Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

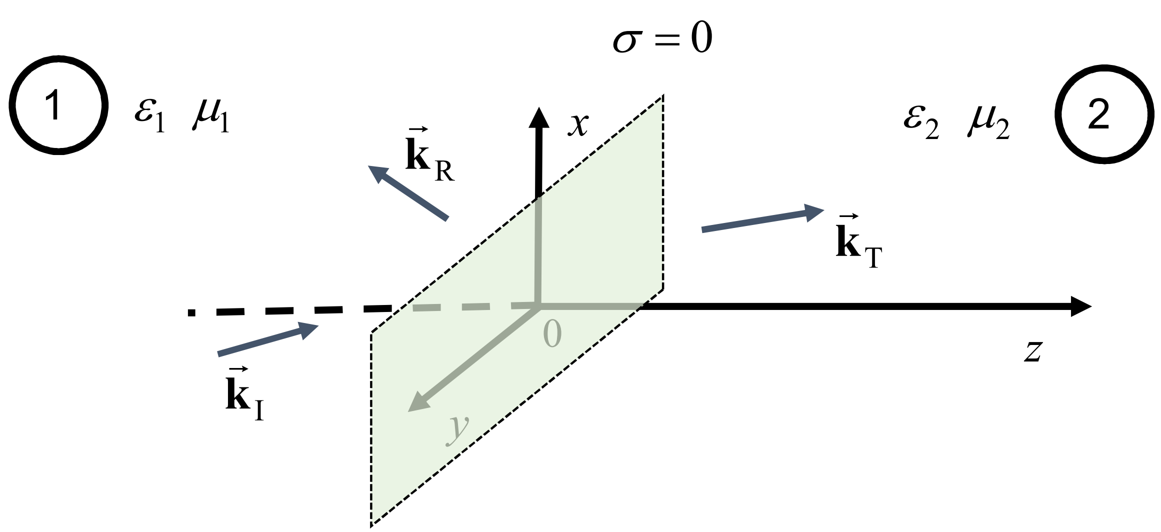
Mathematics

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_\mathrm{I} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{k}}_\mathrm{I} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_\mathrm{R} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{k}}_\mathrm{R} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

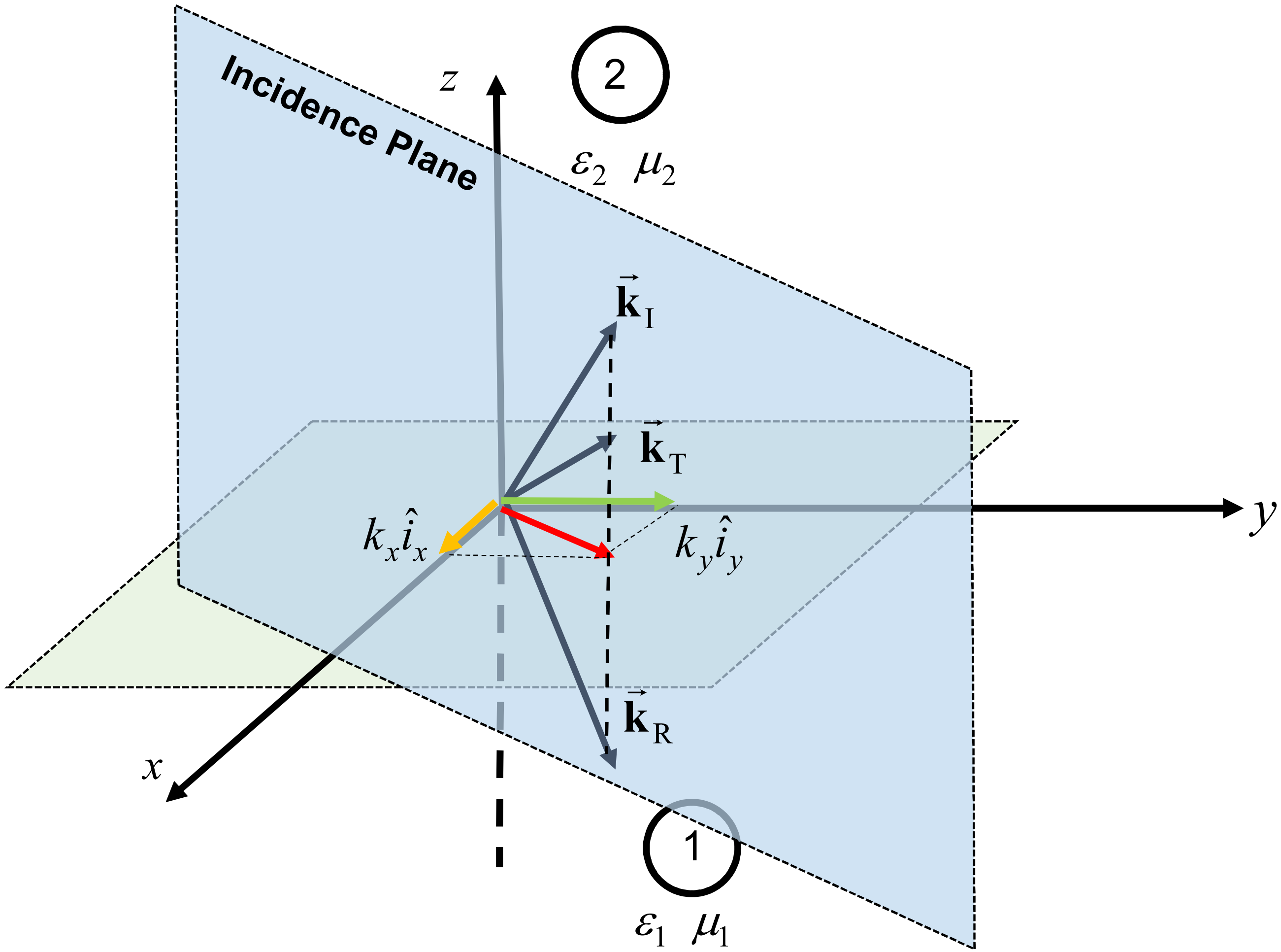
$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_\mathrm{T} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{k}}_\mathrm{T} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

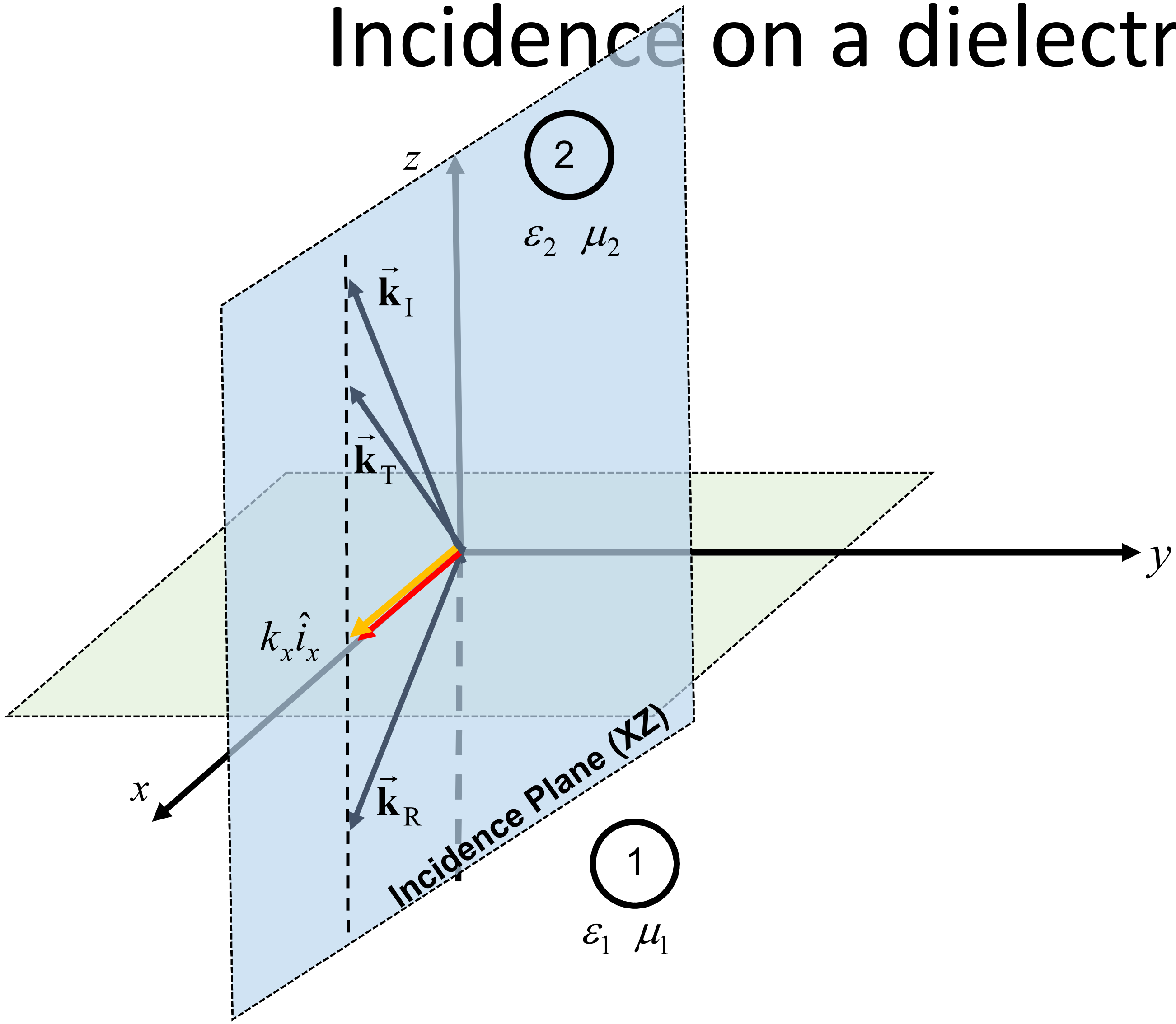


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z$$

Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_\mathrm{I} = k_x\hat{i}_x + k_{\mathrm{I}z}\hat{i}_z$$

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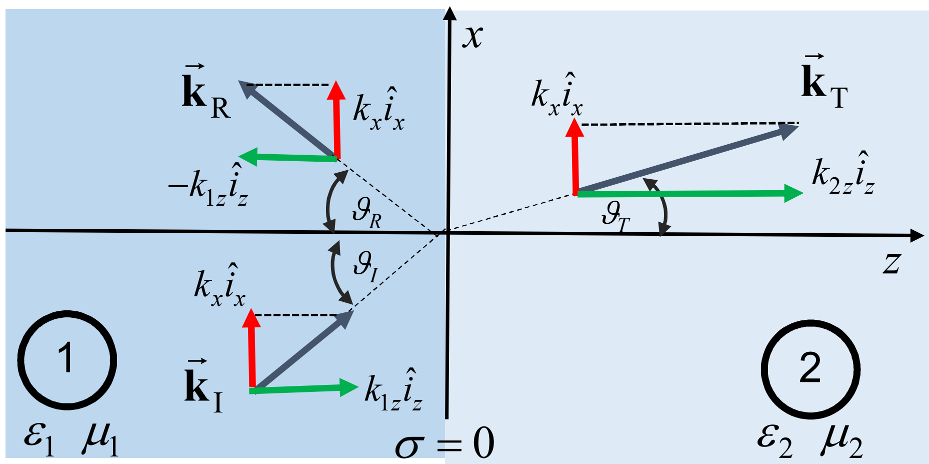

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

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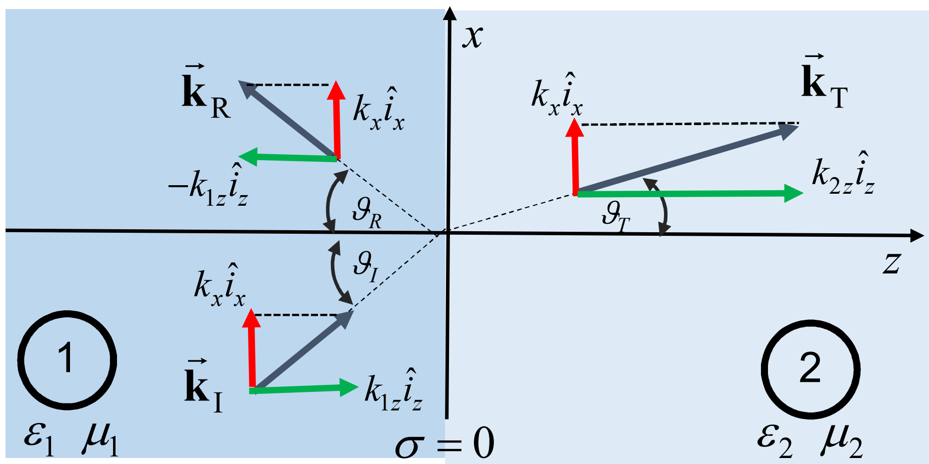
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$



Incidence on a dielectric half-space

$$\frac{\partial}{\partial y}=0$$
$$\frac{\partial}{\partial x}\to -jk_x$$

$$\vec{\mathbf{E}}_1=\vec{\mathbf{E}}_{\mathrm{I}}e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}}+\vec{\mathbf{E}}_{R}e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$
$$\vec{\mathbf{E}}_2=\vec{\mathbf{E}}_{\mathrm{T}}e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}}e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}_{\mathrm{I}}e^{-jk_xx}e^{-jk_{1z}z}$$
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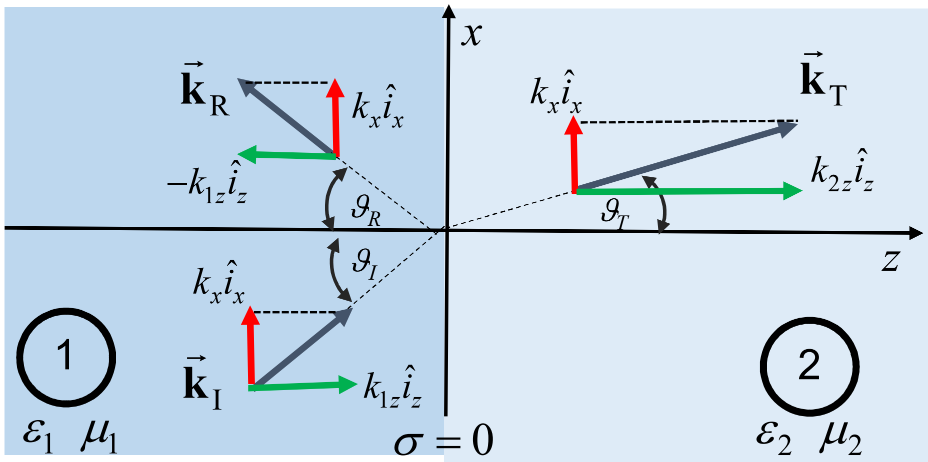
$$k_1\sin\vartheta_{\mathrm{I}}=k_2\sin\vartheta_T$$
$$k_{2z}=\sqrt{k_2^2-k_1^2\sin^2\vartheta_{\mathrm{I}}}$$

$$\frac{\partial}{\partial x}\vec{\mathbf{E}}_{\mathrm{I}}e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}}=\frac{\partial}{\partial x}\vec{\mathbf{E}}_{\mathrm{I}}e^{-jk_xx}e^{-jk_{1z}z}=-jk_x\vec{\mathbf{E}}_{\mathrm{I}}e^{-jk_xx}e^{-jk_{1z}z}=-jk_x\vec{\mathbf{E}}_{\mathrm{I}}e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}}$$



Incidence on a dielectric half-space

$$\frac{\partial}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \to -jk_x$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

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$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

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$$k_x = k_1 \sin\vartheta_I$$

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$$\vartheta_I = \vartheta_R$$

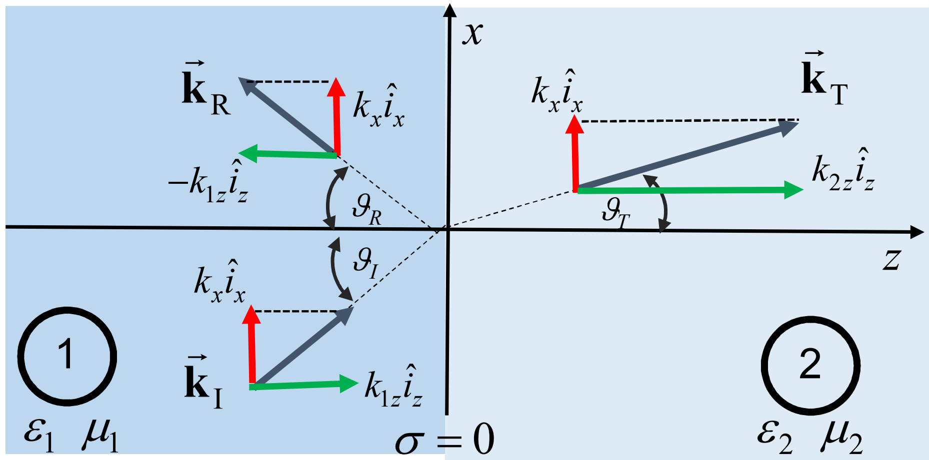
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$$\nabla \times \vec{\mathbf{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{i}_z$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y + \left(-jk_x E_y\right)\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{\partial H_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y + \left(-jk_x H_y\right)\hat{i}_z$$



Incidence on a dielectric half-space

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

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$$\nabla \times \vec{\mathbf{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{i}_z$$

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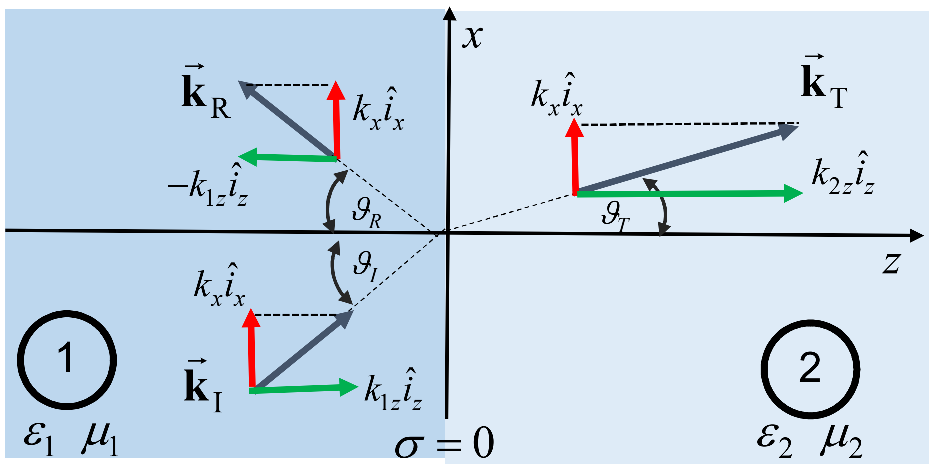

Incidence on a dielectric half-space

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$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial E_y}{\partial z}\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y - jk_x E_y \hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = -\frac{\partial H_y}{\partial z}\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y - jk_x H_y \hat{i}_z$$

Incidence on a dielectric half-space

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$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x \hat{i}_x - j\omega\mu H_y \hat{i}_y - j\omega\mu H_z \hat{i}_z$$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$$\nabla \times \vec{\mathbf{H}} = -\frac{\partial H_y}{\partial z}\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y - jk_x H_y \hat{i}_z$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x \hat{i}_x + j\omega\varepsilon E_y \hat{i}_y + j\omega\varepsilon E_z \hat{i}_z$$

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Incidence on a dielectric half-space

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Incidence on a dielectric half-space

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$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

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$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

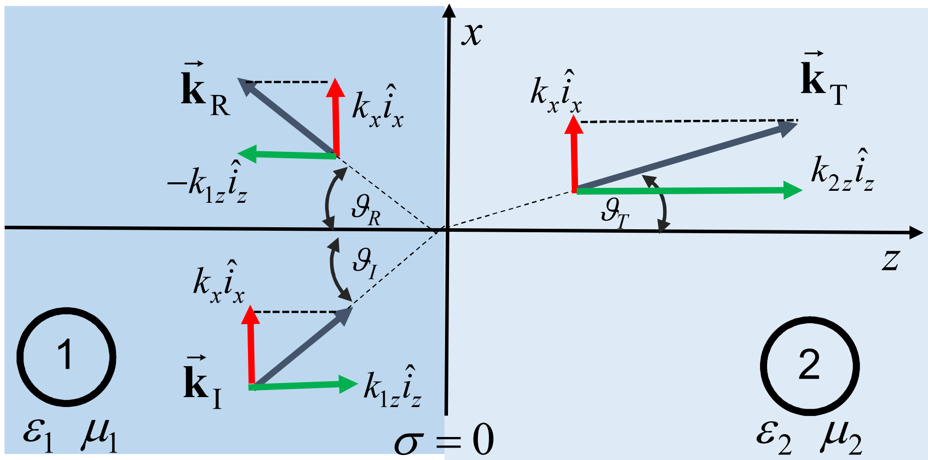
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

Perpendicular Polarization $\perp$

$$\left[H_x, E_y, H_z\right]$$

Parallel Polarization $\parallel$

$$\left[E_x, H_y, E_z\right]$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

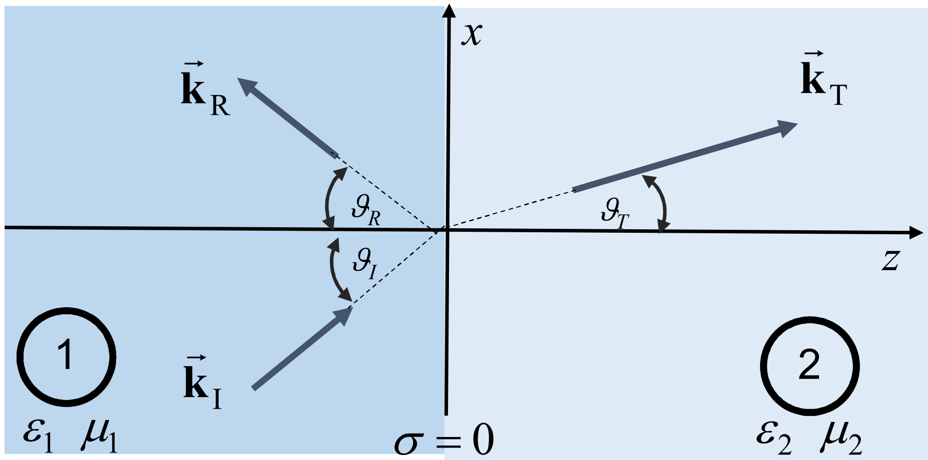
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

Perpendicular Polarization ⊥

$$\left[H_x, E_y, H_z\right]$$

Parallel Polarization ||

$$\left[E_x, H_y, E_z\right]$$



Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

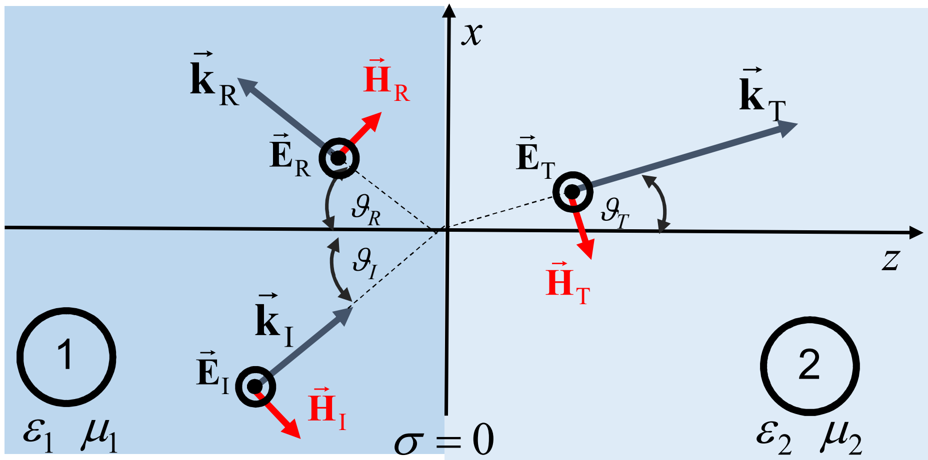
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \Rightarrow \quad H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z \quad \Rightarrow \quad H_z = \frac{k_x}{\omega\mu} E_y$$



Incidence on a dielectric half-space: ⊥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

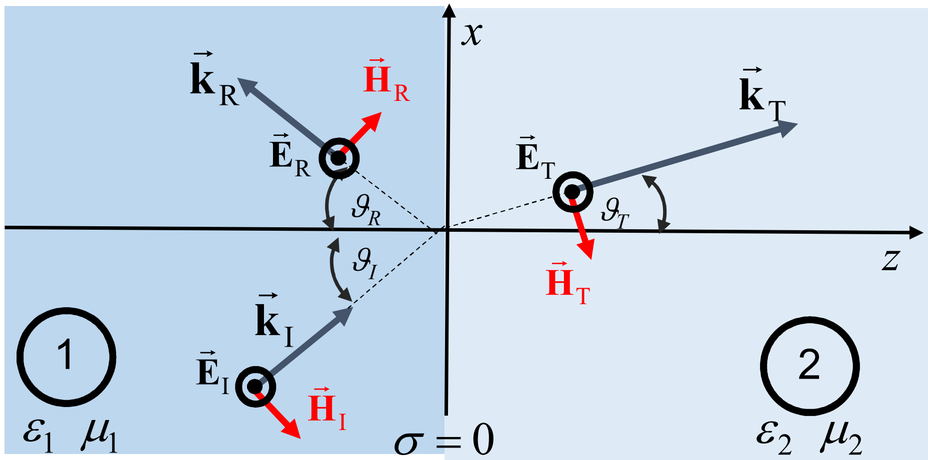
$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \Rightarrow \quad H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z \quad \Rightarrow \quad H_z = \frac{k_x}{\omega\mu} E_y$$



Incidence on a dielectric half-space: ⊥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

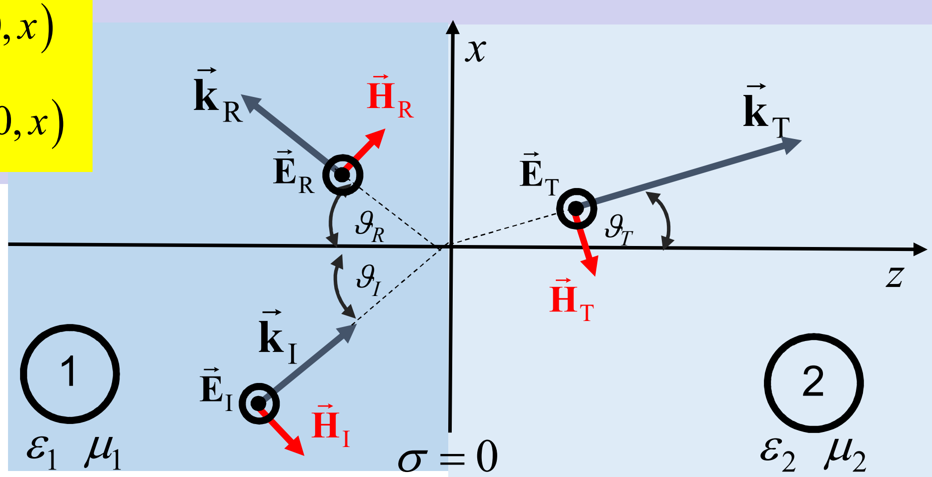
$$k_x = k_1 \sin\vartheta_\mathrm{I}$$
$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

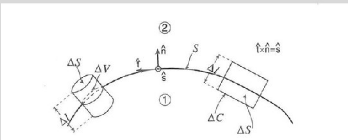
$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$
$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$



MEMO

Fields at boundaries

$$\hat{\mathbf{n}}\times(\dot{\mathbf{e}}_2 - \dot{\mathbf{e}}_1) = 0$$
$$\hat{\mathbf{n}}\times(\dot{\mathbf{h}}_2 - \dot{\mathbf{h}}_1) = \dot{\mathbf{j}}_s$$
$$(\dot{\mathbf{d}}_2 - \dot{\mathbf{d}}_1)\cdot\hat{\mathbf{n}} = \rho_s$$
$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}} = 0$$
$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}} = -\frac{\partial\rho_s}{\partial t}$$



Incidence on a dielectric half-space: ⊥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$
$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

1

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} = E_I e^{-jk_x x} e^{-jk_{1z} z}\hat{i}_y \qquad \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} = E_R e^{-jk_x x} e^{jk_{1z} z}\hat{i}_y$$

$$\vec{\mathbf{E}}_1(z,x) = \left[E_I e^{-jk_{1z} z} + E_R e^{jk_{1z} z}\right] e^{-jk_x x}\hat{i}_y$$

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z} z} + E_R e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z} E_I e^{-jk_{1z} z} + jk_{1z} E_R e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$\vec{\mathbf{E}}_I = E_I \hat{i}_y$$
$$\vec{\mathbf{E}}_R = E_R \hat{i}_y$$
$$\vec{\mathbf{E}}_T = E_T \hat{i}_y$$

2

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}} = E_T e^{-jk_x x} e^{-jk_{2z} z}\hat{i}_y$$

$$\vec{\mathbf{E}}_2(z,x) = E_T e^{-jk_{2z} z} e^{-jk_x x}\hat{i}_y$$

$$E_{2y}(z,x) = E_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z} E_T e^{-jk_{2z} z} e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$

$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

1

$$E_{1y}(z=0,x) = \left[E_\mathrm{I} + E_\mathrm{R}\right] e^{-jk_x x}$$

$$H_{1x}(z=0,x) = -\frac{k_{1z}}{\omega\mu_1}\left[E_\mathrm{I} - E_\mathrm{R}\right] e^{-jk_x x}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z} z} + E_\mathrm{R} e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z} E_\mathrm{I} e^{-jk_{1z} z} + jk_{1z} E_\mathrm{R} e^{jk_{1z} z}\right] e^{-jk_x x}$$

2

$$E_{2y}(z=0,x) = E_\mathrm{T} e^{-jk_x x}$$

$$H_{2x}(z=0,x) = -\frac{k_{2z}}{\omega\mu_2} E_\mathrm{T} e^{-jk_x x}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z} z} e^{-jk_x x}$$

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z} E_\mathrm{T} e^{-jk_{2z} z} e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$

$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

1

$$E_{1y}(z=0,x) = \left[E_{\mathrm{I}} + E_{\mathrm{R}}\right] e^{-jk_x x}$$

$$H_{1x}(z=0,x) = -\frac{1}{Z_1}\left[E_{\mathrm{I}} - E_{\mathrm{R}}\right] e^{-jk_x x}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$E_{1y}(z,x) = \left[E_{\mathrm{I}} e^{-jk_{1z} z} + E_{\mathrm{R}} e^{jk_{1z} z}\right] e^{-jk_x x}$$

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z} E_{\mathrm{I}} e^{-jk_{1z} z} + jk_{1z} E_{\mathrm{R}} e^{jk_{1z} z}\right] e^{-jk_x x}$$

2

$$E_{2y}(z=0,x) = E_{\mathrm{T}} e^{-jk_x x}$$

$$H_{2x}(z=0,x) = -\frac{1}{Z_2} E_{\mathrm{T}} e^{-jk_x x}$$

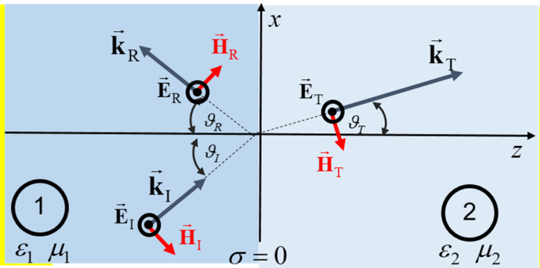
$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$E_{2y}(z,x) = E_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}$$

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z} E_{\mathrm{T}} e^{-jk_{2z} z} e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases} \qquad \Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$



$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1\cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$

$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

$$\begin{cases} E_\mathrm{I} + E_\mathrm{R} = E_\mathrm{T} \\ E_\mathrm{I} - E_\mathrm{R} = \dfrac{Z_1}{Z_2}E_\mathrm{T} \end{cases} \qquad \begin{cases} 1 + \dfrac{E_\mathrm{R}}{E_\mathrm{I}} = \dfrac{E_\mathrm{T}}{E_\mathrm{I}} \\ 1 - \dfrac{E_\mathrm{R}}{E_\mathrm{I}} = \dfrac{Z_1}{Z_2}\dfrac{E_\mathrm{T}}{E_\mathrm{I}} \end{cases}$$

1

$$E_{1y}(z=0,x) = \left[E_\mathrm{I} + E_\mathrm{R}\right]e^{-jk_x x}$$

$$H_{1x}(z=0,x) = -\frac{1}{Z_1}\left[E_\mathrm{I} - E_\mathrm{R}\right]e^{-jk_x x}$$

$$E_{1y}(z,x) = \left[E_\mathrm{I}e^{-jk_{1z}z} + E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z}E_\mathrm{I}e^{-jk_{1z}z} + jk_{1z}E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

2

$$E_{2y}(z=0,x) = E_\mathrm{T}e^{-jk_x x}$$

$$H_{2x}(z=0,x) = -\frac{1}{Z_2}E_\mathrm{T}e^{-jk_x x}$$

$$E_{2y}(z,x) = E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}$$

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z}E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}\right]$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases}$$

$$\Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$

$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

$$2 = \left[\frac{Z_1}{Z_2} + 1\right]\mathrm{T} = \left[\frac{Z_1 + Z_2}{Z_2}\right]\mathrm{T} \quad \Rightarrow \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$\Gamma = \mathrm{T} - 1 = \frac{2Z_2}{Z_1 + Z_2} - 1 = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

1

$$E_{1y}(z=0,x) = \left[E_\mathrm{I} + E_\mathrm{R}\right]e^{-jk_x x}$$

$$H_{1x}(z=0,x) = -\frac{1}{Z_1}\left[E_\mathrm{I} - E_\mathrm{R}\right]e^{-jk_x x}$$

$$E_{1y}(z,x) = \left[E_\mathrm{I}e^{-jk_{1z}z} + E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z}E_\mathrm{I}e^{-jk_{1z}z} + jk_{1z}E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

2

$$E_{2y}(z=0,x) = E_\mathrm{T}e^{-jk_x x}$$

$$H_{2x}(z=0,x) = -\frac{1}{Z_2}E_\mathrm{T}e^{-jk_x x}$$

$$E_{2y}(z,x) = E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}$$

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z}E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases} \qquad \Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$
$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

1

$$E_{1y}(z=0,x) = \left[E_\mathrm{I} + E_\mathrm{R}\right]e^{-jk_x x}$$
$$H_{1x}(z=0,x) = -\frac{1}{Z_1}\left[E_\mathrm{I} - E_\mathrm{R}\right]e^{-jk_x x}$$
$$E_{1y}(z,x) = \left[E_\mathrm{I}e^{-jk_{1z}z} + E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$
$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z}E_\mathrm{I}e^{-jk_{1z}z} + jk_{1z}E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

2

$$E_{2y}(z=0,x) = E_\mathrm{T}e^{-jk_x x}$$
$$H_{2x}(z=0,x) = -\frac{1}{Z_2}E_\mathrm{T}e^{-jk_x x}$$
$$E_{2y}(z,x) = E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}$$
$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z}E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases} \qquad \Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1\cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2\sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2\sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z=0,x) = E_{2y}(z=0,x)$$

$$H_{1x}(z=0,x) = H_{2x}(z=0,x)$$

1

$$E_{1y}(z=0,x) = \left[E_\mathrm{I} + E_\mathrm{R}\right]e^{-jk_x x}$$

$$H_{1x}(z=0,x) = -\frac{1}{Z_1}\left[E_\mathrm{I} - E_\mathrm{R}\right]e^{-jk_x x}$$

$$E_{1y}(z,x) = \left[E_\mathrm{I}e^{-jk_{1z}z} + E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

1

$$H_{1x}(z,x) = \frac{1}{j\omega\mu_1}\left[-jk_{1z}E_\mathrm{I}e^{-jk_{1z}z} + jk_{1z}E_\mathrm{R}e^{jk_{1z}z}\right]e^{-jk_x x}$$

2

$$E_{2y}(z=0,x) = E_\mathrm{T}e^{-jk_x x}$$

$$H_{2x}(z=0,x) = -\frac{1}{Z_2}E_\mathrm{T}e^{-jk_x x}$$

$$E_{2y}(z,x) = E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}$$

2

$$H_{2x}(z,x) = \frac{1}{j\omega\mu_2}\left[-jk_{2z}E_\mathrm{T}e^{-jk_{2z}z}e^{-jk_x x}\right]$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases} \qquad \Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$\Gamma = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

Fresnel reflection coefficient

$$\Gamma = \frac{\dfrac{\omega\mu_2}{k_{2z}} - \dfrac{\omega\mu_1}{k_{1z}}}{\dfrac{\omega\mu_2}{k_{2z}} + \dfrac{\omega\mu_1}{k_{1z}}} \frac{k_{2z}\ k_{1z}}{k_{2z}\ k_{1z}} = \frac{k_{1z}\mu_2 - k_{2z}\mu_1}{k_{1z}\mu_2 + k_{2z}\mu_1} = \frac{k_{1z} - k_{2z}(\mu_1/\mu_2)}{k_{1z} + k_{2z}(\mu_1/\mu_2)} = \frac{k_1\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}}{k_1\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}}$$

$$= \frac{k_1\cos\vartheta_I - (\mu_1/\mu_2)k_1\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{k_1\cos\vartheta_I + (\mu_1/\mu_2)k_1\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}} = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1+\Gamma = \mathrm{T} \\ 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \end{cases} \qquad \Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$\Gamma \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad \mathrm{T} \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}}$$

$$\Gamma = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}} \qquad \mathrm{T} = 1 + \Gamma$$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma \triangleq \frac{E_R}{E_I} \qquad T \triangleq \frac{E_T}{E_I}$$

$$\Gamma = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T = 1 + \Gamma$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$\Gamma \triangleq \frac{E_R}{E_I} \qquad T \triangleq \frac{E_T}{E_I}$$

$$\Gamma = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T = 1 + \Gamma$$

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma \triangleq \frac{E_R}{E_I}$$

$$T \triangleq \frac{E_T}{E_I}$$

$$\Gamma = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T = 1 + \Gamma$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma_\perp \triangleq \frac{E_R}{E_I}$$

$$T_\perp \triangleq \frac{E_T}{E_I}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T_\perp = 1 + \Gamma_\perp$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$

$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right] e^{-jk_x x}$$

1

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\Gamma_\perp \triangleq \frac{E_R}{E_I}$$

$$T_\perp \triangleq \frac{E_T}{E_I}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$T_\perp = 1 + \Gamma_\perp$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

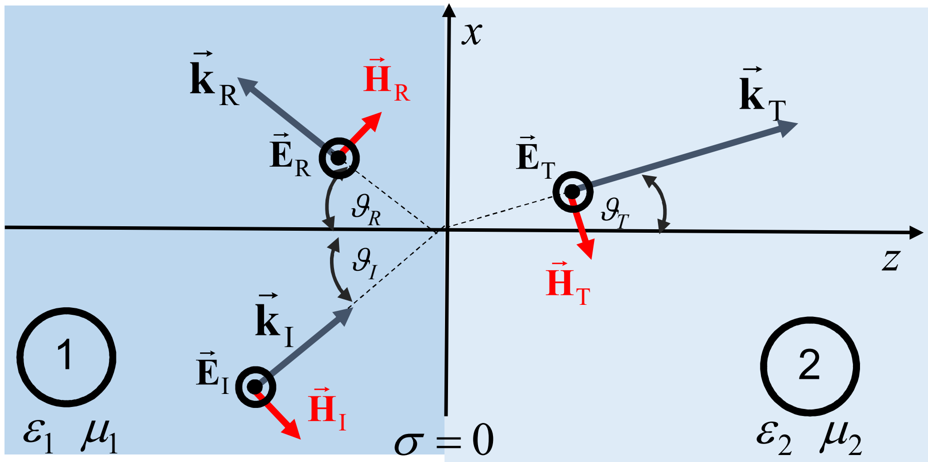
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$H_x = \frac{1}{j\omega\mu}\frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu}E_y$$



$$E_{1y}(z,x) = \left[E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}\right]e^{-jk_x x} = E_I e^{-jk_x x}e^{-jk_{1z}z} + E_R e^{-jk_x x}e^{jk_{1z}z}$$

1

$$E_{2y}(z,x) = E_T e^{-jk_{2z}z}e^{-jk_x x}$$

2

Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

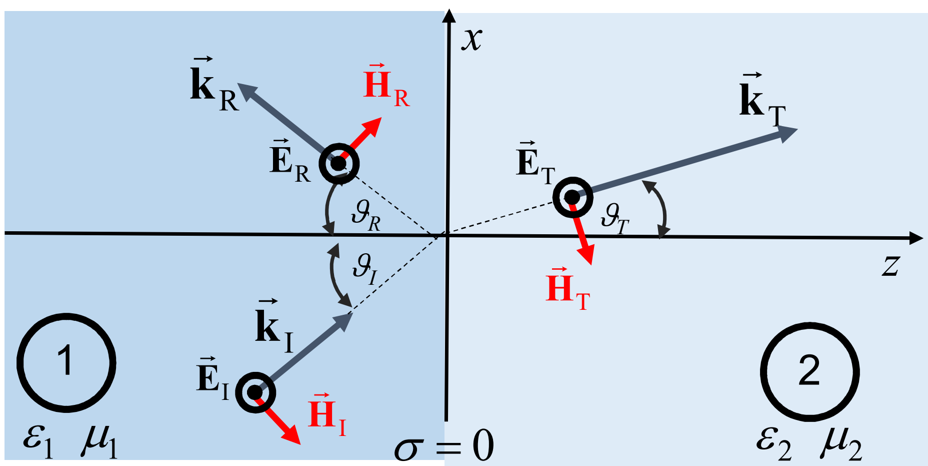
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** ⊥

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$



Incidence on a dielectric half-space: ∥ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

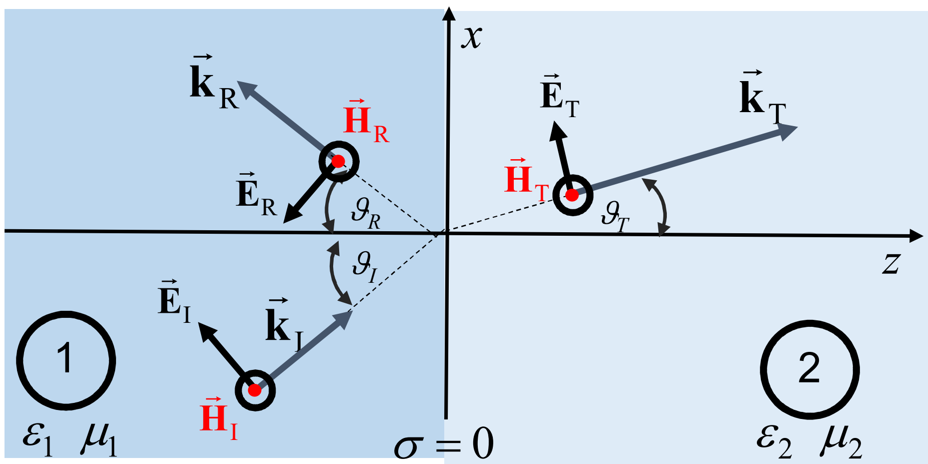
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ∥

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \Rightarrow E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z \Rightarrow E_z = -\frac{k_x}{\omega\varepsilon} H_y$$



Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$$\left[E_x, H_y, E_z\right]$$

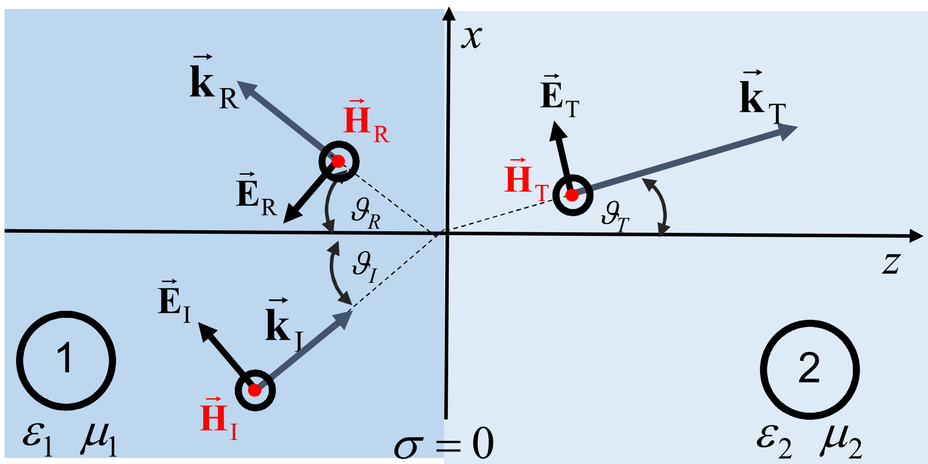
Parallel Polarization ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \Rightarrow E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z \Rightarrow E_z = -\frac{k_x}{\omega\varepsilon} H_y$$



Incidence on a dielectric half-space: ‖ polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

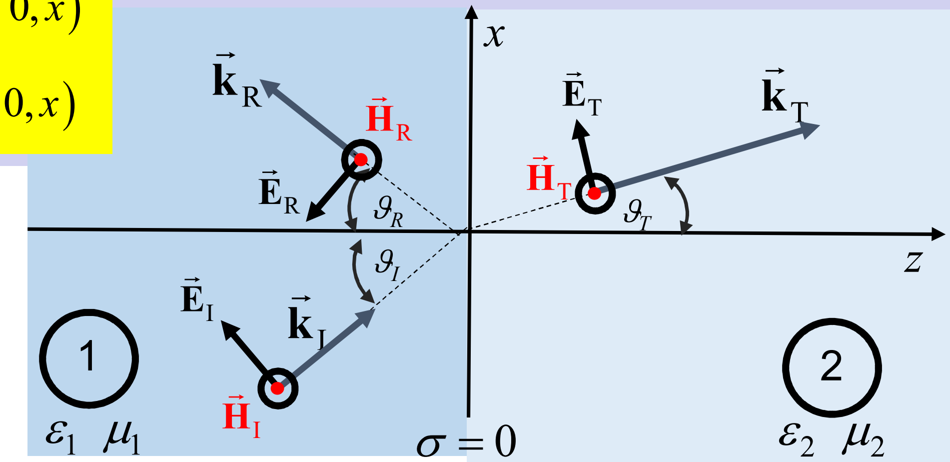
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ‖

$$E_x = -\frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$H_{1y}(z=0,x) = H_{2y}(z=0,x)$$

$$E_{1x}(z=0,x) = E_{2x}(z=0,x)$$



MEMO

Fields at boundaries

$$\hat{\mathbf{n}}\times(\dot{\mathbf{e}}_2 - \dot{\mathbf{e}}_1) = 0$$

$$\hat{\mathbf{n}}\times(\dot{\mathbf{h}}_2 - \dot{\mathbf{h}}_1) = \dot{\mathbf{j}}_s$$

$$(\dot{\mathbf{d}}_2 - \dot{\mathbf{d}}_1)\cdot\hat{\mathbf{n}} = \rho_s$$

$$(\dot{\mathbf{b}}_2 - \dot{\mathbf{b}}_1)\cdot\hat{\mathbf{n}} = 0$$

$$(\dot{\mathbf{j}}_2 - \dot{\mathbf{j}}_1)\cdot\hat{\mathbf{n}} = -\frac{\partial\rho_s}{\partial t}$$