Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

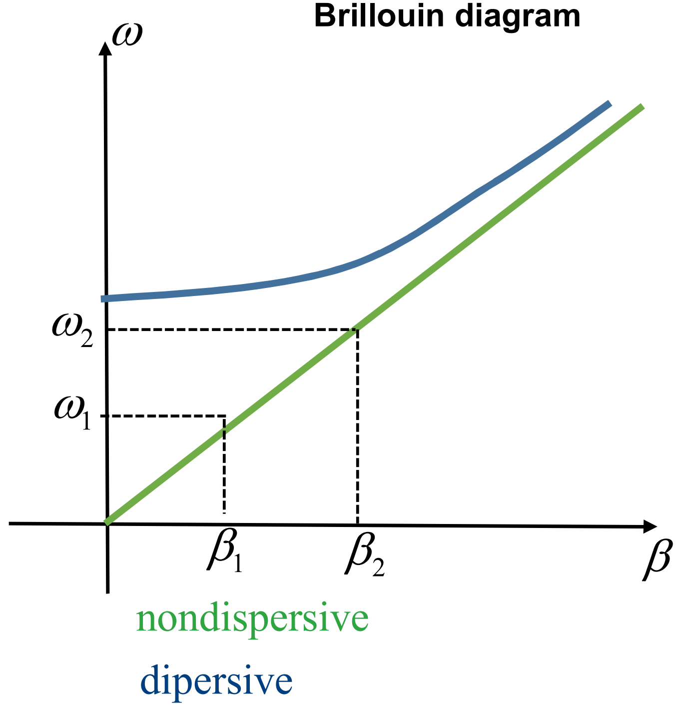
Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

Brillouin diagram



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

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Plane Waves (Phasor Domain)

Time nondispersive & lossless

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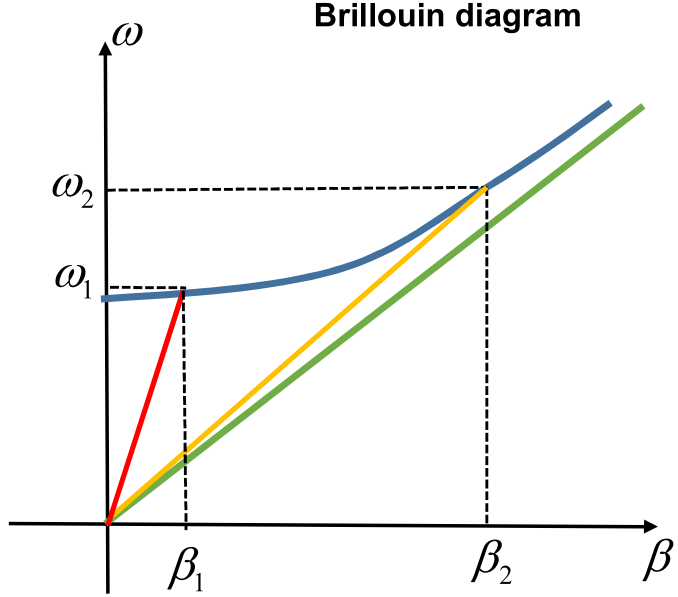
Brillouin diagram



nondispersive

dipersive

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Plane Waves (Spectral Domains)

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y,H_x\}$ $\{E_x,H_y\}$ **Independent each other**

$$k=\omega\sqrt{\mu\varepsilon}$$

$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x,H_y\}$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega_0\mu H_y\\[2mm]\dfrac{dH_y}{dz}=-j\omega_0\varepsilon E_x\end{cases}\qquad \frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x(z)=E_x^+e^{-jkz}+E_x^-e^{jkz}$$

$$\zeta H_y(z)=E_x^+e^{-jkz}-E_x^-e^{jkz}$$

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu(\omega_0)=\mu_1(\omega_0)-j\mu_2(\omega_0)\end{cases}$$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0)=\beta(\omega_0)-j\alpha(\omega_0)$$

Phasor Domain

$\{E_x,H_y\}$

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Fourier Domain

Plane Waves (Fourier Domain)

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

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$$\left\{E_x, H_y\right\}$$

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Fourier Domain

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

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$$\left\{E_y, H_x\right\}$$
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Independent each other

Plane Waves (Fourier Domain)

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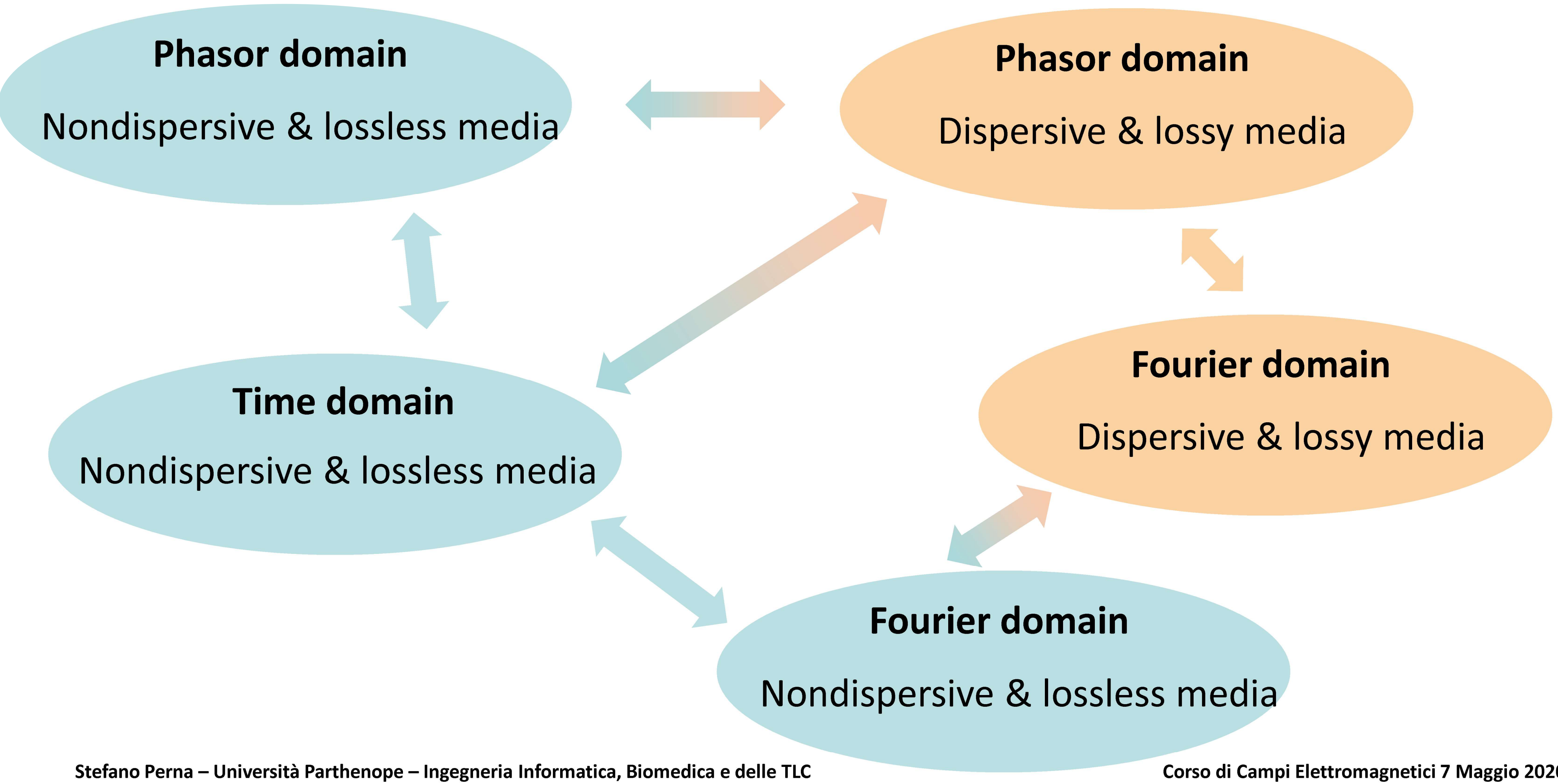
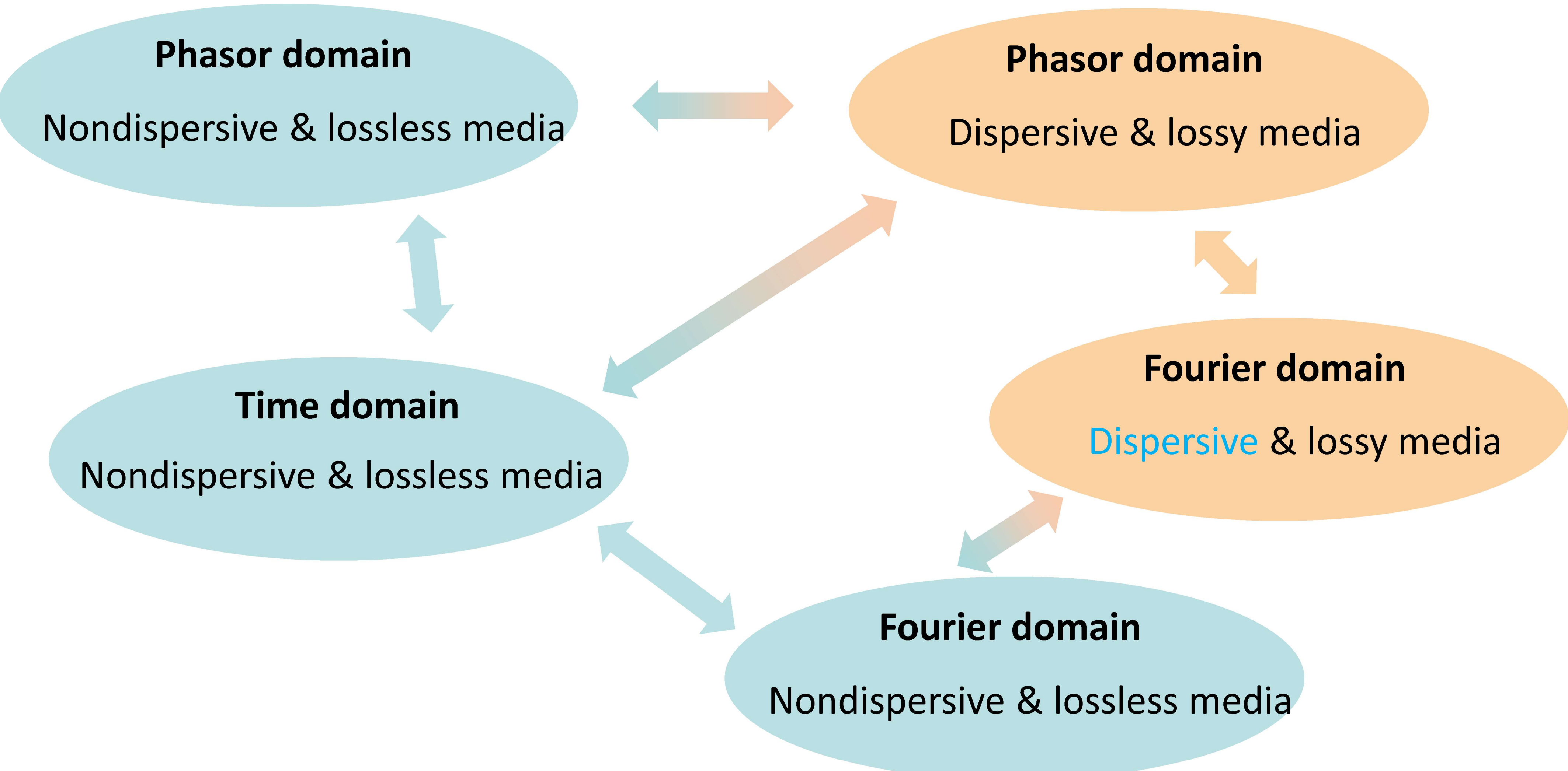
Independent each other

Rationale


Phasor domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Time domain
Nondispersive & lossless media
Fourier domain
Dispersive & lossy media
Fourier domain
Nondispersive & lossless media

Rationale



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Plane Waves (Fourier Domain)

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega)=\varepsilon_1(\omega)-j\varepsilon_2(\omega) \\ \mu=\mu_1(\omega)-j\mu_2(\omega) \\ \sigma : real \end{cases}$$

$$\varepsilon_{eq}(\omega)=\varepsilon(\omega)\left[1-\frac{j\sigma}{\omega\varepsilon(\omega)}\right]$$

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Independent each other

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Time nondispersive & lossless

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Time nondispersive & lossless

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Fourier Domain

Plane Waves (Fourier Domain)

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$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega$$

$$= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega \quad = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

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$\left\{E_y, H_x\right\}$

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Independent each other

Plane Waves (Fourier Domain)

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Time nondispersive & lossless

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$\left\{E_y, H_x\right\}$

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Plane Waves (Fourier Domain)

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$$e_x^+(z,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

Time nondispersive & lossless

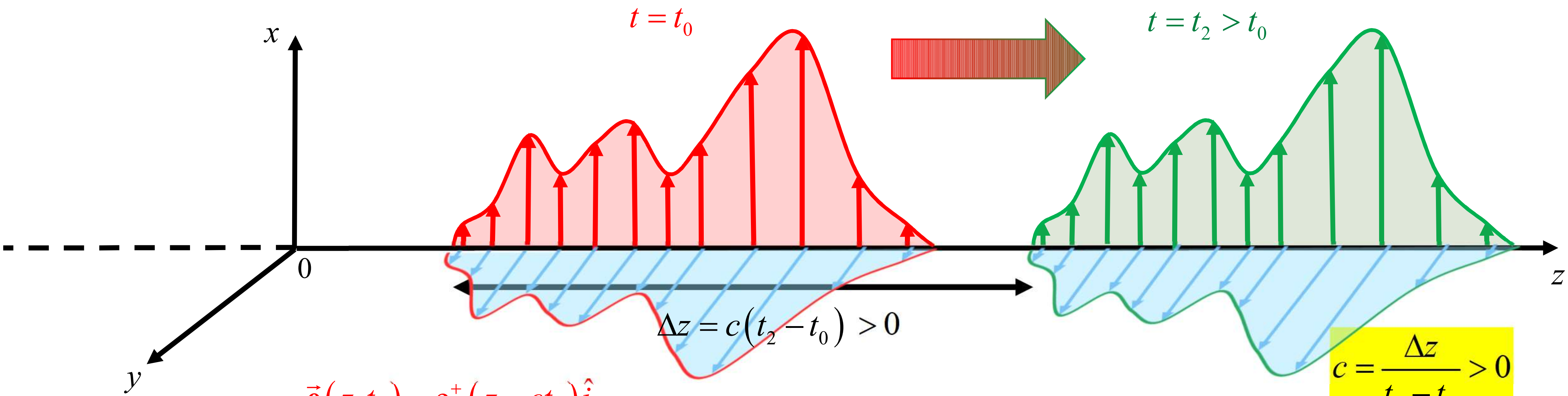
$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

<u>Progressive</u> plane wave

$$e_x^+(z=0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega t}d\omega = f(t)$$

$$e_x^+(z>0,t) = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega = f\left(t-\frac{z}{c}\right) = f\left[-\frac{1}{c}(z-ct)\right] = f\left[(z-ct)\right]$$

Plane Waves



$$\vec{\mathbf{e}}(z,t_0) = \mathrm{e}^+(z-ct_0)\hat{i}_x$$

$$\vec{\mathbf{e}}(z,t_2) = \mathrm{e}^+(z-ct_2)\hat{i}_x = \mathrm{e}^+(z-ct_0+ct_0-ct_2)\hat{i}_x = \mathrm{e}^+(z-ct_0-c[t_2-t_0])\hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed c along the positive sense of the z-axis

$\begin{cases} e^+(z-ct) \\ h^+(z-ct) \end{cases}$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\Downarrow$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Fourier Domain)

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} \quad = \frac{\omega}{c}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega \quad \boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\beta z}e^{j\omega t}d\omega}$$

$$\boxed{= \frac{1}{2\pi}\int E^+(\omega)e^{-j\frac{\omega}{c}z}e^{j\omega t}d\omega} \quad = \frac{1}{2\pi}\int E^+(\omega)e^{j\omega\left(t-\frac{z}{c}\right)}d\omega$$

~~Time nondispersive & lossless~~

$$\cancel{\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \quad \zeta = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Mathematical tools

$$f(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow F(\omega)$$

$$f(t)\ \text{real}\ \Rightarrow \boxed{F(\omega)=F^*(-\omega)}$$

$$f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}d\omega \quad =\frac{1}{2\pi}\left[\boxed{\int_{-\infty}^{0}F(\omega)e^{j\omega t}d\omega}+\int_{0}^{\infty}F(\omega)e^{j\omega t}d\omega\right] \quad =\frac{1}{2\pi}\left[\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega+\int_{0}^{\infty}F(\omega)e^{j\omega t}d\omega\right]$$

$$\eta=-\omega$$

$$\boxed{\int_{-\infty}^{0}F(\omega)e^{j\omega t}d\omega}=-\int_{\infty}^{0}F(-\eta)e^{-j\eta t}d\eta \quad =\int_{0}^{\infty}F(-\eta)e^{-j\eta t}d\eta \quad =\int_{0}^{\infty}F^*(\eta)\left[e^{j\eta t}\right]^* d\eta =\int_{0}^{\infty}\left[F(\eta)e^{j\eta t}\right]^* d\eta =\boxed{\int_{0}^{\infty}\left[F(\omega)e^{j\omega t}\right]^* d\omega}$$

Mathematical tools

$$f(t) \longrightarrow \boxed{\textbf{FT}} \longrightarrow F(\omega)$$

$$f(t) \text{ real } \Rightarrow F(\omega) = F^*(-\omega)$$

$$\boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega} = \frac{1}{2\pi}\left[\int_{-\infty}^{0} F(\omega) e^{j\omega t} d\omega + \int_{0}^{\infty} F(\omega) e^{j\omega t} d\omega\right] = \frac{1}{2\pi}\left[\int_{0}^{\infty} \left[F(\omega) e^{j\omega t}\right]^* d\omega + \int_{0}^{\infty} F(\omega) e^{j\omega t} d\omega\right]$$

$$= \frac{1}{2\pi}\int_{0}^{\infty} F(\omega) e^{j\omega t} + \left[F(\omega) e^{j\omega t}\right]^* d\omega \quad = \frac{1}{2\pi}\left[\int_{0}^{\infty} 2\,\mathrm{Re}\left\{F(\omega) e^{j\omega t}\right\} d\omega\right] = \boxed{\mathrm{Re}\left\{\frac{1}{\pi}\int_{0}^{\infty} F(\omega) e^{j\omega t} d\omega\right\}}$$

$$q = a + jb$$

$$q + q^* = (a + jb) + (a - jb) = 2a \; = 2\,\mathrm{Re}\{q\}$$

Plane Waves : dispersion

$\{E_x, H_y\}$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega) - j\alpha(\omega)$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta H_y^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+(z,\omega) = E^+(\omega)e^{-jkz} = E^+(\omega)e^{-j\beta(\omega)z}e^{-\alpha(\omega)z}$$

$$e_x^+(z,t) = \frac{1}{2\pi}\int E_x^+(z,\omega)e^{j\omega t}d\omega = \mathrm{Re}\left\{\frac{1}{\pi}\int_0^\infty E_x^+(z,\omega)e^{j\omega t}d\omega\right\}$$

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$$\alpha(\omega) \approx 0$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

Plane Waves : dispersion

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega)e^{-j\beta z}$$

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Plane Waves : dispersion

$\{E_x, H_y\}$

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$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

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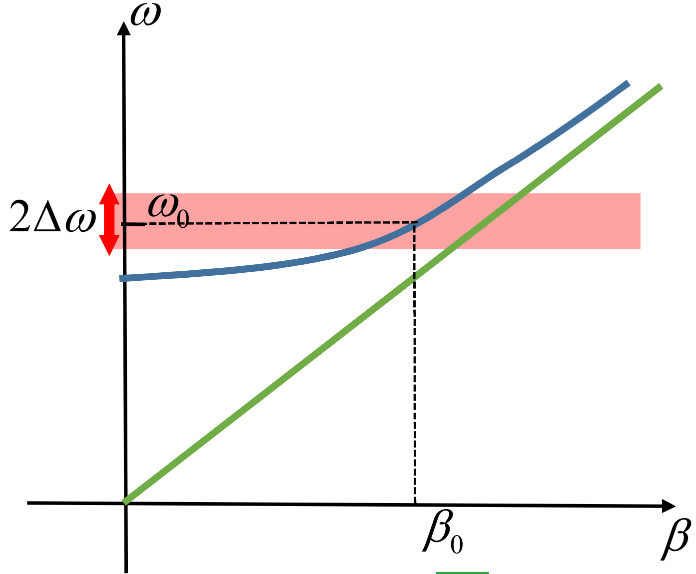

nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

dipersive : $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega-\omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2} + \cdots$$

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$$\beta(\omega) \approx \beta_0 + \frac{(\omega-\omega_0)}{v_g}$$

$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)}$$

Plane Waves : dispersion

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\left[\beta_0+\frac{(\omega-\omega_0)}{v_g}\right]z}e^{j\omega t}d\omega$$

$$=\frac{1}{\pi}\int_{-\Delta\omega}^{\Delta\omega} E^+(\eta+\omega_0)e^{-j\left[\beta_0+\frac{\eta}{v_g}\right]z}e^{j(\eta+\omega_0)t}d\eta = \frac{1}{\pi}\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)\boxed{e^{-j\frac{\eta}{v_g}z}e^{j\eta t}}d\eta$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

$$\eta=\omega-\omega_0$$

$$\boxed{e^{-j\beta_0 z}e^{j\omega_0 t}} = e^{j\omega_0\left(t-\frac{\beta_0}{\omega_0}z\right)} = \boxed{e^{j\omega_0\left(t-\frac{z}{v_p}\right)}}$$

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$$\beta(\omega)\approx\beta_0+\frac{(\omega-\omega_0)}{v_g}$$

$$\beta_0=\beta(\omega_0)$$

$$v_g=\frac{1}{\beta'(\omega_0)}$$

$$v_p=\frac{\omega_0}{\beta_0}$$

Plane Waves : dispersion

$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$=\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta$$

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$$\frac{1}{\pi}\int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega = \frac{1}{\pi} e^{j\omega_0\left(t - \frac{z}{v_p}\right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{j\left(t - \frac{z}{v_g}\right)\eta} d\eta$$

Plane Waves : dispersion

$\left\{E_x, H_y\right\}$

$$E_x^+(z,\omega) = E^+(\omega) e^{-j\beta(\omega)z}$$

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$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)} = \beta(\omega)$$

$$\boxed{e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi} e^{j\omega_0\left(t-\frac{z}{v_p}\right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta) e^{j\left(t-\frac{z}{v_g}\right)\eta} d\eta\right\}}$$

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$$e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi} e^{j\omega_0\left(t-\frac{z}{v_p}\right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0+\eta) e^{j\left(t-\frac{z}{v_g}\right)\eta} d\eta\right\}$$

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$$\beta_0 = \beta(\omega_0)$$

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Plane Waves : dispersion

$\{E_x, H_y\}$

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$$\boxed{e_x^+(z,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta\right\}}$$

$$e_x^+(z=0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}\underbrace{\boxed{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\eta t}d\eta}}_{f(t)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0 t}f(t)\right\} = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t) = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}\underbrace{\boxed{\int_{-\Delta\omega}^{\Delta\omega}E^+(\omega_0+\eta)e^{j\left(t-\frac{z}{v_g}\right)\eta}d\eta}}_{f\left(t-\frac{z}{v_g}\right)}\right\} = \mathrm{Re}\left\{\frac{1}{\pi}e^{j\omega_0\left(t-\frac{z}{v_p}\right)}f\left(t-\frac{z}{v_g}\right)\right\}$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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$$e_x^+(z=0,t) \qquad = \frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t)$$

$$= \frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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$$e_x^+(z=0,t)=\frac{1}{\pi}f(t)\cos[\omega_0 t]$$

$$e_x^+(z>0,t)=\frac{1}{\pi}f\left(t-\frac{z}{v_g}\right)\cos\left[\omega_0\left(t-\frac{z}{v_p}\right)\right]$$

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Plane Waves : dispersion

$$e_x^+(z,t) = \frac{1}{\pi} f\left(t - \frac{z}{v_g}\right) \cos\left[\omega_0\left(t - \frac{z}{v_p}\right)\right]$$

$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)}$$

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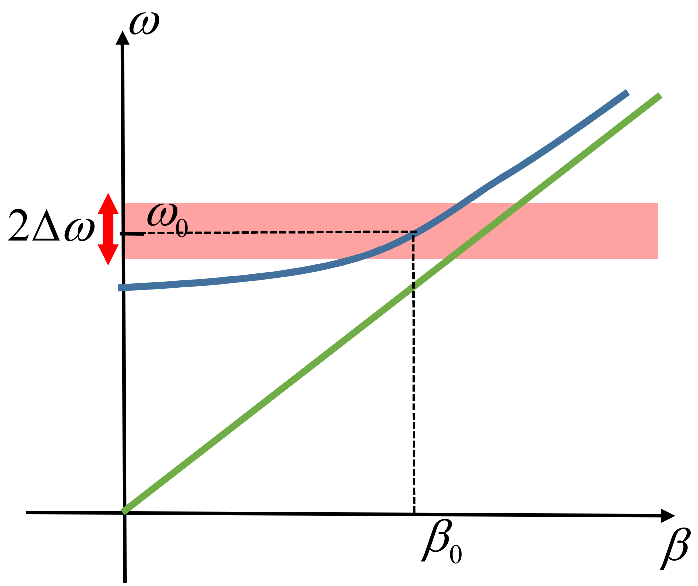

nondispersive : $\beta = \omega\sqrt{\mu\varepsilon}$

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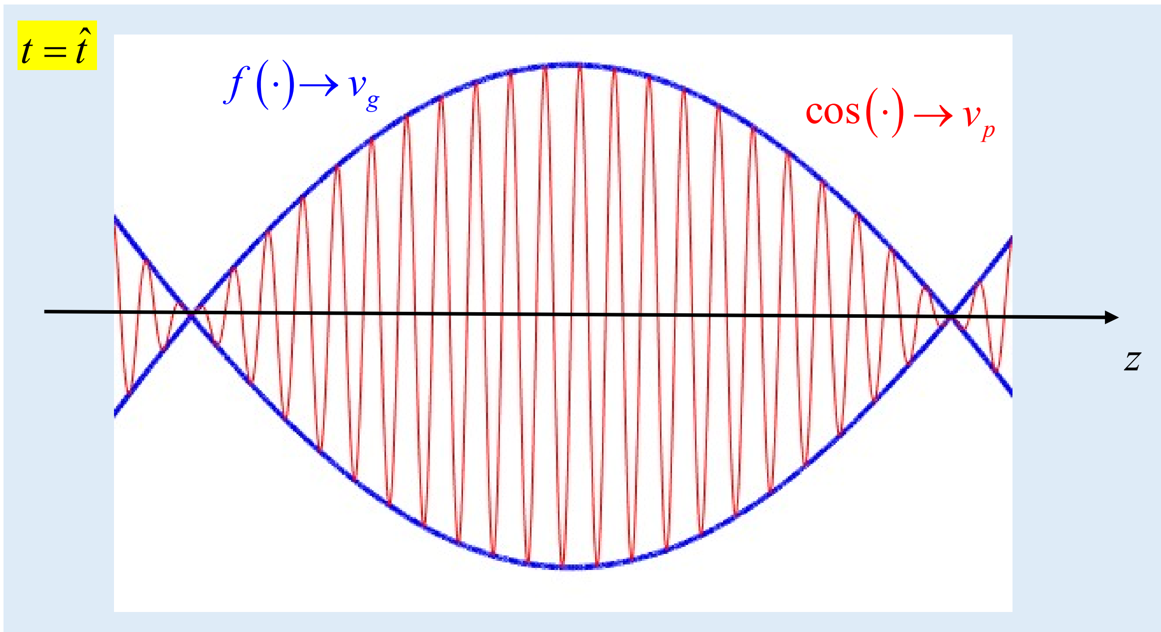
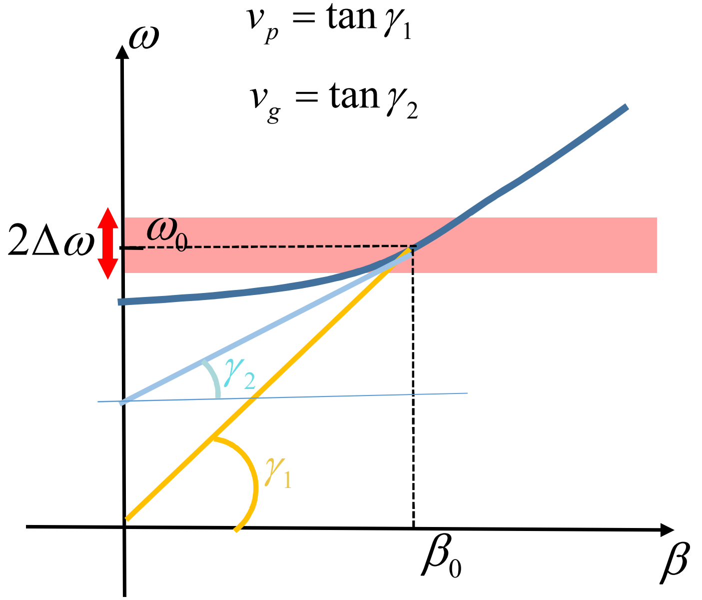


Plane Waves : dispersion

$$v_p = \tan\gamma_1$$

$$v_g = \tan\gamma_2$$



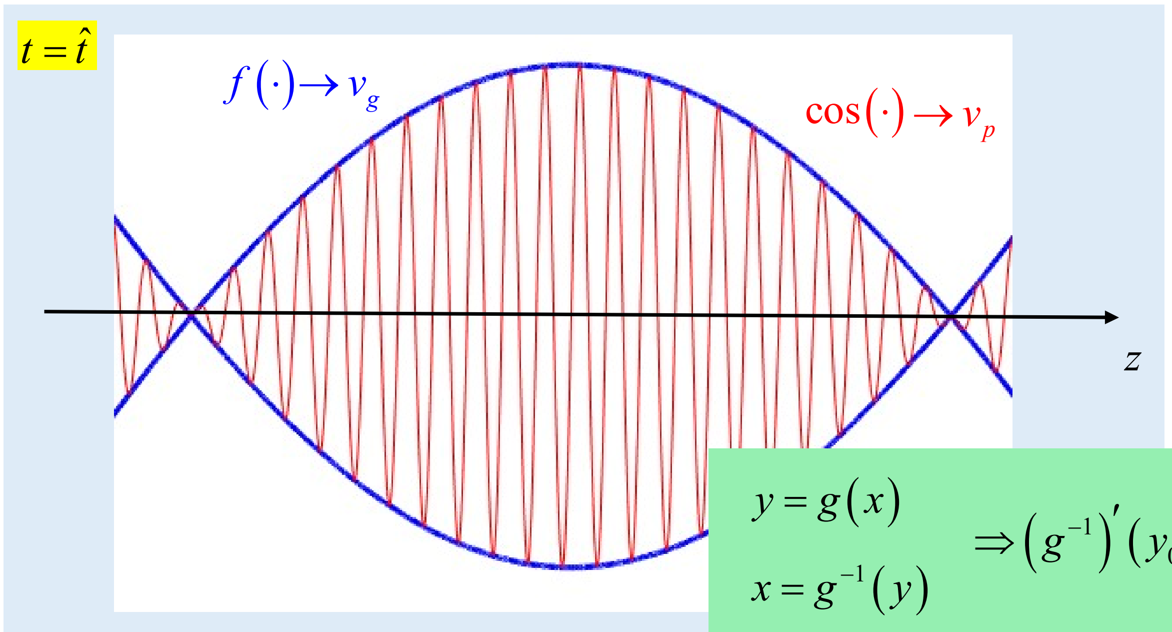
$$\text{dipersive}: \beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$e_x^+(z,t) = \frac{1}{\pi} f\left(t - \frac{z}{v_g}\right)\cos\left[\omega_0\left(t - \frac{z}{v_p}\right)\right]$$

$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)} = \omega'(\beta_0)$$

$$v_p = \frac{\omega_0}{\beta_0}$$



$$y = g(x)$$

$$x = g^{-1}(y)$$

$$\Rightarrow \left(g^{-1}\right)'(y_0) = \frac{1}{g'(x_0)}$$

Plane Waves : dispersion

Source-free

Medium
- Linear
- $\boxed{\text{Time dispersive}}$
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

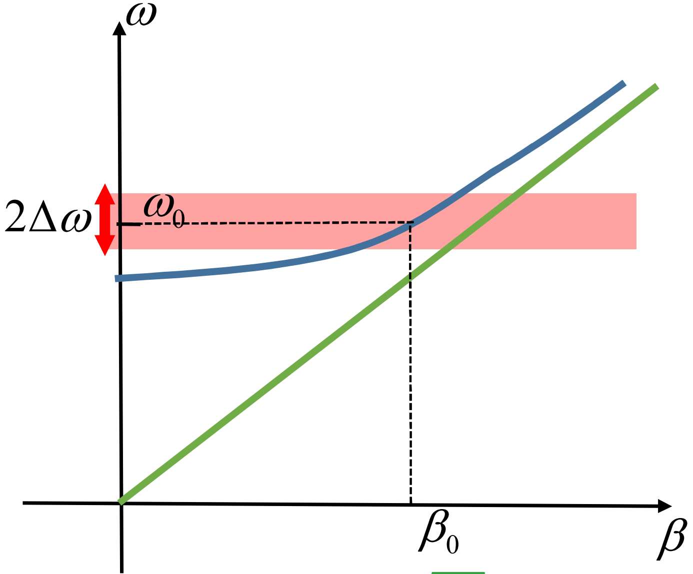
$$\begin{cases} \varepsilon(\omega)=\varepsilon_1(\omega) \\ \mu(\omega)=\mu_1(\omega) \\ \sigma=0 \end{cases}$$

$\{E_x, H_y\}$

$$E_x^+(z,\omega)=E^+(\omega)e^{-j\beta(\omega)z}$$

$$\zeta H_y^+(z,\omega)=E^+(\omega)e^{-j\beta(\omega)z}$$

$$k(\omega)=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}=\beta(\omega)$$



nondispersive : $\beta=\omega\sqrt{\mu\varepsilon}$

dipersive : $\beta=\omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi}\int_0^\infty E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega \approx \frac{1}{\pi}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega)e^{-j\beta(\omega)z}e^{j\omega t}d\omega$$

$$\beta(\omega)\approx\beta(\omega_0)+\beta'(\omega_0)(\omega-\omega_0)+\cancel{\frac{1}{2}\beta''(\omega_0)(\omega-\omega_0)^2}+\cdots$$

$$\beta(\omega)\approx\beta_0+\frac{(\omega-\omega_0)}{v_g}$$

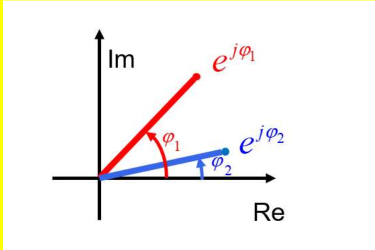
$$\beta_0=\beta(\omega_0)$$

$$v_g=\frac{1}{\beta'(\omega_0)}$$

Plane Waves : dispersion

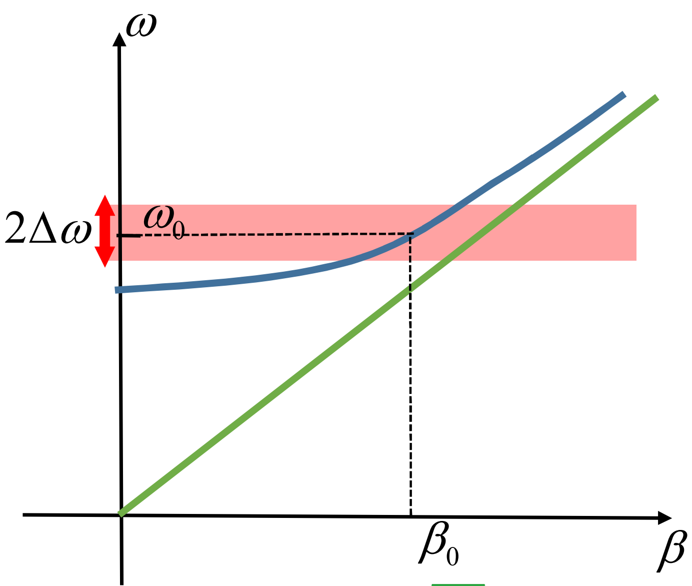
$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \cdots$$

$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\boxed{\beta''(\omega_0)}\Delta\omega^2 z << 2\pi$$

Channel &
carrier frequency



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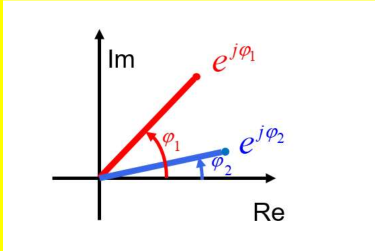
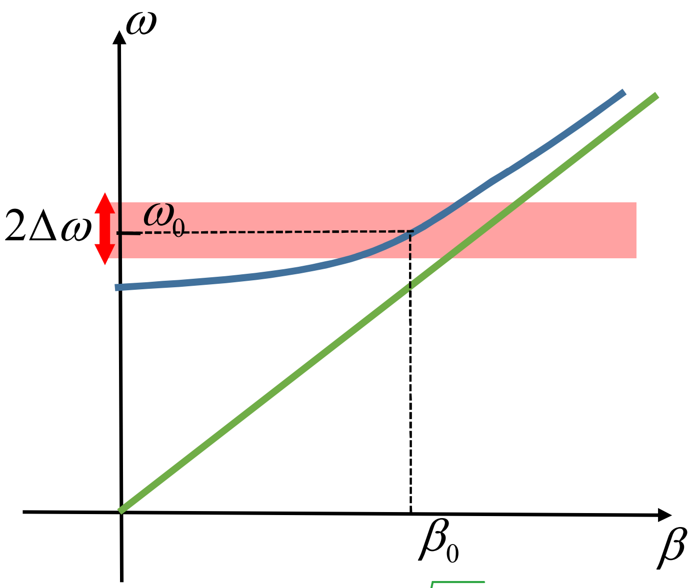


$$\frac{1}{2}\beta''(\omega_0)\boxed{\Delta\omega^2}z << 2\pi$$

Channel & carrier frequency

Bandwidth



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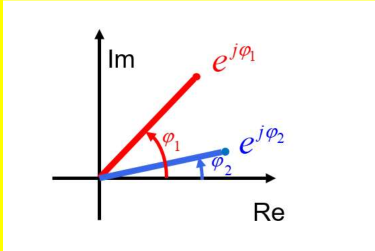
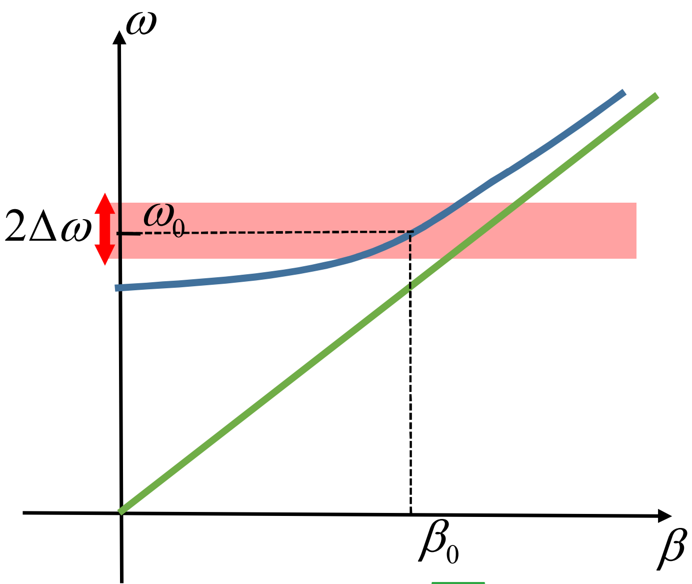


$$\frac{1}{2}\beta''(\omega_0)\Delta\omega^2 \boxed{z} << 2\pi$$

Channel & carrier frequency

Bandwidth

Distance



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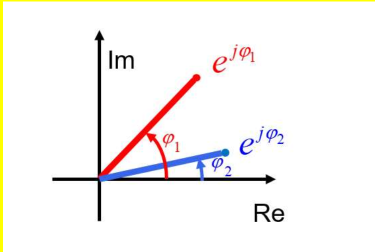

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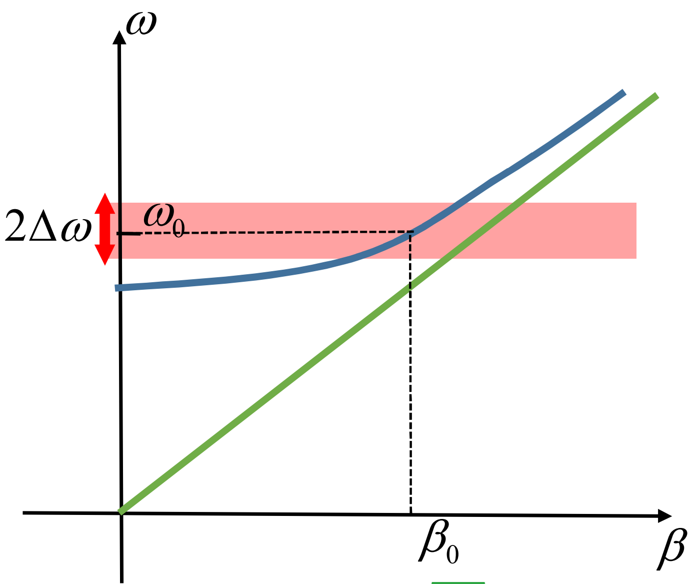

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