Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

Plane Waves

General expression of plane waves (PD)

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

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$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

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$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

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Plane Waves (Spectral Domains)

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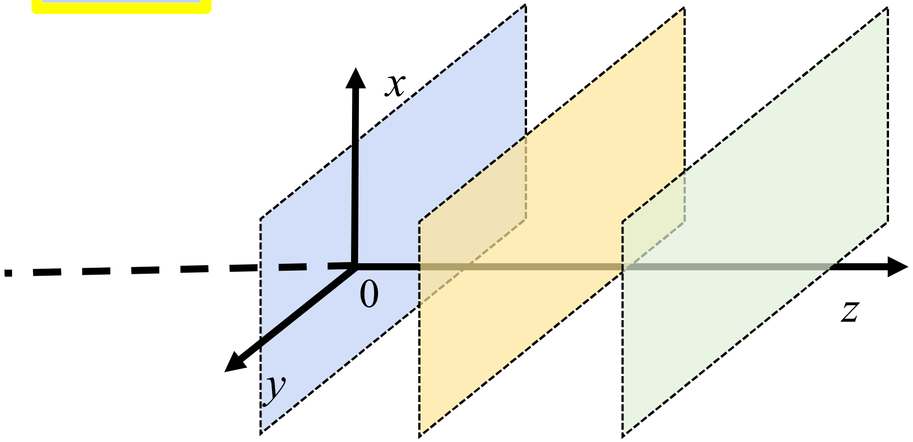
Phasor Domain



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$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

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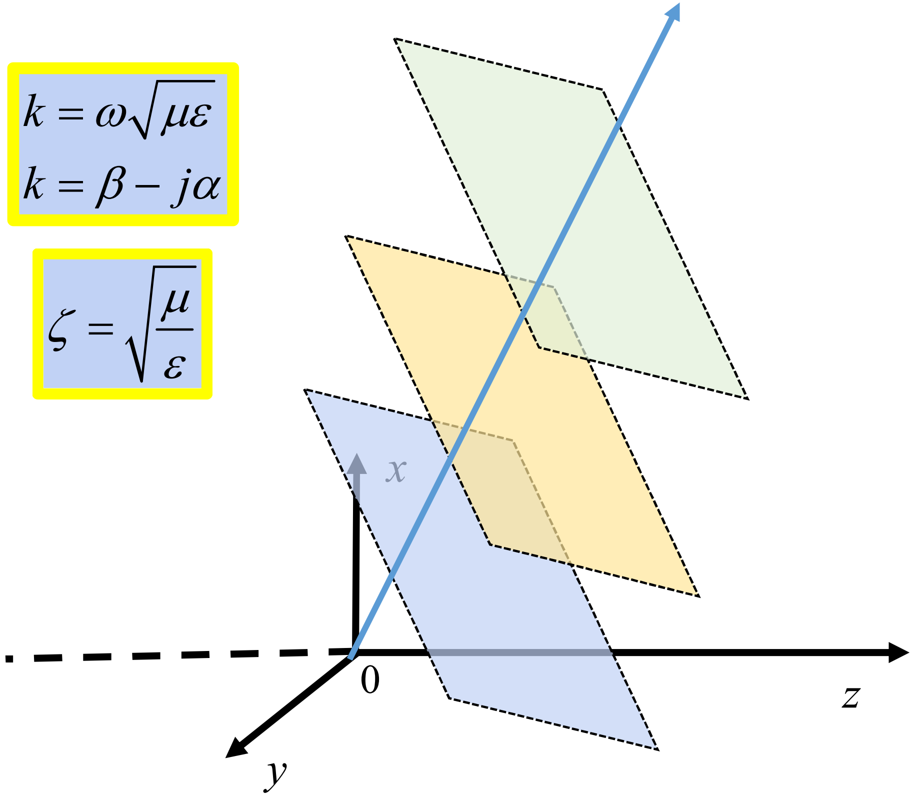

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$\{E_y, H_x\}$

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Independent each other

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

$$\frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$$

General expression of plane waves (PD)

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

⬇

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{\mathbf{k}}\times\vec{\mathbf{E}}$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}}\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}}\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \ldots\ldots \\ \ldots\ldots\ldots \end{cases}$$

$$\frac{\partial}{\partial x}\rightarrow -jk_x \qquad \frac{\partial}{\partial y}\rightarrow -jk_y \qquad \frac{\partial}{\partial z}\rightarrow -jk_z$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

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General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

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$$\frac{\partial}{\partial x} \rightarrow -jk_x \qquad \frac{\partial}{\partial y} \rightarrow -jk_y \qquad \frac{\partial}{\partial z} \rightarrow -jk_z$$

⬇

$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$$

$$\nabla \cdot \vec{\mathbf{E}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \cdot \vec{\mathbf{E}} = -jk_x E_x - jk_y E_y - jk_z E_z = -j\vec{\mathbf{k}} \cdot \vec{\mathbf{E}}$$

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⬇

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Source-free

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Source-free

$$\rightarrow \begin{cases} -j\vec{\mathbf{k}} \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}} \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ -j\vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

General expression of plane waves (PD)

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$$\begin{cases} \vec{\mathbf{k}}\times\vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}}\times\vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{k}}\times\left(\vec{\mathbf{k}}\times\vec{\mathbf{E}}\right) = \omega\mu\left(\vec{\mathbf{k}}\times\vec{\mathbf{H}}\right) = \omega\mu\left(-\omega\varepsilon\vec{\mathbf{E}}\right) = -\omega^2\mu\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}\right) - \vec{\mathbf{E}}\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}}\right)$$

$$-\left(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}}\right)\vec{\mathbf{E}} = -\omega^2\mu\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right) = \vec{\mathbf{B}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right) - \vec{\mathbf{C}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\vec{\mathbf{k}}\times\vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}}$$

$$\vec{\mathbf{k}}\times\vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$\vec{\mathbf{k}}$:propagation vector

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MEMO

The electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ **points to the propagation direction and** $\zeta = \sqrt{\mu/\varepsilon}$

$\vec{\mathbf{k}} = k_z\hat{i}_z$ ⇒ $\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_z z$

$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = k_z^2$ ⇒ $k_z^2 = \omega^2\mu\varepsilon$ ⇒ $k_z = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_z z} = \vec{\mathbf{E}}^{+} e^{-j\beta z} e^{-\alpha z}$$

$$\begin{cases} \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = k_z E_z = 0 \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = k_z H_z = 0 \end{cases} \Rightarrow E_z = H_z = 0$$

$$\omega\mu\vec{\mathbf{H}} = \vec{\mathbf{k}}\times\vec{\mathbf{E}} = k_z\hat{i}_z\times\vec{\mathbf{E}} = \omega\sqrt{\mu\varepsilon}\,\hat{i}_z\times\vec{\mathbf{E}} \Rightarrow \sqrt{\frac{\mu}{\varepsilon}}\vec{\mathbf{H}} = \hat{i}_z\times\vec{\mathbf{E}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \vec{\mathbf{k}}\times\vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}} \\ \vec{\mathbf{k}}\times\vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$\vec{\mathbf{k}}$:propagation vector

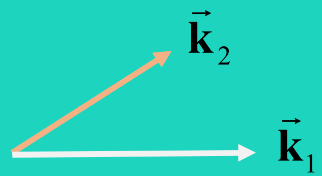
Medium

- Linear - Isotropic - Space nondispersive - Time dispersive - Lossy
- Homogeneous (Time-invariant & Space-invariant)

$$k = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon = (\beta - j\alpha)^2 \Rightarrow \vec{\mathbf{k}} = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j(\vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2)\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}$$



General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\overrightarrow{\mathbf{H}} \\ \vec{\mathbf{k}} \times \overrightarrow{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}}\cdot\overrightarrow{\mathbf{H}} = 0 \end{cases}$$

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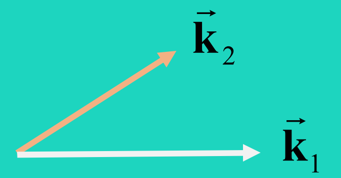
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-j(\vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2)\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}$$

Propagation ($e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}}$)

Attenuation ($e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}$)



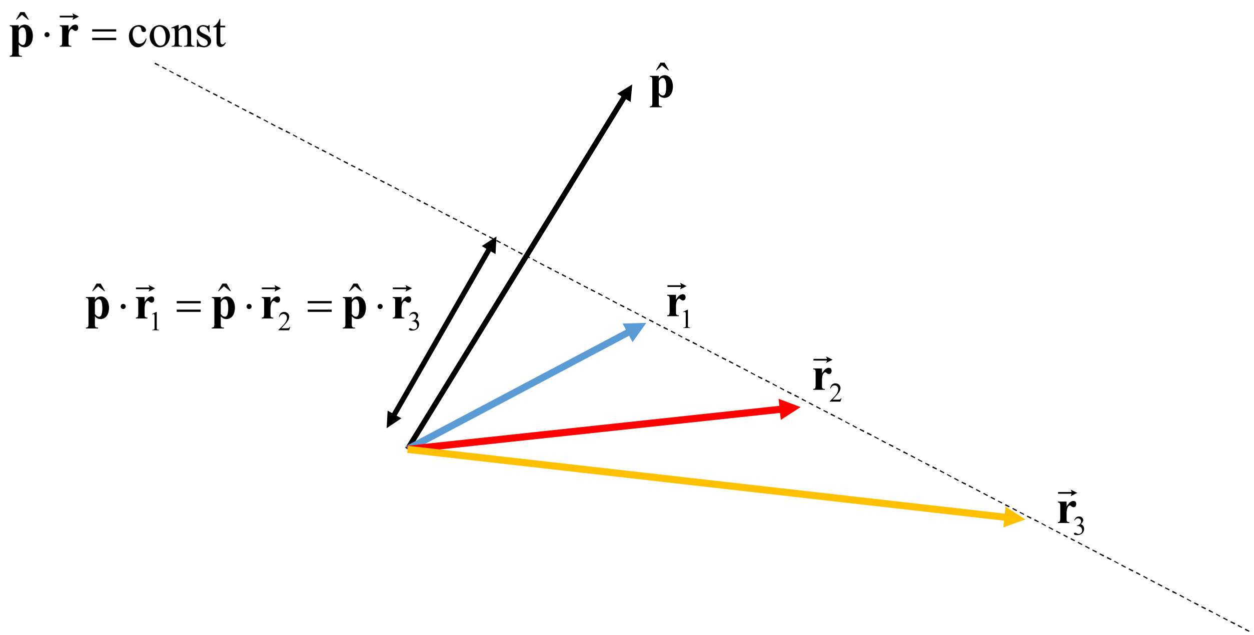
The wave propagates along $\vec{\mathbf{k}}_1$

The wave attenuates along $\vec{\mathbf{k}}_2$

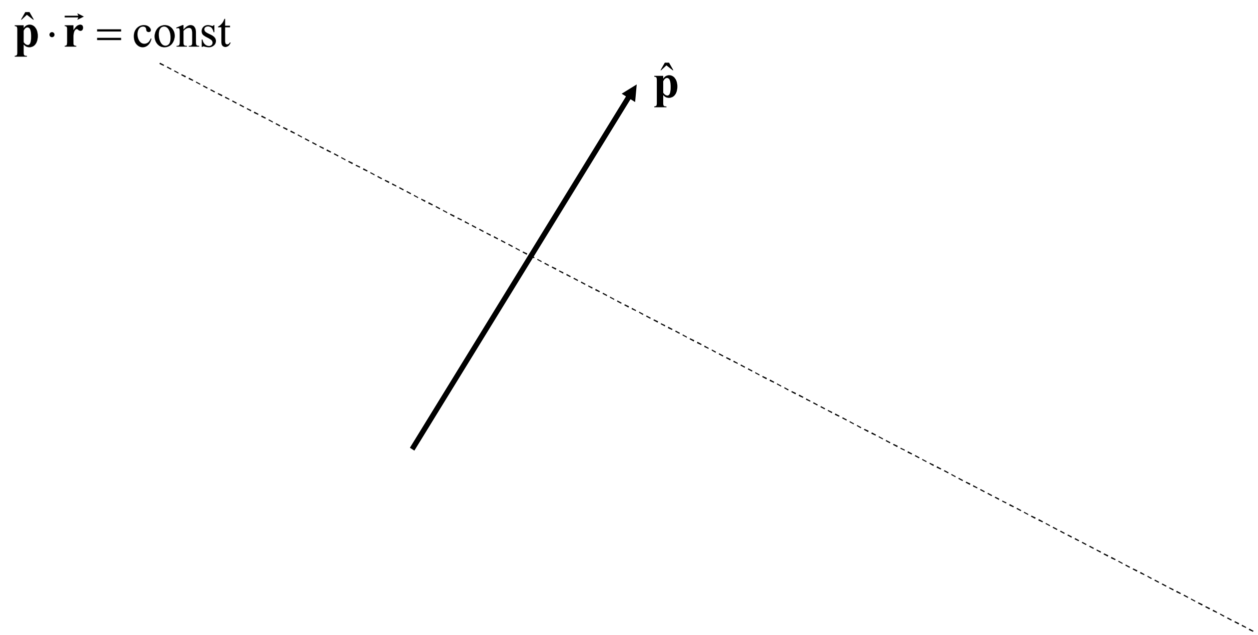
$\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-phase planes

$\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-amplitude planes

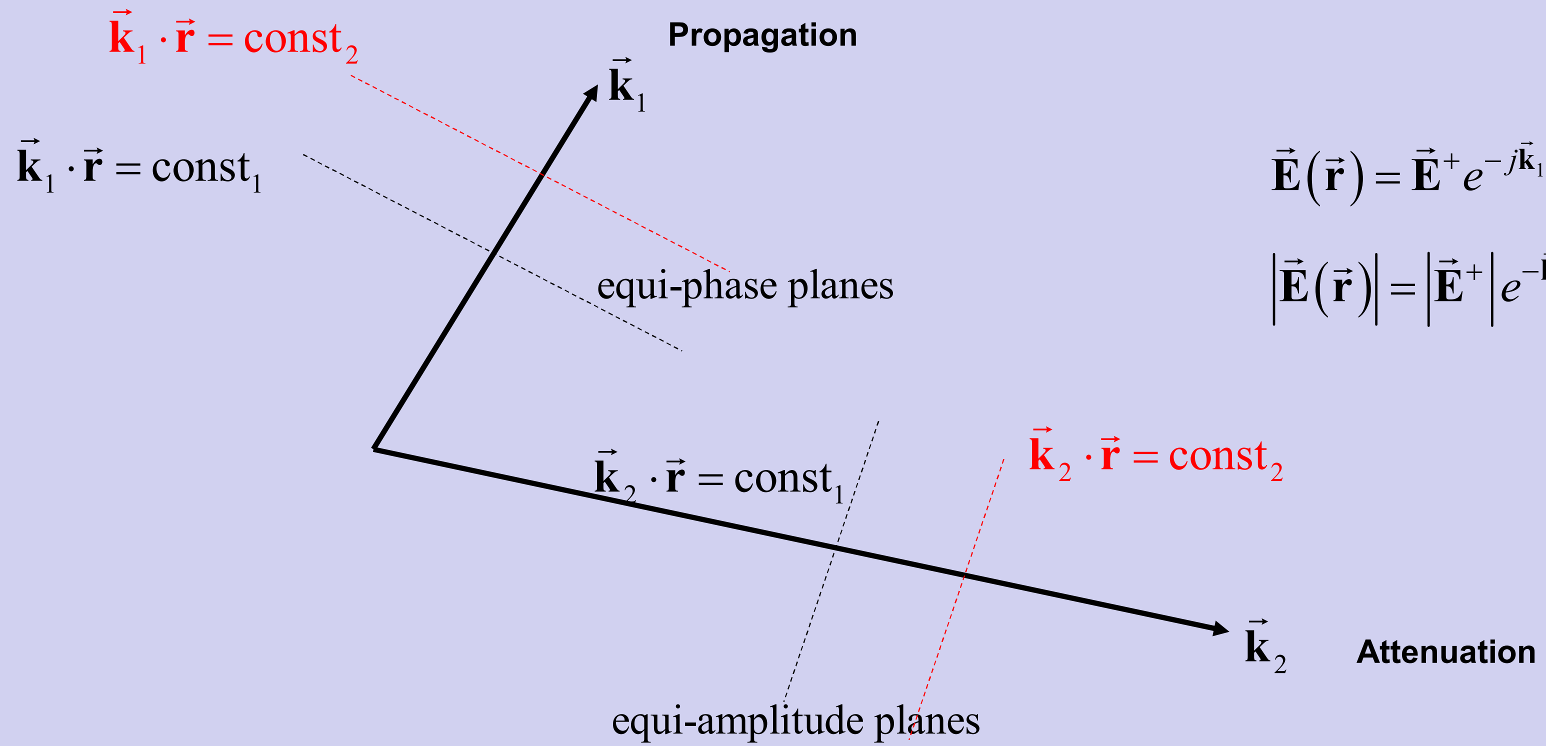
General expression of plane waves (PD)



General expression of plane waves (PD)



General expression of plane waves (PD)



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$

$$\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right| = \left|\vec{\mathbf{E}}^+\right| e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$

When $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ are proportional, equi-amplitude and equi-phase planes become coincident: the plane wave is said HOMOGENEOUS

More generally, equi-amplitude and equi-phase planes may be not coincident: in this the plane wave is said NOT-HOMOGENEOUS

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2 \mu\varepsilon$$

$\vec{\mathbf{k}}$:propagation vector

Medium

- Linear - Isotropic - Space nondispersive - Time dispersive - Lossy
- Homogeneous (Time-invariant & Space-invariant)

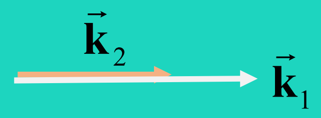
$$k = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{k}} = k_z \hat{i}_z \Rightarrow \vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_z z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = k_z^2 \Rightarrow k_z^2 = \omega^2\mu\varepsilon \Rightarrow k_z = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_z z} = \vec{\mathbf{E}}^{+} e^{-j\beta z} e^{-\alpha z}$$

$$\vec{\mathbf{k}} = (\beta - j\alpha)\hat{i}_z = \beta\hat{i}_z - j\alpha\hat{i}_z = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$



HOMOGENEOUS PLANE-WAVE

Plane Waves

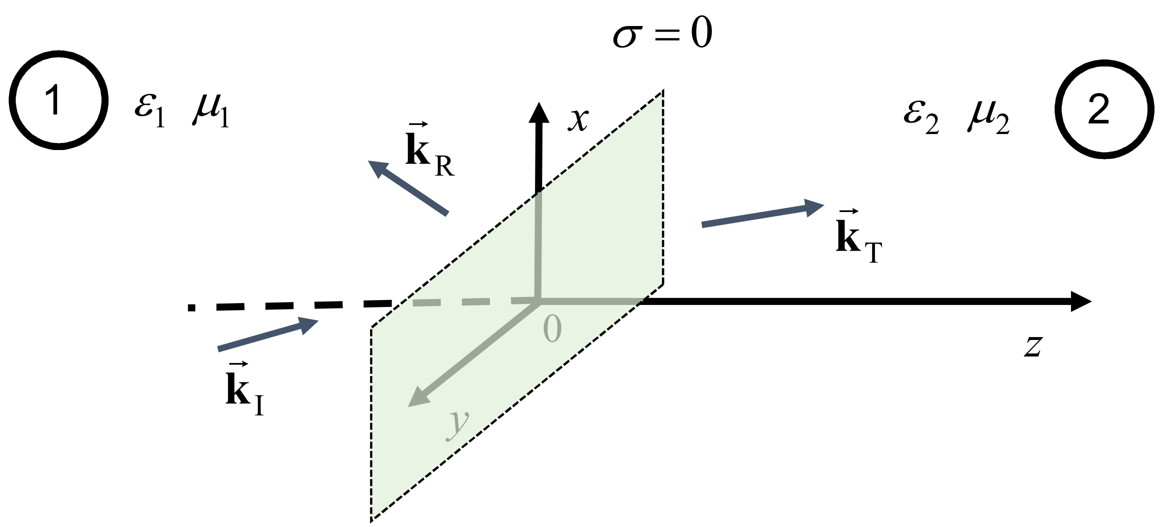
Incidence on a dielectric half-space

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

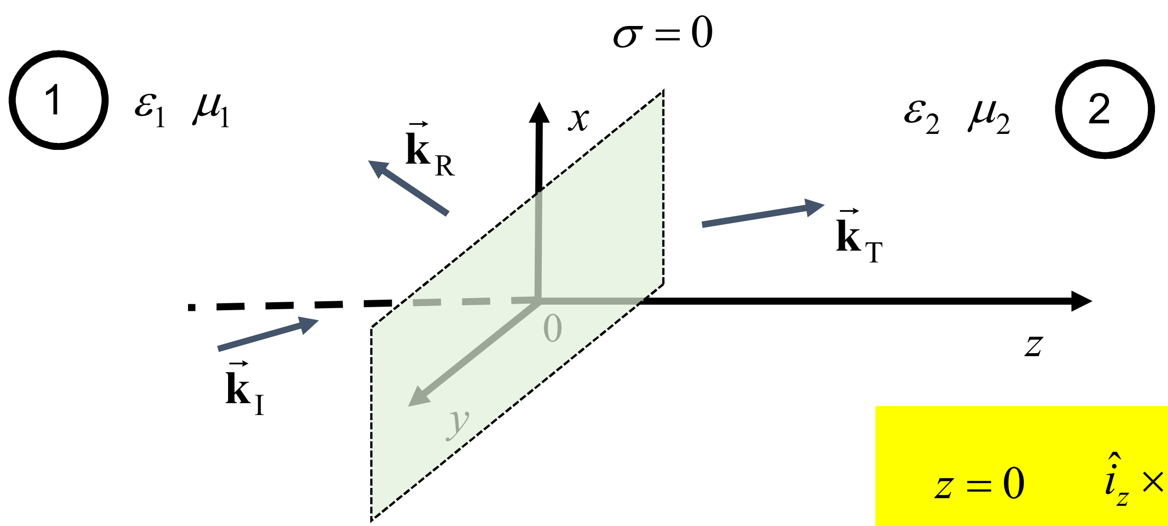
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

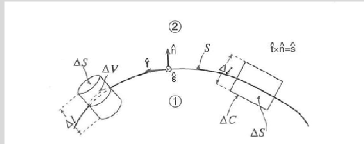
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



MEMO

Fields at boundaries

$$\hat{\mathbf{n}}\times(\vec{\mathbf{e}}_2-\vec{\mathbf{e}}_1)=0$$
$$\hat{\mathbf{n}}\times(\vec{\mathbf{h}}_2-\vec{\mathbf{h}}_1)=\vec{\mathbf{j}}_s$$
$$(\vec{\mathbf{d}}_2-\vec{\mathbf{d}}_1)\cdot\hat{\mathbf{n}}=\rho_s$$
$$(\vec{\mathbf{b}}_2-\vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}}=0$$
$$(\vec{\mathbf{j}}_2-\vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}}=-\frac{\partial\rho_s}{\partial t}$$



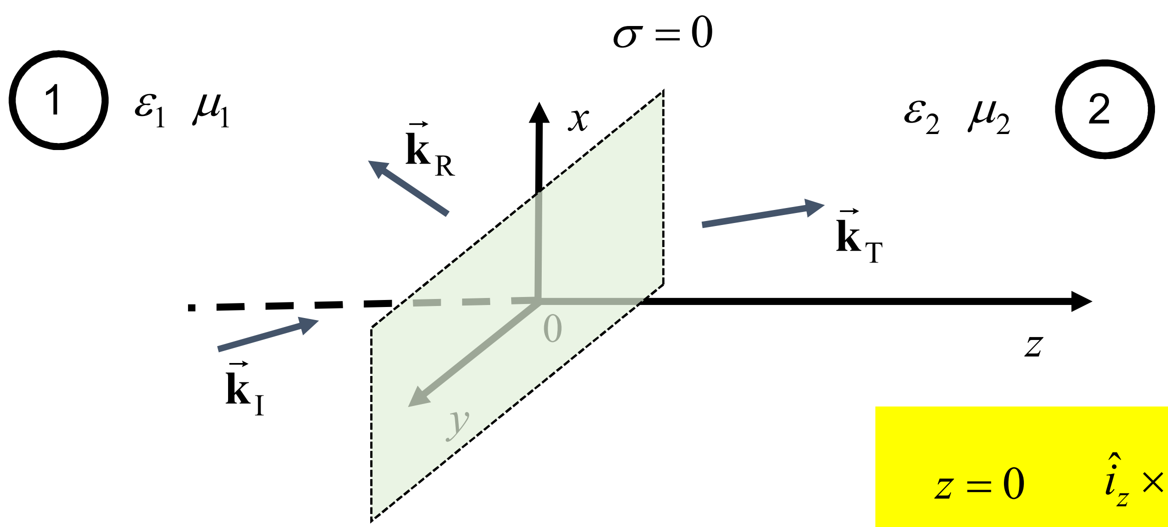
$$z=0 \qquad \hat{i}_z\times\left(\vec{\mathbf{E}}_2-\vec{\mathbf{E}}_1\right)=\mathbf{0} \qquad \hat{i}_z\times\vec{\mathbf{E}}_1=\hat{i}_z\times\vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$z = 0 \qquad \hat{i}_z \times \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z}$$

$$z = 0 \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

$z = 0 \qquad y = 0$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x}$$

$$\hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} + \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} = \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} \quad \Rightarrow \quad k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$z = 0 \qquad x = 0$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}y}y}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}y}y}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}z}z}$$

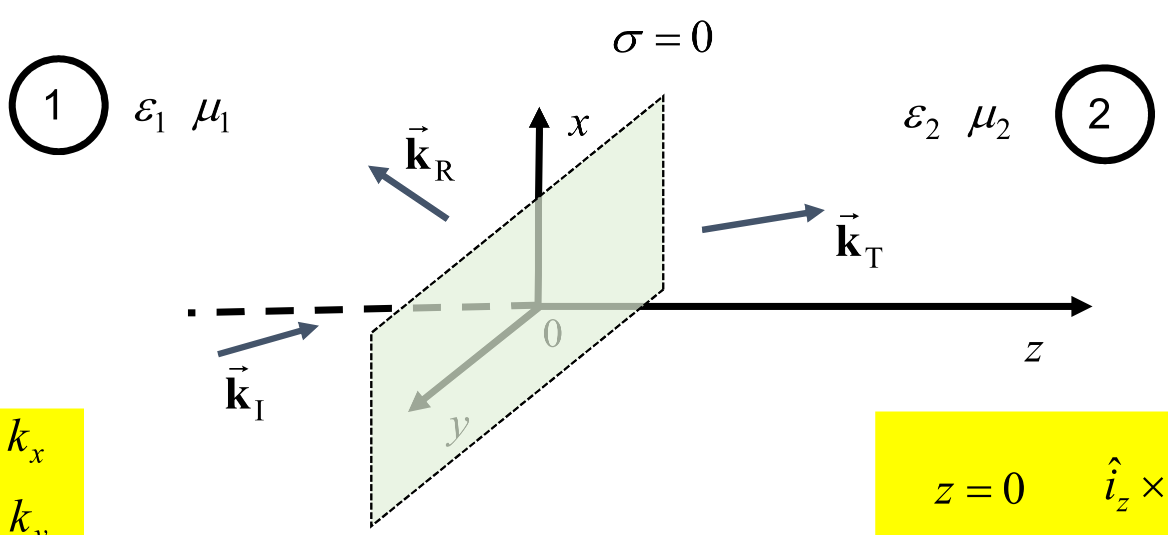
$$\hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}y}y} + \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}y}y} = \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}y}y} \quad \Rightarrow \quad k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$$k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

$$z = 0 \qquad \hat{i}_z \times \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$$k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

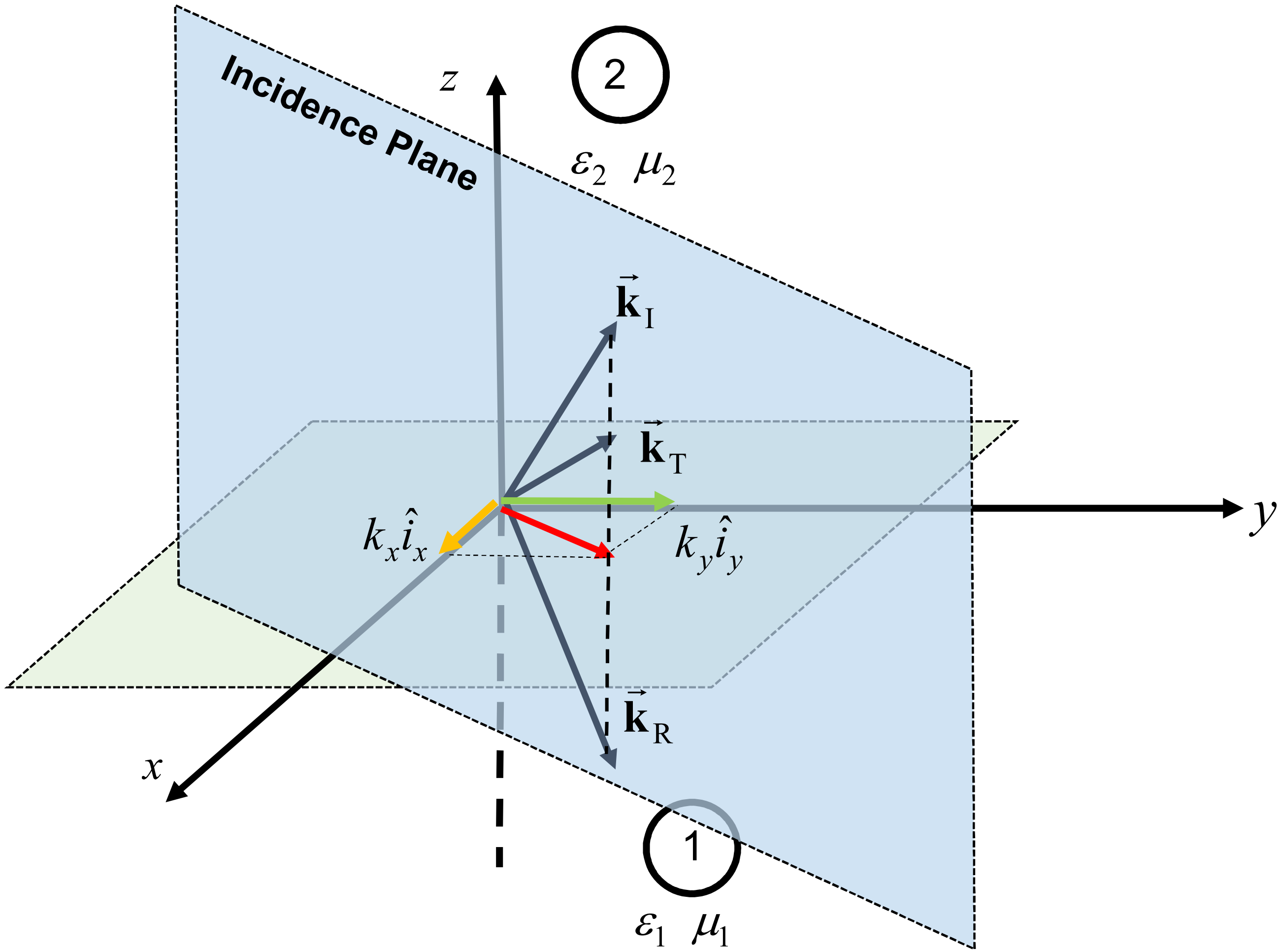
$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_y^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_y^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_y^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

Incidence on a dielectric half-space

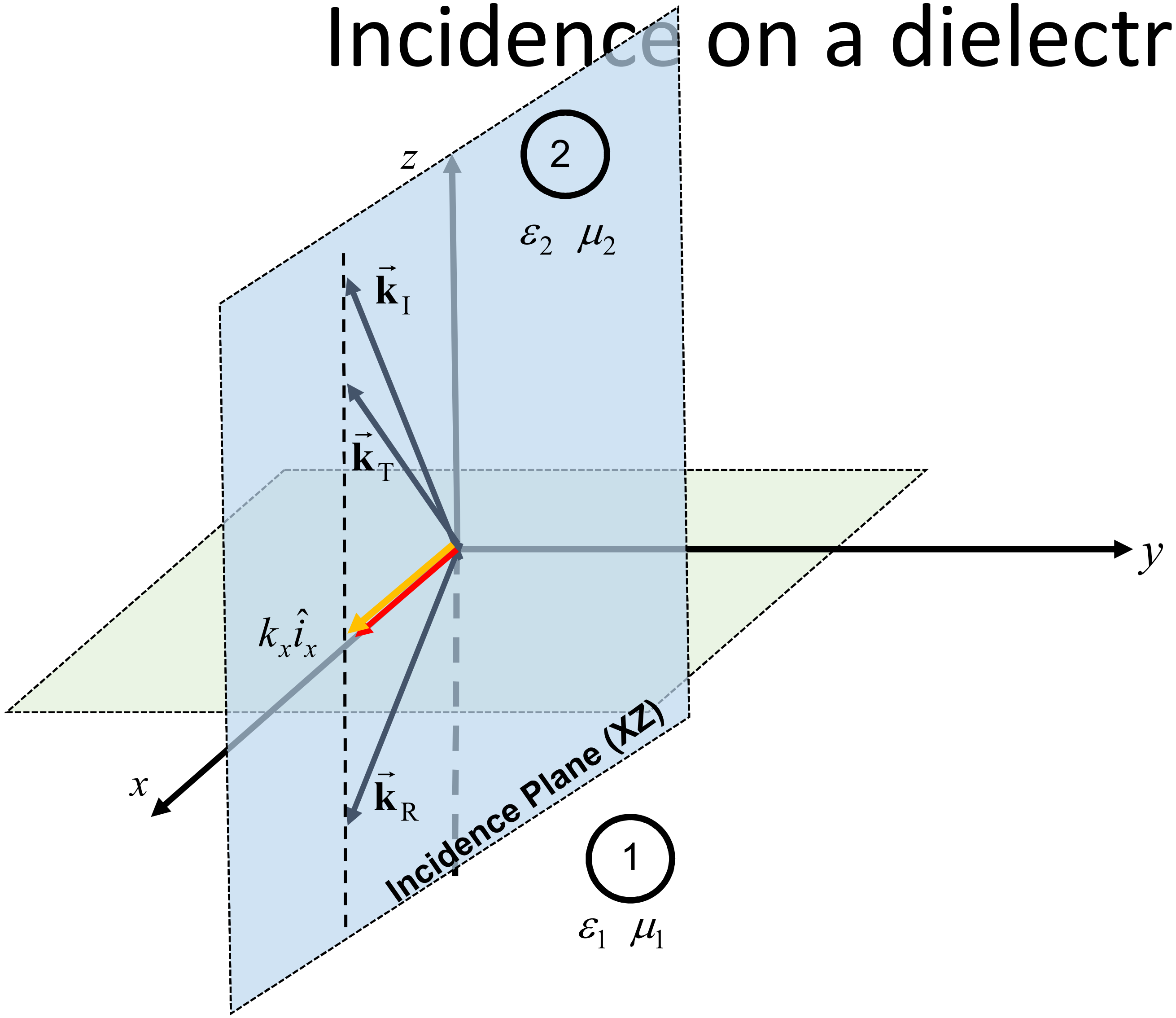


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{I}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{R}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{T}z} \hat{i}_z$$

Incidence on a dielectric half-space

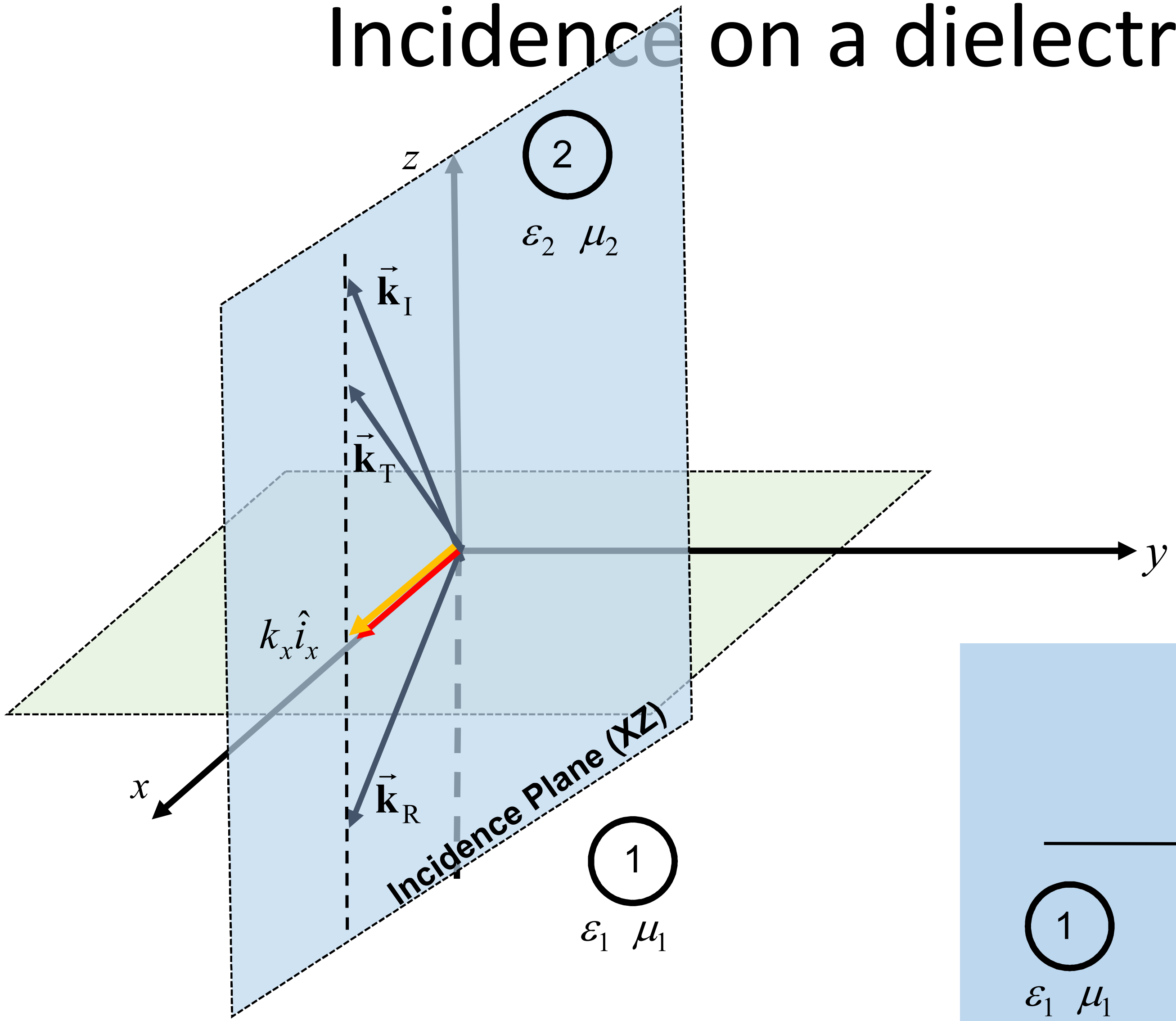


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z$$

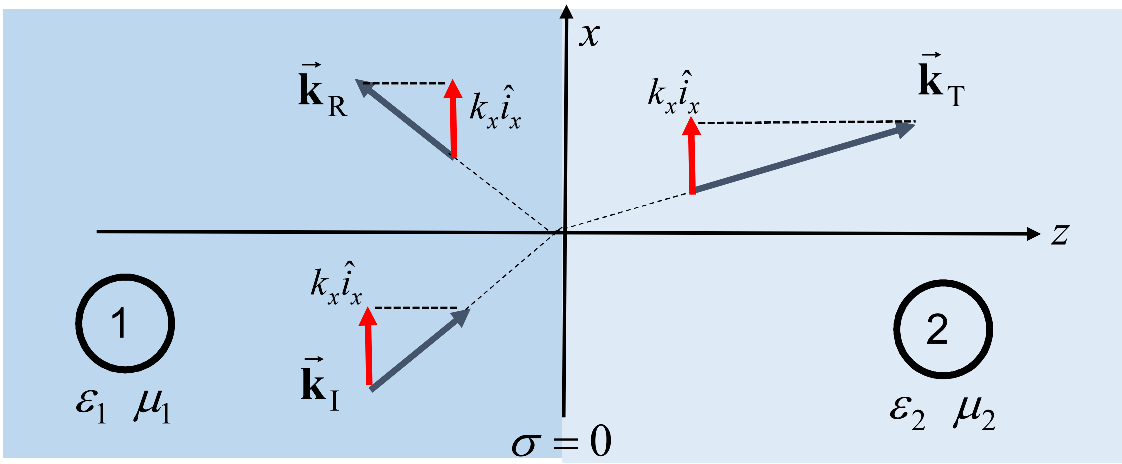
Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{\mathrm{I}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_{\mathrm{R}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z} \hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{\mathrm{I}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{-jk_{\mathrm{R}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{T}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

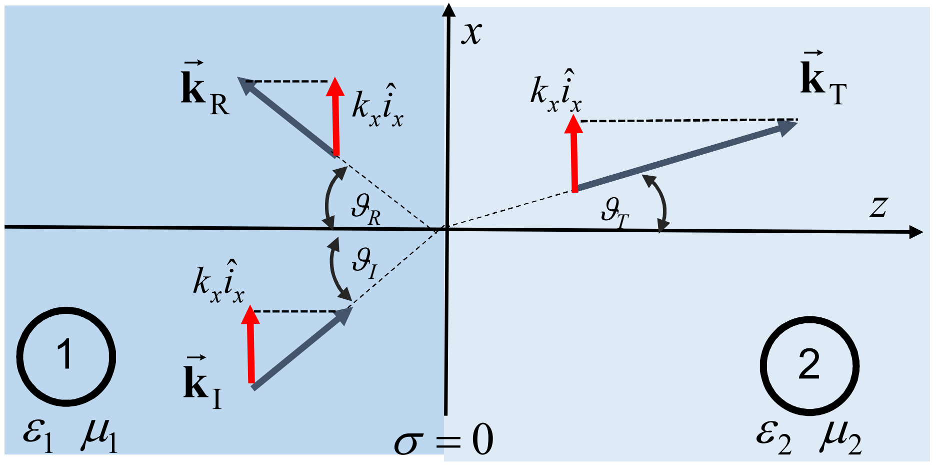
$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$\Rightarrow \quad \vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

Snellius Law

$$\sin\vartheta_{\mathrm{I}} = \frac{k_2}{k_1}\sin\vartheta_T \qquad \sin\vartheta_{\mathrm{I}} = \frac{\omega\sqrt{\mu_2\varepsilon_2}}{\omega\sqrt{\mu_1\varepsilon_1}}\sin\vartheta_T \qquad n = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{\mathrm{Iz}} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{-jk_{\mathrm{Rz}} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{Tz}} z}$$

$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{\mathrm{Iz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_{\mathrm{Rz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{Tz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{I}} \cdot \vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{\mathrm{Iz}}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_{\mathrm{R}} \cdot \vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{\mathrm{Rz}}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_{\mathrm{T}} \cdot \vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{Tz}}^2 = \omega^2 \mu_2 \varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

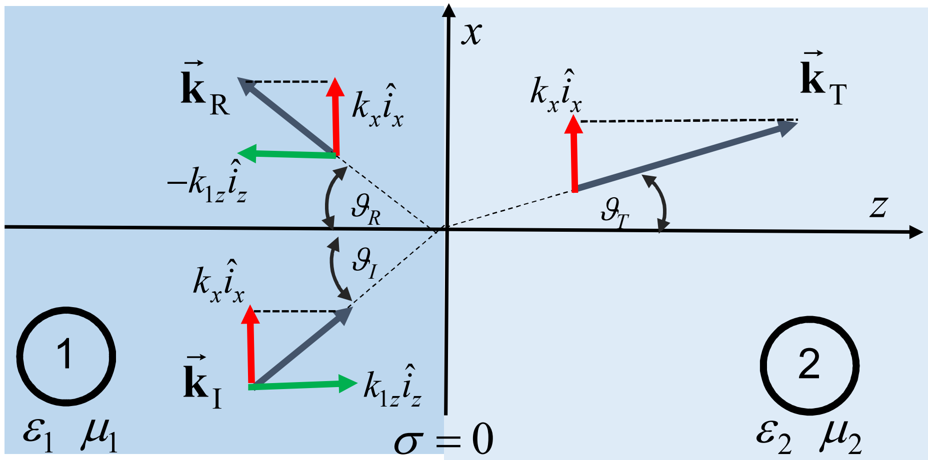
$$k_x = k_1 \sin\vartheta_I$$

$$k_x = k_1 \sin\vartheta_R$$

$$k_x = k_2 \sin\vartheta_T$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$



$$k_{\mathrm{Iz}}^2 = k_{\mathrm{Rz}}^2 \Rightarrow k_{\mathrm{Iz}} = -k_{\mathrm{Rz}} = k_{1z} \Rightarrow$$

$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z} \hat{i}_z$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$
$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$
$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{Tz} z}$$

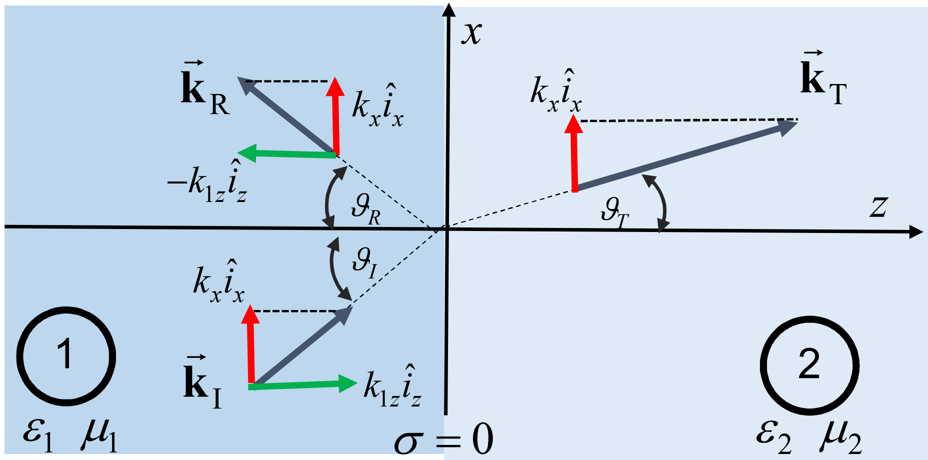
$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z}\hat{i}_z$$
$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z}\hat{i}_z$$
$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{Tz}\hat{i}_z$$

$$\vec{\mathbf{k}}_I\cdot\vec{\mathbf{k}}_I = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$
$$\vec{\mathbf{k}}_R\cdot\vec{\mathbf{k}}_R = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$
$$\vec{\mathbf{k}}_T\cdot\vec{\mathbf{k}}_T = k_x^2 + k_{Tz}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$



$$k_{Iz}^2 = k_{Rz}^2 \Rightarrow k_{Iz} = -k_{Rz} = k_{1z} \Rightarrow \begin{cases}\vec{\mathbf{k}}_I = k_x\hat{i}_x + k_{1z}\hat{i}_z \\ \vec{\mathbf{k}}_R = k_x\hat{i}_x - k_{1z}\hat{i}_z\end{cases}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

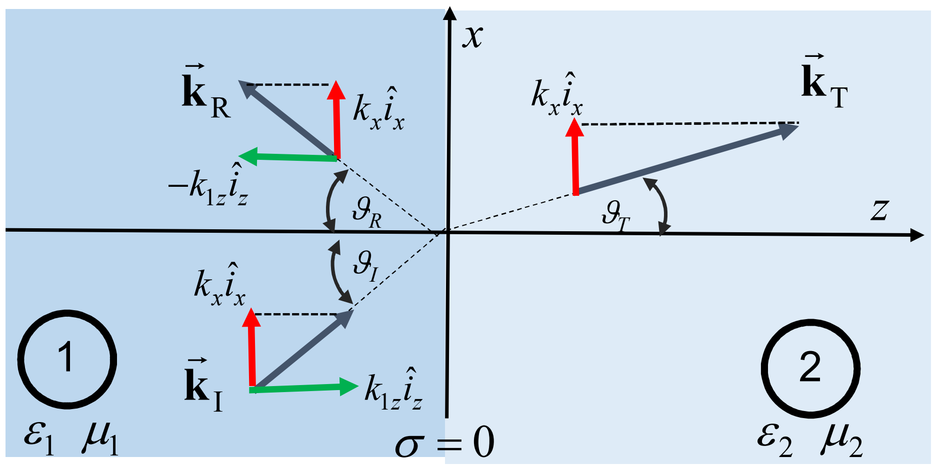
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{T}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z} \qquad \vec{\mathbf{k}}_\mathrm{I} = k_x \hat{i}_x + k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{k}}_\mathrm{I} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z} \qquad \vec{\mathbf{k}}_\mathrm{R} = k_x \hat{i}_x - k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{k}}_\mathrm{R} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z} \qquad \vec{\mathbf{k}}_\mathrm{T} = k_x \hat{i}_x + k_{2z}\hat{i}_z \qquad \vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{k}}_\mathrm{T} = k_x^2 + k_{2z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

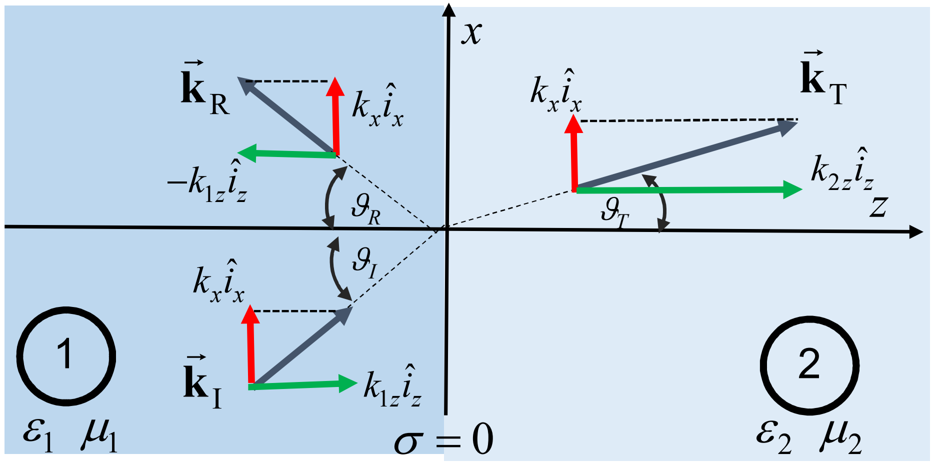
$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = k_2 \cos\vartheta_\mathrm{T}$$

$$k_{2z} = \sqrt{k_2^2 - k_x^2} = \sqrt{k_2^2 - k_1^2 \sin\vartheta_\mathrm{I}}$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$