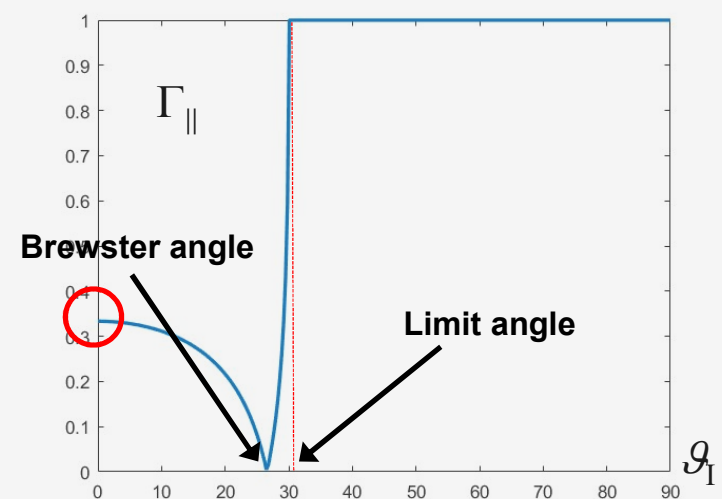
$$\frac{k_2}{k_1} > 1$$



$$\mu_1 = \mu_2$$

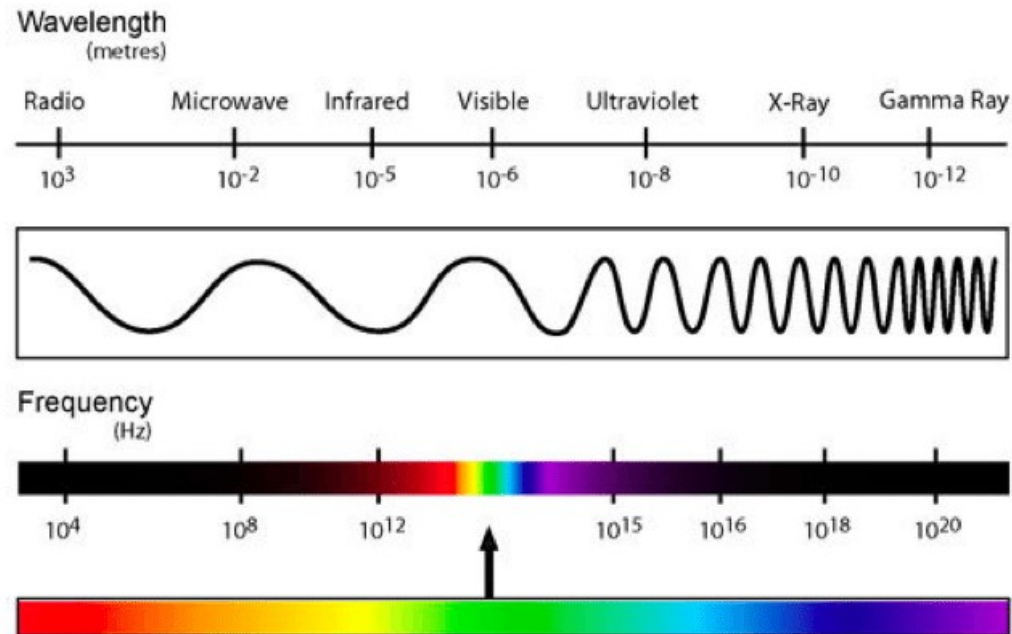
$$4\varepsilon_2 = \varepsilon_1$$

$$\frac{k_2}{k_1} < 1$$





# Electromagnetic spectrum



**Free space (Linear, isotropic, local, homogeneous)**

$$\lambda = \frac{c}{f}$$

$f$ : frequency  
 $\lambda$ : wavelength  
 $c$ : light speed

# Incidence: Limit angle

2  
 $\epsilon_2 \mu_2$

1  
 $\epsilon_1 \mu_1$

