Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma_\perp \triangleq \frac{E_R}{E_I}$$

$$T_\perp \triangleq \frac{E_T}{E_I}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T_\perp = 1 + \Gamma_\perp$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right]e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

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Incidence on a dielectric half-space: ⊥ polarization

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$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

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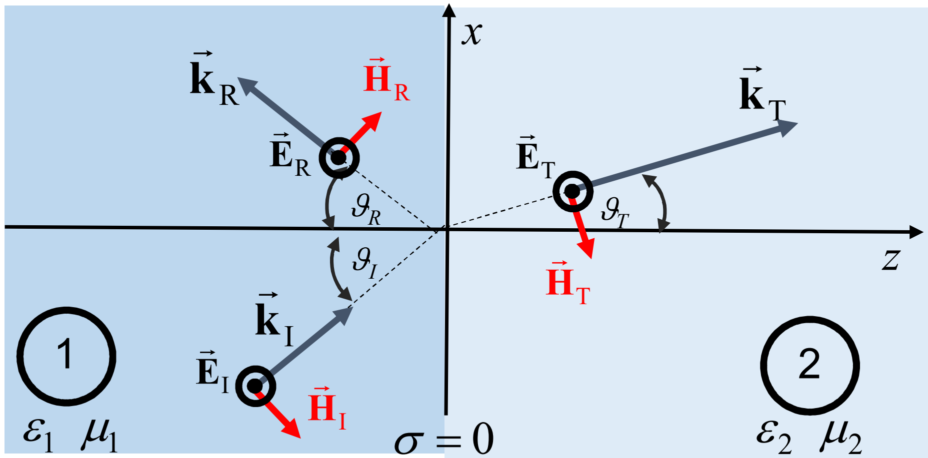

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right] e^{-jk_x x} = E_I e^{-jk_x x} e^{-jk_{1z}z} + E_R e^{-jk_x x} e^{jk_{1z}z}$$

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$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

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Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

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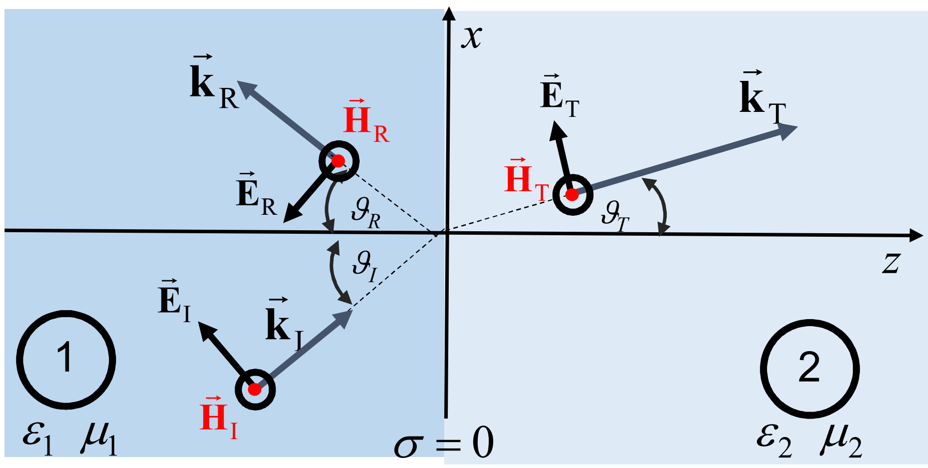
$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ‖

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \Rightarrow E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z \Rightarrow E_z = -\frac{k_x}{\omega\varepsilon} H_y$$



Incidence on a dielectric half-space: ∥ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

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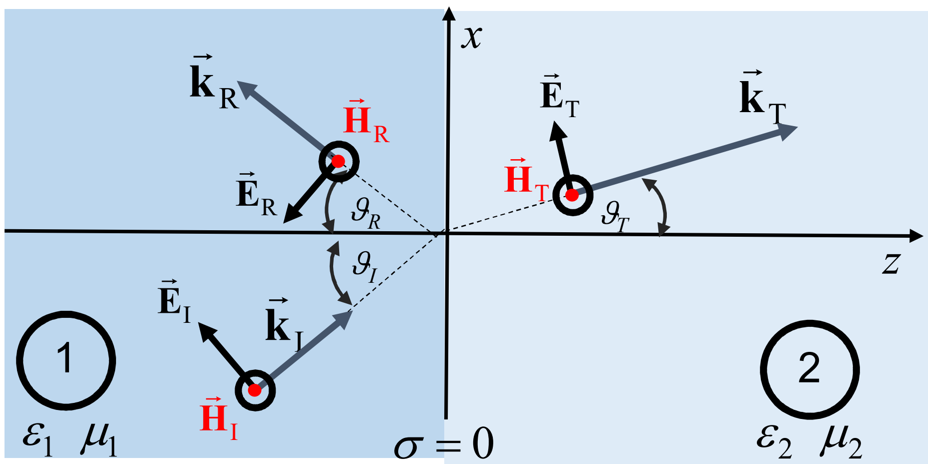

Incidence on a dielectric half-space: ‖ polarization

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$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

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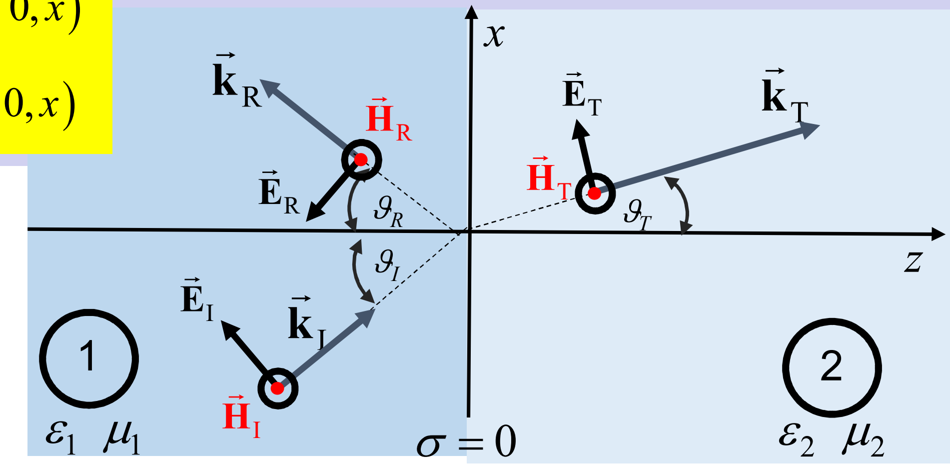


MEMO

Fields at boundaries

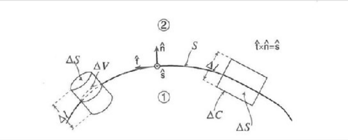
$$\hat{\mathbf{n}}\times(\dot{\mathbf{e}}_2 - \dot{\mathbf{e}}_1) = \mathbf{0}$$

$$\hat{\mathbf{n}}\times(\dot{\mathbf{h}}_2 - \dot{\mathbf{h}}_1) = \dot{\mathbf{j}}_s$$

$$(\dot{\mathbf{d}}_2 - \dot{\mathbf{d}}_1)\cdot\hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}} = -\frac{\partial\rho_s}{\partial t}$$



Incidence on a dielectric half-space: ∥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

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$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
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$$\left[E_x, H_y, E_z\right]$$

Parallel Polarization ∥

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z};\quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$H_{1y}(z=0,x) = H_{2y}(z=0,x)$$
$$E_{1x}(z=0,x) = E_{2x}(z=0,x)$$

1

$$\vec{\mathbf{H}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = H_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}\hat{i}_y \qquad \vec{\mathbf{H}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = H_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}\hat{i}_y$$

$$\vec{\mathbf{H}}_1(z,x) = \left[H_{\mathrm{I}} e^{-jk_{1z} z} + H_{\mathrm{R}} e^{jk_{1z} z}\right] e^{-jk_x x}\hat{i}_y$$

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$$\vec{\mathbf{H}}_{\mathrm{I}} = H_{\mathrm{I}}\hat{i}_y$$
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$$\vec{\mathbf{H}}_{\mathrm{T}} = H_{\mathrm{T}}\hat{i}_y$$

2

$$\vec{\mathbf{H}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = H_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}\hat{i}_y$$

$$\vec{\mathbf{H}}_2(z,x) = H_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}\hat{i}_y$$

$$H_{2y}(z,x) = H_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}$$

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$$E_{1x}(z=0,x) = \frac{k_{1z}}{\omega\varepsilon_1}\left[H_{\mathrm{I}} - H_{\mathrm{R}}\right] e^{-jk_x x}$$

$$H_{1y}(z,x) = \left[H_{\mathrm{I}} e^{-jk_{1z} z} + H_{\mathrm{R}} e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$E_{1x}(z,x) = -\frac{1}{j\omega\varepsilon_1}\left[-jk_{1z} H_{\mathrm{I}} e^{-jk_{1z} z} + jk_{1z} H_{\mathrm{R}} e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

2

$$H_{2y}(z=0,x) = H_{\mathrm{T}} e^{-jk_x x}$$

$$E_{2x}(z=0,x) = \frac{k_{2z}}{\omega\varepsilon_2} H_{\mathrm{T}} e^{-jk_x x}$$

$$H_{2y}(z,x) = H_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z,x) = -\frac{1}{j\omega\varepsilon_2}\left[-jk_{2z} H_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}\right]$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

Incidence on a dielectric half-space: ∥ polarization

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$$\left[E_x, H_y, E_z\right]$$

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$$H_{1y}(z=0,x) = \left[H_\mathrm{I} + H_\mathrm{R}\right] e^{-jk_x x}$$

$$E_{1x}(z=0,x) = Z_1\left[H_\mathrm{I} - H_\mathrm{R}\right] e^{-jk_x x}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z} z} + H_\mathrm{R} e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$E_{1x}(z,x) = -\frac{1}{j\omega\varepsilon_1}\left[-jk_{1z} H_\mathrm{I} e^{-jk_{1z} z} + jk_{1z} H_\mathrm{R} e^{jk_{1z} z}\right] e^{-jk_x x}$$

2

$$H_{2y}(z=0,x) = H_\mathrm{T} e^{-jk_x x}$$

$$E_{2x}(z=0,x) = Z_2 H_\mathrm{T} e^{-jk_x x}$$

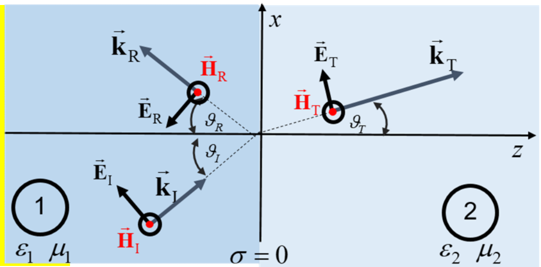
$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$H_{2y}(z,x) = H_\mathrm{T} e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z,x) = -\frac{1}{j\omega\varepsilon_2}\left[-jk_{2z} H_\mathrm{T} e^{-jk_{2z} z} e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}}$$



$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1\cos\vartheta_\mathrm{I}$$

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$$H_{1y}(z=0,x) = H_{2y}(z=0,x)$$

$$E_{1x}(z=0,x) = E_{2x}(z=0,x)$$

$$\begin{cases} H_\mathrm{I} + H_\mathrm{R} = H_\mathrm{T} \\ H_\mathrm{I} - H_\mathrm{R} = \dfrac{Z_2}{Z_1} H_\mathrm{T} \end{cases} \qquad \begin{cases} 1 + \dfrac{H_\mathrm{R}}{H_\mathrm{I}} = \dfrac{H_\mathrm{T}}{H_\mathrm{I}} \\ 1 - \dfrac{H_\mathrm{R}}{H_\mathrm{I}} = \dfrac{Z_2}{Z_1}\dfrac{H_\mathrm{T}}{H_\mathrm{I}} \end{cases}$$

1

$$H_{1y}(z=0,x) = [H_\mathrm{I} + H_\mathrm{R}]e^{-jk_x x}$$

$$E_{1x}(z=0,x) = Z_1[H_\mathrm{I} - H_\mathrm{R}]e^{-jk_x x}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

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$$E_{2x}(z=0,x) = Z_2 H_\mathrm{T}e^{-jk_x x}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$H_{2y}(z,x) = H_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}$$

$$E_{2x}(z,x) = -\frac{1}{j\omega\varepsilon_2}\left[-jk_{2z}H_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \frac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma \triangleq -\frac{H_R}{H_I} \qquad \mathrm{T} \triangleq \frac{Z_2 H_T}{Z_1 H_I}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1\cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon}H_y$$

$$H_{1y}(z=0,x) = H_{2y}(z=0,x)$$

$$E_{1x}(z=0,x) = E_{2x}(z=0,x)$$

1

$$H_{1y}(z=0,x) = [H_I + H_R]e^{-jk_x x}$$

$$E_{1x}(z=0,x) = Z_1[H_I - H_R]e^{-jk_x x}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$H_{1y}(z,x) = \left[H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}\right]e^{-jk_x x}$$

$$E_{1x}(z,x) = -\frac{1}{j\omega\varepsilon_1}\left[-jk_{1z}H_I e^{-jk_{1z}z} + jk_{1z}H_R e^{jk_{1z}z}\right]e^{-jk_x x}$$

2

$$H_{2y}(z=0,x) = H_T e^{-jk_x x}$$

$$E_{2x}(z=0,x) = Z_2 H_T e^{-jk_x x}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$H_{2y}(z,x) = H_T e^{-jk_{2z}z}e^{-jk_x x}$$

$$E_{2x}(z,x) = -\frac{1}{j\omega\varepsilon_2}\left[-jk_{2z}H_T e^{-jk_{2z}z}e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma \triangleq -\frac{H_{\mathrm{R}}}{H_{\mathrm{I}}} \qquad \mathrm{T} \triangleq \frac{Z_2 H_{\mathrm{T}}}{Z_1 H_{\mathrm{I}}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$$\left[E_x, H_y, E_z\right]$$

Parallel Polarization ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

1

$$H_{1y}(z,x) = \left[H_{\mathrm{I}} e^{-jk_{1z}z} + H_{\mathrm{R}} e^{jk_{1z}z}\right] e^{-jk_x x}$$

2

$$H_{2y}(z,x) = H_{\mathrm{T}} e^{-jk_{2z}z} e^{-jk_x x}$$

Incidence on a dielectric half-space: ∥ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma \triangleq -\frac{H_R}{H_I} \qquad \mathrm{T} \triangleq \frac{Z_2 H_T}{Z_1 H_I}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ∥

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon}H_y$$

$$\Gamma_{\|} = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$H_{1y}(z,x) = \left[H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$H_{2y}(z,x) = H_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ∥ polarization

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

Fresnel reflection coefficient

$$\Gamma_{\parallel} = \frac{\dfrac{k_{2z}}{\omega\varepsilon_2} - \dfrac{k_{1z}}{\omega\varepsilon_1}}{\dfrac{k_{2z}}{\omega\varepsilon_2} + \dfrac{k_{1z}}{\omega\varepsilon_1}} = \frac{k_{2z}\varepsilon_1 - k_{1z}\varepsilon_2}{k_{2z}\varepsilon_1 + k_{1z}\varepsilon_2} = -\frac{k_{1z} - k_{2z}(\varepsilon_1/\varepsilon_2)}{k_{1z} + k_{2z}(\varepsilon_1/\varepsilon_2)} = -\frac{k_1\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}}{k_1\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}}$$

$$= -\frac{k_1\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)k_1\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{k_1\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)k_1\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}} = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$$\left[E_x, H_y, E_z\right]$$

Parallel Polarization ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}} \qquad \mathrm{T}_\parallel = 1 + \Gamma_\parallel$$

$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z}z} + H_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$H_{2y}(z,x) = H_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$$\left[E_x, H_y, E_z\right]$$

Parallel Polarization ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon}H_y$$

$$\Gamma \triangleq -\frac{H_{\mathrm{R}}}{H_{\mathrm{I}}}$$

$$\mathrm{T} \triangleq \frac{Z_2 H_{\mathrm{T}}}{Z_1 H_{\mathrm{I}}}$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_{\|} = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$$\mathrm{T}_{\|} = 1 + \Gamma_{\|}$$

$$H_{1y}(z,x) = \left[H_{\mathrm{I}} e^{-jk_{1z}z} + H_{\mathrm{R}} e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$H_{2y}(z,x) = H_{\mathrm{T}} e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ∥ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1+Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1\cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2\sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ||

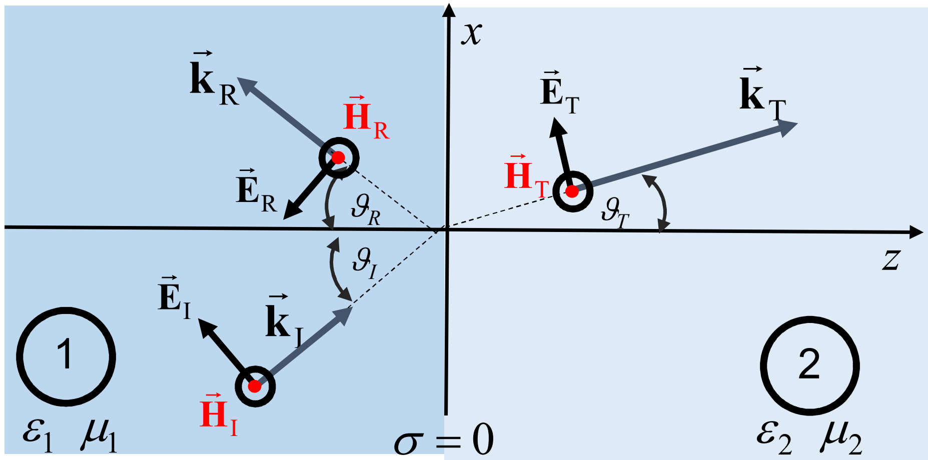
$$H_{1y}(z,x) = \left[H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}\right]e^{-jk_x x} \qquad (1)$$

$$H_{2y}(z,x) = H_T e^{-jk_{2z}z} e^{-jk_x x} \qquad (2)$$

$$\Gamma_{\parallel} \triangleq -\frac{H_R}{H_I} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_{\parallel} \triangleq \frac{Z_2 H_T}{Z_1 H_I} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_{\parallel} = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$



Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

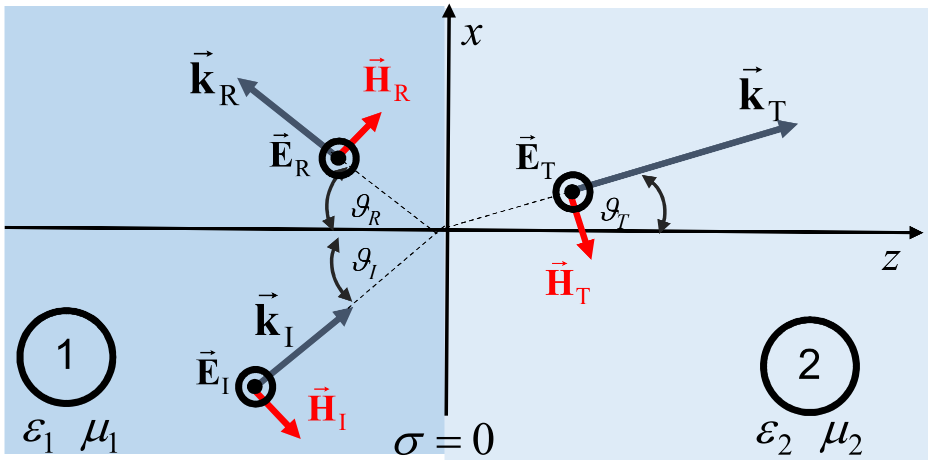
$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\perp \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$



Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \frac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1\cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\perp \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z}z} + H_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \quad (1)$$

$$H_{2y}(z,x) = H_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_\| \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_\| \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \frac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

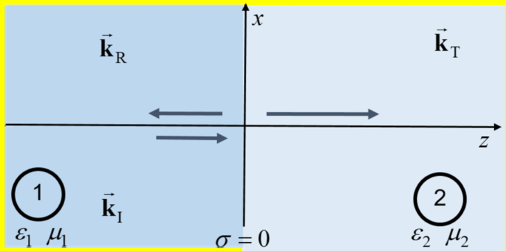
$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow \quad k_{1z} = k_1 \qquad k_{2z} = k_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1\cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

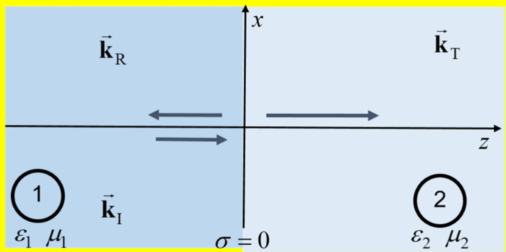
$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow k_{1z} = k_1 \qquad k_{2z} = k_2$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

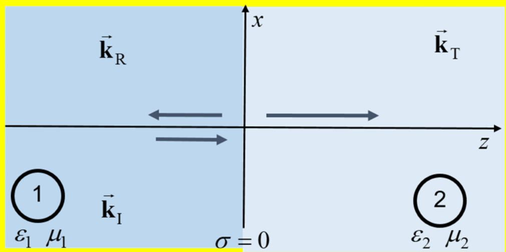
$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow k_{1z} = k_1 \qquad k_{2z} = k_2$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

$\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} = \frac{k_1}{\omega\varepsilon_1} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\varepsilon_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2} = \zeta_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

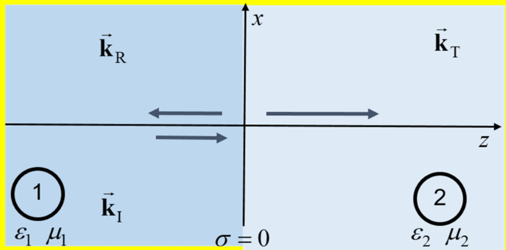
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

In the case of normal incidence, perpendicular and parallel polarizations behave the same

Normal incidence



$$\vartheta_I = 0° \Rightarrow \quad k_{1z} = k_1, \quad k_{2z} = k_2$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

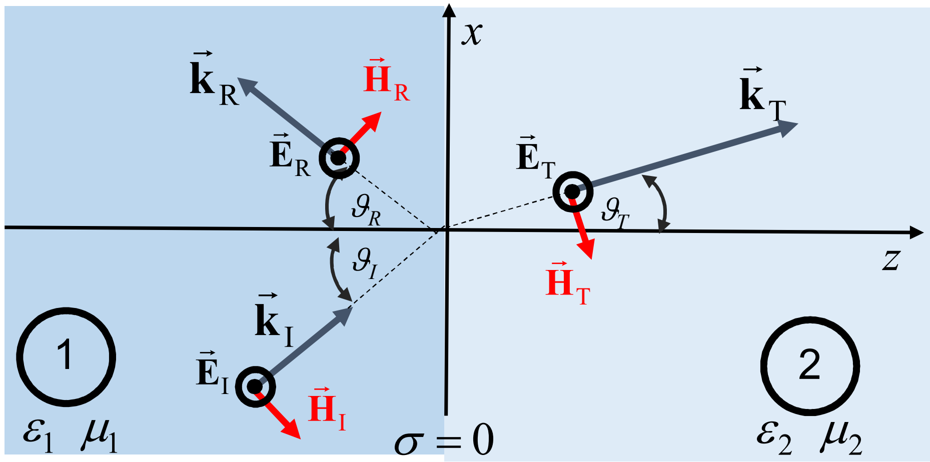
$\parallel$

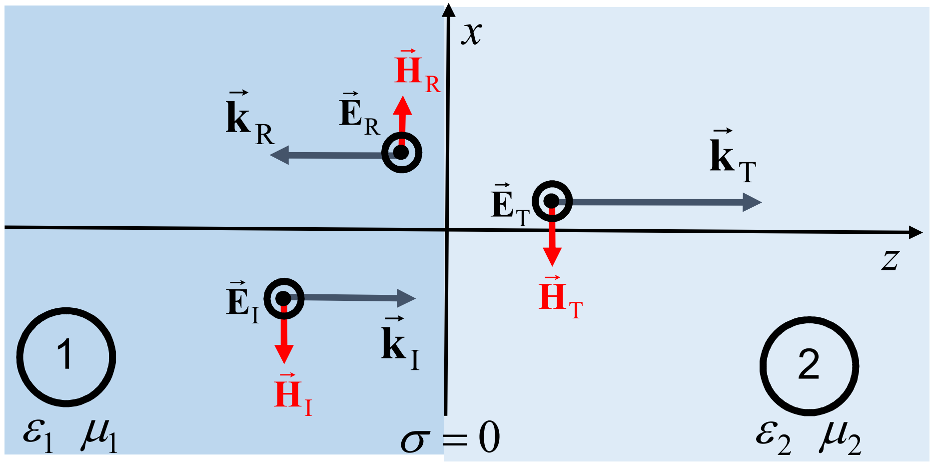
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} = \frac{k_1}{\omega\varepsilon_1} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\varepsilon_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2} = \zeta_2$$

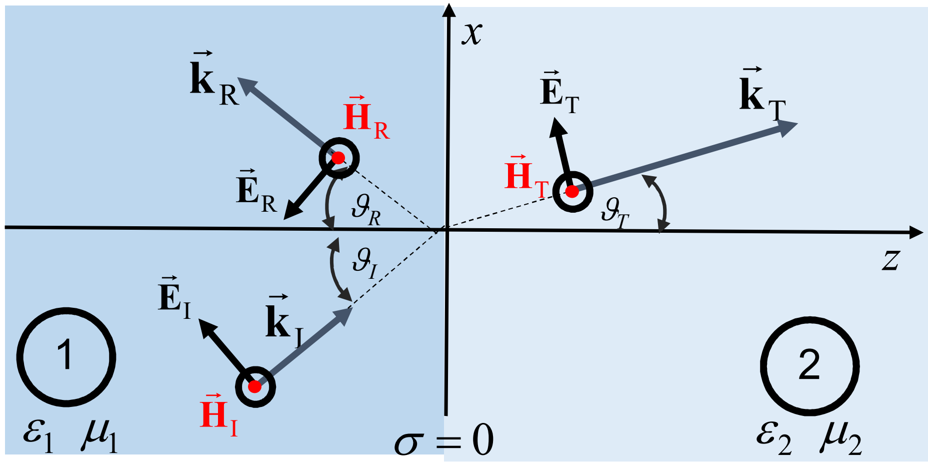
Normal Incidence

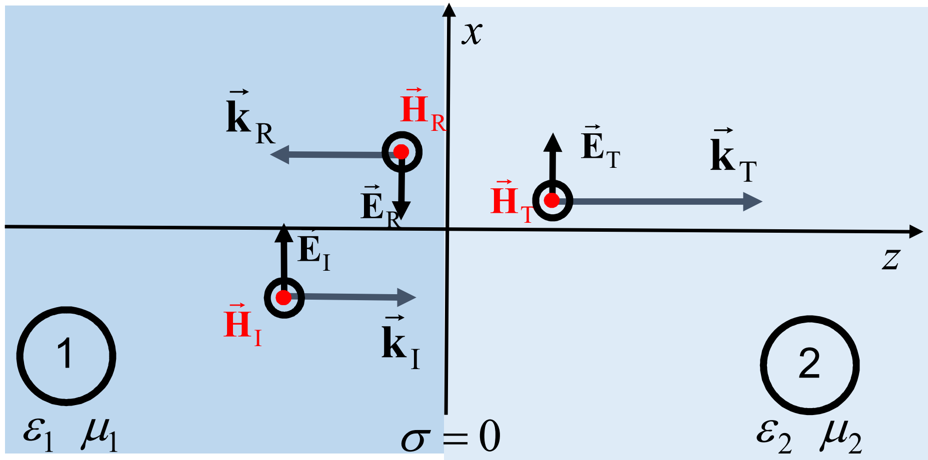
Perpendicular Polarization $\perp$





Parallel Polarization $\parallel$



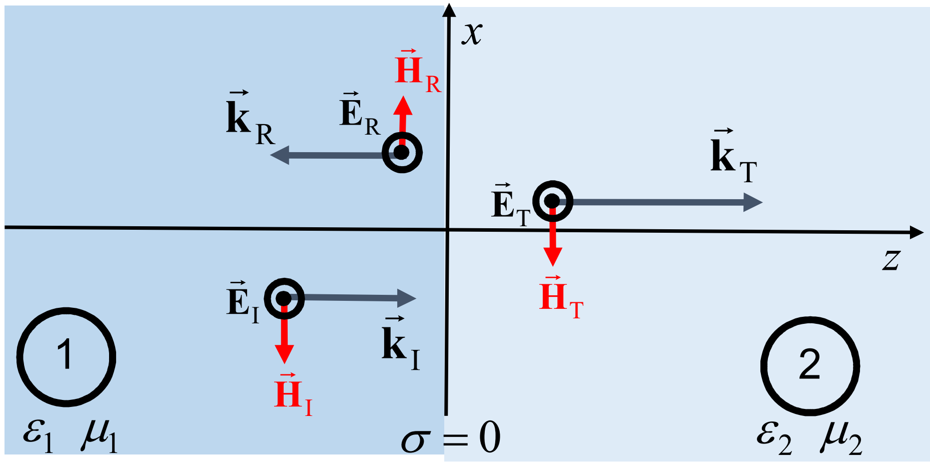


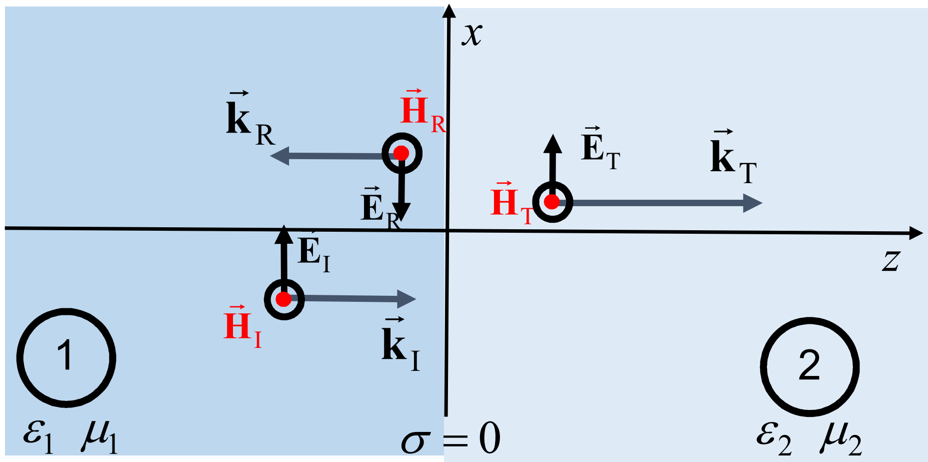
Normal Incidence

Perpendicular Polarization $\perp$

Parallel Polarization $\parallel$

In the case of normal incidence, perpendicular and parallel polarizations behave the same





Limit angle

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z}\hat{i}_z$$
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1\cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}$$

$\left[H_x, E_y, H_z\right]$ Perpendicular Polarization $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

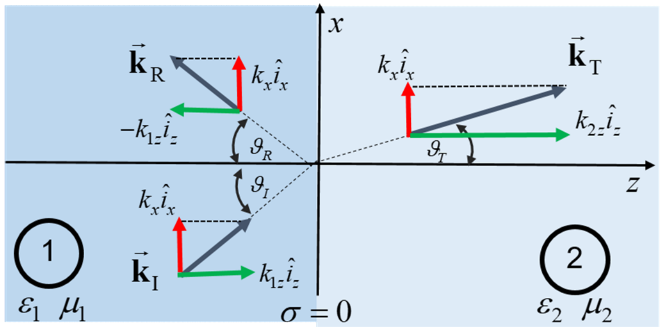
$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$\left[E_x, H_y, E_z\right]$ Parallel Polarization $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$\vec{\mathbf{k}}_I = k_x\hat{i}_x + k_{1z}\hat{i}_z$$
$$\vec{\mathbf{k}}_R = k_x\hat{i}_x - k_{1z}\hat{i}_z$$
$$\vec{\mathbf{k}}_T = k_x\hat{i}_x + k_{2z}\hat{i}_z$$



Limit angle

$$\vec{\mathbf{k}}_T = k_x\hat{i}_x + k_{2z}\hat{i}_z$$
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

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$[E_x, H_y, E_z]$ Parallel Polarization $\parallel$

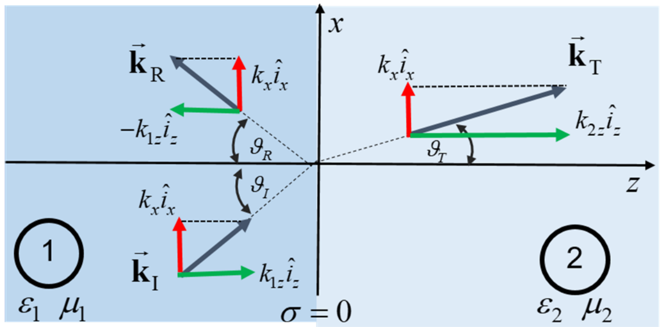
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

if $\dfrac{k_2}{k_1} < 1$, an angle $\bar{\vartheta}_I$ exists such that $\sin\bar{\vartheta}_I = \dfrac{k_2}{k_1}$

$$\vartheta = \bar{\vartheta}_I \;\Rightarrow\; \sin\vartheta_I = \frac{k_2}{k_1} \;\Rightarrow\; k_{2z} = \sqrt{k_2^2 - k_1^2\frac{k_2^2}{k_1^2}} = 0$$

$$\Rightarrow\; \vec{\mathbf{k}}_T = k_x\hat{i}_x \;\Rightarrow\; \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-jk_x x} \qquad |\Gamma_\parallel| = |\Gamma_\perp| = 1$$



Limit angle

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$
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$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2} \mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_I \qquad k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

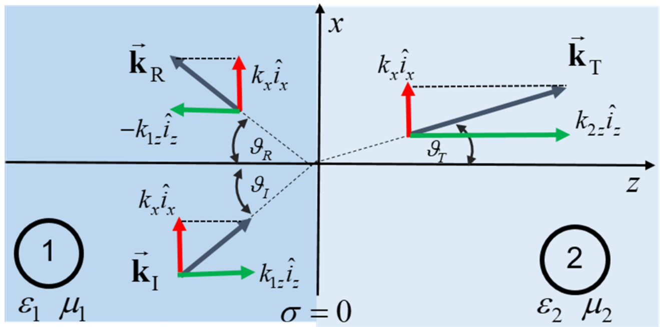
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

if $\dfrac{k_2}{k_1} < 1$, an angle $\bar{\vartheta}_I$ exists such that $\sin\bar{\vartheta}_I = \dfrac{k_2}{k_1}$

$$\vartheta > \bar{\vartheta}_I \Rightarrow \sin\vartheta_I > \frac{k_2}{k_1} \Rightarrow k_2^2 - k_1^2 \sin^2\vartheta_I < 0 \Rightarrow k_{2z} = -ja$$

$$\Rightarrow \vec{\mathbf{k}}_T = k_x \hat{i}_x - ja\hat{i}_z \Rightarrow \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-az} \qquad |\Gamma_\parallel| = |\Gamma_\perp| = 1$$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
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$$k_x = k_1 \sin\vartheta_\mathrm{I}$$
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$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$
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$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

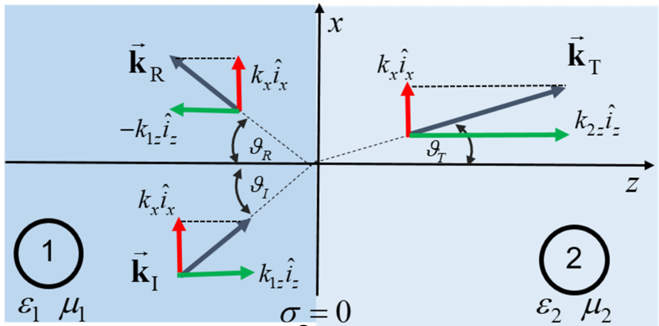
$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case ($\mu_1=\mu_2$). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

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$$k_x = k_1 \sin\vartheta_I$$

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$$\vartheta_I = \vartheta_R$$

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$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

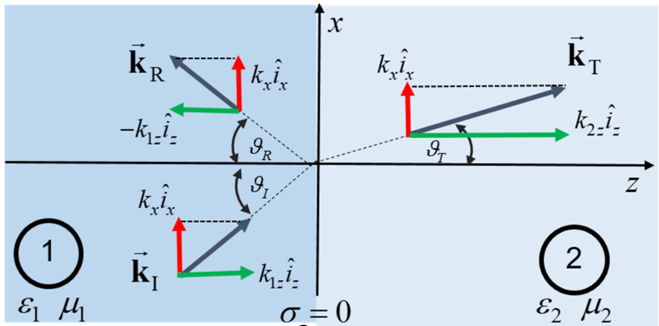
$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case ($\mu_1=\mu_2$). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

Perpendicular Polarization $\perp$

$$Z_2 - Z_1 \quad \Rightarrow k_{1z} = k_{2z} \quad \Rightarrow k_1 = k_2$$

This condition cannot be enforced, since $\mu_1=\mu_2$ and $\varepsilon_1 \neq \varepsilon_2$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$
$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

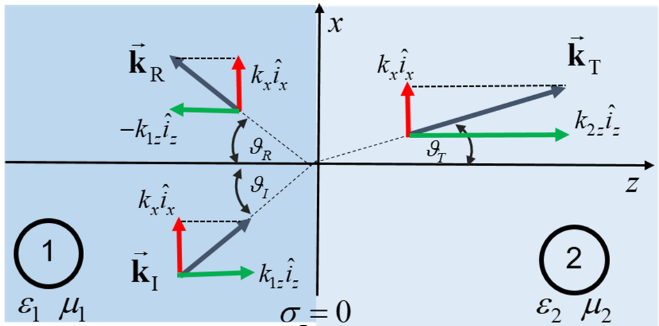
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case (μ_1=μ_2). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

Parallel Polarization $\parallel$

$$Z_2 - Z_1 \quad \Rightarrow \frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2} \quad \Rightarrow \sin^2\vartheta = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$



Brewster angle

$$\begin{cases} 1-\Gamma = \frac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

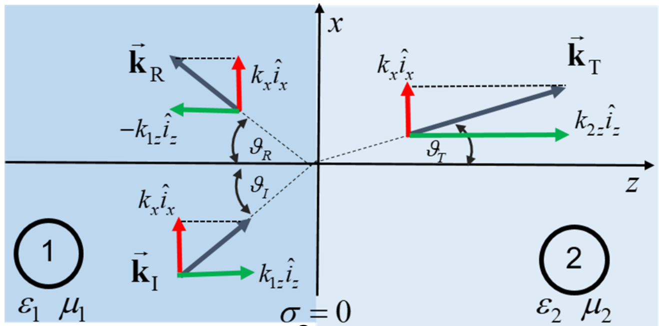
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case (μ_1=μ_2). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

$$\sin^2\vartheta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2} \Rightarrow \begin{matrix} \Gamma_\parallel = 0 \\ \Gamma_\perp \neq 0 \end{matrix}$$

An unpolarized plane wave incident at angle ϑ_B is reflected with perpendicular polarization



Brewster angle

$$k_{1z} = k_1 \cos\vartheta_I \qquad k_1^2 = \omega^2 \mu \varepsilon_1 \qquad \frac{k_1^2}{k_2^2} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I} \qquad k_2^2 = \omega^2 \mu \varepsilon_2$$

Parallel Polarization $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$Z_1 = Z_2 \quad \Rightarrow \frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2} \quad \Rightarrow \left(\frac{k_{1z}}{\varepsilon_1}\right)^2 = \left(\frac{k_{2z}}{\varepsilon_2}\right)^2$$

$$\Rightarrow k_{1z}{}^2 = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 k_{2z}{}^2 \quad \Rightarrow k_1{}^2 \cos^2\vartheta_I = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(k_2^2 - k_1^2 \sin^2\vartheta_I\right) \quad \Rightarrow k_1{}^2\left(1-\sin^2\vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 k_2^2 \left(1 - \frac{k_1^2}{k_2^2}\sin^2\vartheta_I\right)$$

$$\Rightarrow \frac{k_1{}^2}{k_2^2}\left(1-\sin^2\vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(1 - \frac{k_1^2}{k_2^2}\sin^2\vartheta_I\right) \quad \Rightarrow \frac{\varepsilon_1}{\varepsilon_2}\left(1-\sin^2\vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(1 - \frac{\varepsilon_1}{\varepsilon_2}\sin^2\vartheta_I\right) \quad \Rightarrow 1-\sin^2\vartheta_I = \frac{\varepsilon_1}{\varepsilon_2} - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \sin^2\vartheta_I$$

$$\Rightarrow 1 - \frac{\varepsilon_1}{\varepsilon_2} = \left[1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\right]\sin^2\vartheta_I \quad \Rightarrow 1 - \frac{\varepsilon_1}{\varepsilon_2} = \left[1 - \frac{\varepsilon_1}{\varepsilon_2}\right]\left[1 + \frac{\varepsilon_1}{\varepsilon_2}\right]\sin^2\vartheta_I \quad \Rightarrow 1 = \left[1 + \frac{\varepsilon_1}{\varepsilon_2}\right]\sin^2\vartheta_I \quad \Rightarrow 1 = \left[\frac{\varepsilon_2 + \varepsilon_1}{\varepsilon_2}\right]\sin^2\vartheta_I$$