Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\boxed{\begin{gathered} k = \omega\sqrt{\mu\varepsilon} \\ k = \beta - j\alpha \end{gathered}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\boxed{\frac{d^2E_x}{dz^2} + k^2 E_x = 0}$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

$$\boxed{\frac{d^2E_y}{dz^2} + k^2 E_y = 0}$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$\frac{d^2 f(z)}{dz^2} + k^2 f(z) = 0$$

$$\xi^2 + k^2 = 0 \qquad \xi = \pm jk$$

$$f(z) = C_1 e^{-jkz} + C_2 e^{+jkz}$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \boxed{\dfrac{dE_x}{dz} = -j\omega\mu H_y} \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$$\boxed{E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

$$\frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$$-j\omega\mu H_y = \frac{dE_x}{dz} = -jkE_x^+ e^{-jkz} + jkE_x^- e^{jkz}$$

$$\omega\mu H_y = kE_x^+ e^{-jkz} - kE_x^- e^{jkz}$$

$$\frac{\omega\mu}{k} H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

ζ : intrinsic impedance of the medium

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \boxed{\dfrac{dE_y}{dz} = j\omega\mu H_x} \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$\boxed{E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\Downarrow$$

$$E_z = H_z = 0$$

$$j\omega\mu H_x = \frac{dE_y}{dz} = -jkE_y^+ e^{-jkz} + jkE_y^- e^{jkz}$$

$$-\omega\mu H_x = kE_y^+ e^{-jkz} - kE_y^- e^{jkz} \qquad -\frac{\omega\mu}{k} H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz} \qquad -\zeta H_y = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\Downarrow$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$$\left\{E_x, H_y\right\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\left\{E_x, H_y\right\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$$\left\{E_y, H_x\right\}$$

$$\left\{E_x, H_y\right\}$$

Independent each other

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

$\{E_x, H_y\}$

$$\boxed{\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Fourier Domain

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\boxed{\begin{aligned} E_x &= E_x^+ e^{-jkz} + E_x^- e^{jkz} \\ \zeta H_y &= E_x^+ e^{-jkz} - E_x^- e^{jkz} \end{aligned}}$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\boxed{\begin{aligned} E_x(z) &= E_x^+ e^{-jkz} + E_x^- e^{jkz} \\ \zeta H_y(z) &= E_x^+ e^{-jkz} - E_x^- e^{jkz} \end{aligned}}$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$
$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\boxed{\begin{aligned} E_x &= E_x^+ e^{-jkz} + E_x^- e^{jkz} \\ \zeta H_y &= E_x^+ e^{-jkz} - E_x^- e^{jkz} \end{aligned}}$$

Fourier Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\{E_y,H_x\}$
$\{E_x,H_y\}$
Independent each other

$\{E_x,H_y\}$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega_0\mu H_y\\ \dfrac{dH_y}{dz}=-j\omega_0\varepsilon E_x\end{cases}\qquad \frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$E_x(z)=E_x^+e^{-jkz}+E_x^-e^{jkz}$$

$$\zeta H_y(z)=E_x^+e^{-jkz}-E_x^-e^{jkz}$$

Phasor Domain

$$k=\omega\sqrt{\mu\varepsilon}$$
$$k=\beta-j\alpha$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$\{E_x,H_y\}$

$$\begin{cases}\dfrac{dE_x}{dz}=-j\omega\mu H_y\\ \dfrac{dH_y}{dz}=-j\omega\varepsilon E_x\end{cases}\qquad \frac{d^2E_x}{dz^2}+k^2E_x=0$$

$$\boxed{\begin{aligned}E_x(z,\omega)&=E_x^+(\omega)e^{-jkz}+E_x^-(\omega)e^{jkz}\\ \zeta H_y(z,\omega)&=E_x^+(\omega)e^{-jkz}-E_x^-(\omega)e^{jkz}\end{aligned}}$$

Fourier Domain

Plane Waves (Spectral Domains)

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\{E_x, H_y\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\} \qquad \{E_x, H_y\}$$

Independent each other

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

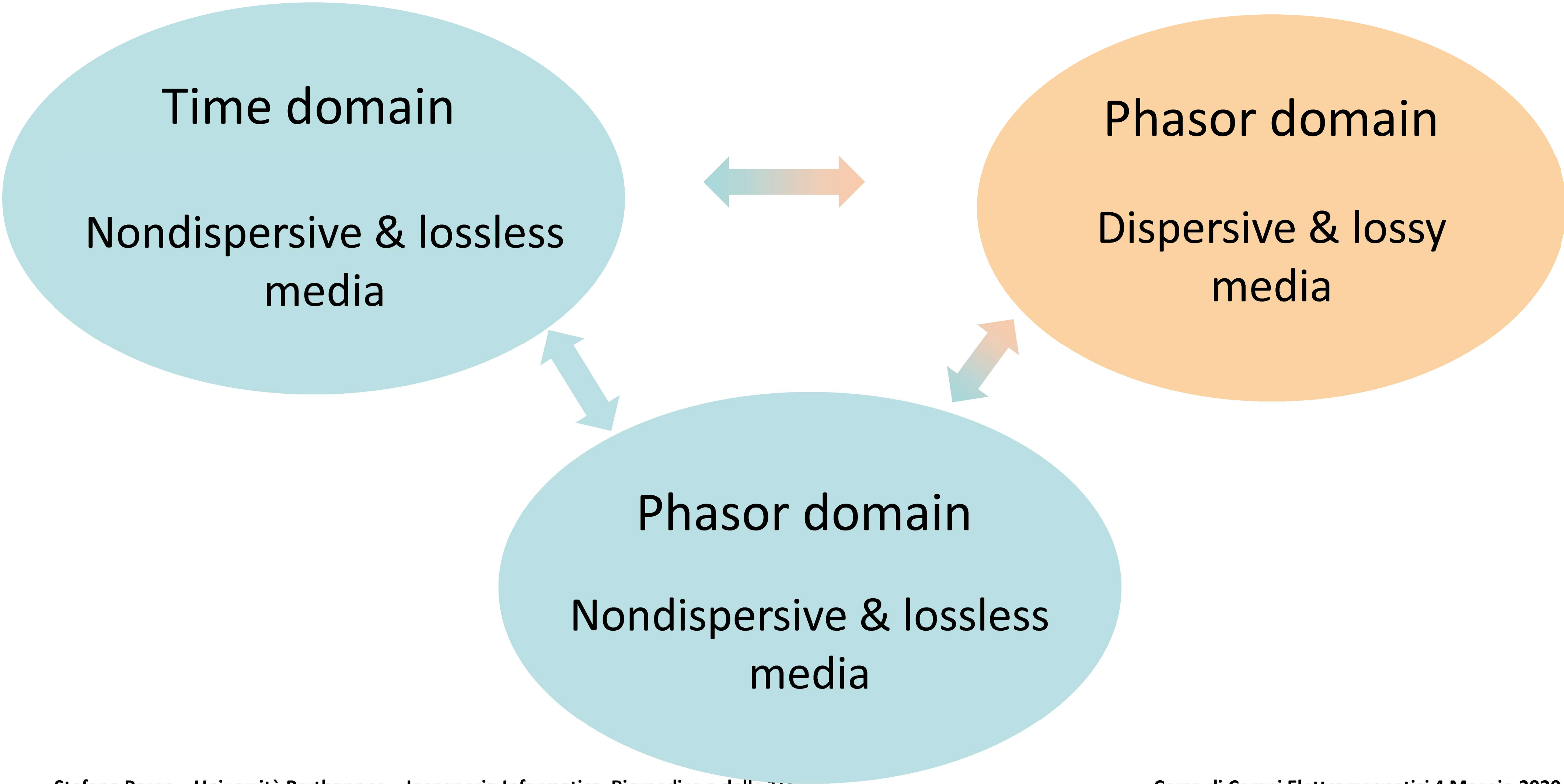
General expression of plane waves (PD)

Incidence

Plane Waves

Phasor domain

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$$\left\{ E_x, H_y \right\}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

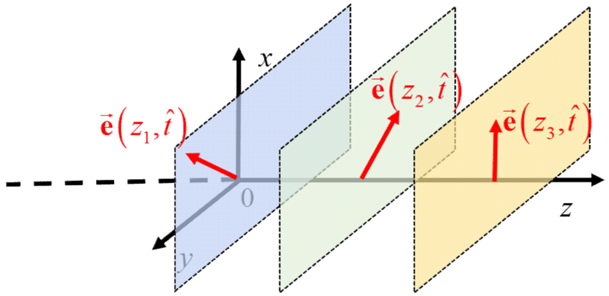
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

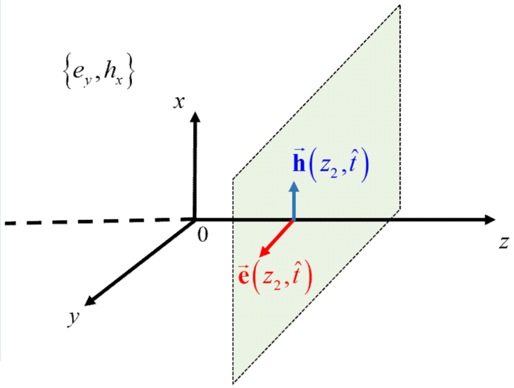
Phasor Domain

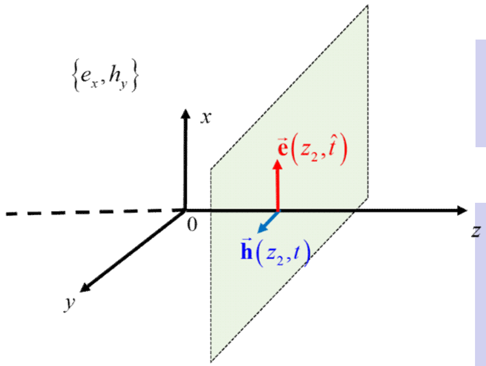
$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$







Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$\left\{ E_y, H_x \right\} \quad \left\{ E_x, H_y \right\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$E_x(z)=E_x^+e^{-jkz}+E_x^-e^{jkz}$$

$$\zeta H_y(z)=E_x^+e^{-jkz}-E_x^-e^{jkz}$$

$$\zeta(\omega_0)=\sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$\varepsilon_{eq}(\omega_0)=\varepsilon(\omega_0)\left[1-\frac{j\sigma}{\omega_0\varepsilon(\omega_0)}\right]$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Source-free

Medium

- **- Linear**
- **- Time nondispersive**
- **- Space non-dispersive**
- **- Isotropic**
- **- Homogeneous (TI – SI)**
- **- Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$\varepsilon_{eq}(\omega_0) = \varepsilon(\omega_0)\left[1 - \frac{j\sigma}{\omega_0\varepsilon(\omega_0)}\right]$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega_0)=\omega_0\sqrt{\mu(\cancel{\omega_0})\varepsilon(\cancel{\omega_0})}=\beta(\omega_0)\cancel{-j\alpha(\omega_0)}$$

$$E_x(z)=E_x^+e^{-jkz}+E_x^-e^{jkz}$$

$$\zeta H_y(z)=E_x^+e^{-jkz}-E_x^-e^{jkz}$$

$$\zeta(\cancel{\omega_0})=\sqrt{\frac{\mu(\cancel{\omega_0})}{\varepsilon(\cancel{\omega_0})}}$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon : real\\ \mu : real\\ \sigma = 0\end{cases}$$

$$\varepsilon_{eq}=\varepsilon$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$
$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$
$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = \boxed{E_x^+ e^{-j\beta z}} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = \boxed{E_x^+ e^{-j\beta z}} - E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ = \left|E_x^+\right| e^{j\varphi^+}$$

$$E_x^+ e^{-j\beta z} = \left|E_x^+\right| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{\left|E_x^+\right| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = \left|E_x^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right)$$

$$= \left|E_x^+\right| \cos\left(-\beta\left[z - \frac{\omega_0}{\beta} t\right] + \varphi^+\right) = \left|E_x^+\right| \cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E_x^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$E_x^+ e^{-j\beta z} = \left|E_x^+\right| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{\left|E_x^+\right| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = \left|E_x^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

$$= \left|E_x^+\right|\cos\left(-\beta\left[z - \frac{\omega_0}{\beta}t\right] + \varphi^+\right) = \left|E_x^+\right|\cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+\left(z - v_p t\right) = e_x^+\left(z - ct\right)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- = |E_x^-| e^{j\varphi^-}$$

$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z,t) = \mathrm{Re}\left\{|E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$
$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E_x^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = \left|E_x^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-(z + v_p t)$$

$$E_x^- e^{j\beta z} = \left|E_x^-\right| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z,t) = \mathrm{Re}\left\{\left|E_x^-\right| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = \left|E_x^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = \left|E_x^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \quad \rightarrow e_x^-(z,t) = \left|E_x^-\right| \cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-\left(z + v_p t\right)$$

<u>Regressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E_x^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E_x^+ e^{-j\beta z}$$

$$E_x^-(z) = E_x^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = \left|E^-\right| \cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \to e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \to e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \quad \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}}^+(\vec{\mathbf{r}}) = \frac{1}{2}\vec{\mathbf{E}}^+(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{+*}(\vec{\mathbf{r}}) = \frac{1}{2}E_x^+(z)\hat{i}_x \times H_y^{+*}(z)\hat{i}_y = \frac{1}{2}\left(E^+ e^{-j\beta z}\right)\cdot\frac{\left(E^+ e^{-j\beta z}\right)^*}{\zeta}\hat{i}_z$$

$$= \frac{|E^+|^2}{2\zeta}\hat{i}_z = \frac{|E_x^+(z)|^2}{2\zeta}\hat{i}_z = \zeta\frac{|H_y^+(z)|^2}{2}\hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$
$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$
$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$\varepsilon_{\ } = \varepsilon$

$\beta(\omega_0)$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$

$$E_x^+(z)=E^+e^{-j\beta z}$$
$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$E_x^-(z)=E^-e^{j\beta z}$$
$$\zeta H_y^-(z)=-E^-e^{j\beta z}$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu\varepsilon}}=c$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+e^{-j\beta z}\ \rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+(z-v_p t)$$

Progressive plane wave

$$E^-e^{j\beta z}\ \rightarrow e_x^-(z,t)=\left|E^-\right|\cos\left(\omega_o t+\beta z+\varphi^-\right)=e_x^-(z+v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases}\varepsilon : real\\ \mu : real\\ \sigma=0\end{cases}$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$\varepsilon \ \ =\varepsilon$

$\beta(\omega_0)$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}})=\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}}^-(\vec{\mathbf{r}})=\frac{1}{2}\vec{\mathbf{E}}^-(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^{-*}(\vec{\mathbf{r}})=\frac{1}{2}E_x^-(z)\hat{i}_x\times H_y^{-*}(z)\hat{i}_y=-\frac{\left|E_x^-(z)\right|^2}{2\zeta}\hat{i}_z$$
$$=-\zeta\frac{\left|H_y^-(z)\right|^2}{2}\hat{i}_z$$

$$\hat{i}_z=\hat{i}_x\times\hat{i}_y$$
$$\hat{i}_y=\hat{i}_z\times\hat{i}_x$$
$$\hat{i}_x=\hat{i}_y\times\hat{i}_z$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = \left|E^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-(z + v_p t)$$

<u>Regressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

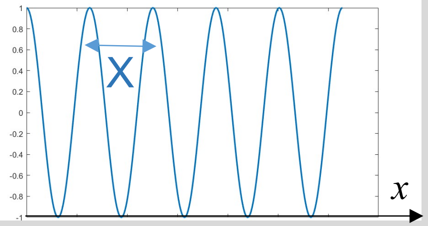
$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \to e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Memo



$$\cos(2\pi\nu x) = \cos\left(\frac{2\pi}{X} x\right)$$

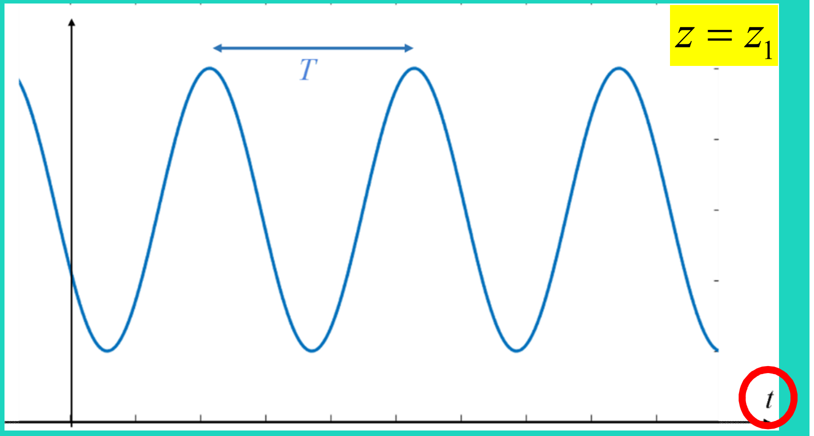
ν: frequency

$X = \frac{1}{\nu}$: period

$$e_x^+(z = z_1, t) = |E^+|\cos(\omega_o t - \beta z_1 + \varphi^+) = |E^+|\cos\left(2\pi\left[\frac{\omega_o}{2\pi}\right]t - \beta z_1 + \varphi^+\right)$$

frequency: $f_0 = \frac{\omega_o}{2\pi}$

period: $T = \frac{1}{f_0} = \frac{2\pi}{\omega_o}$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$\boxed{k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)}$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

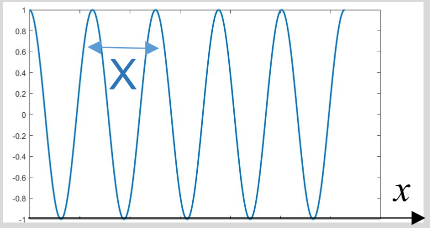
$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \to \boxed{e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)}$$

Progressive plane wave

Memo



$$\cos(2\pi\nu x) = \cos\left(\frac{2\pi}{X}x\right)$$

$\nu :$ frequency

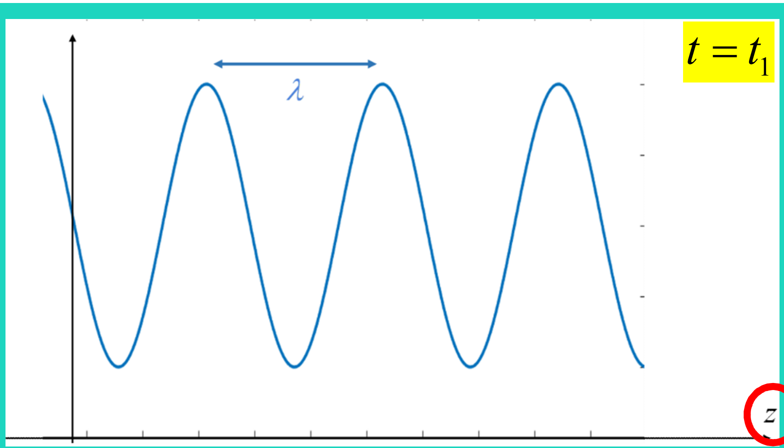
$X = \frac{1}{\nu} :$ period

$$e_x^+(z, t = t_1) = |E^+|\cos(\omega_o t_1 - \beta z + \varphi^+) = |E^+|\cos(-\omega_o t_1 + \beta z - \varphi^+)$$

$$= |E^+|\cos\left(2\pi\left[\frac{\beta}{2\pi}\right]z - \omega_o t_1 - \varphi^+\right)$$

λ : wavelength

$$\boxed{\begin{aligned} &\text{frequency}: \ \nu = \frac{\beta}{2\pi} \\ &\text{period}: \ \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0\sqrt{\mu\varepsilon}} = \frac{c}{f_0} \end{aligned}}$$



Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0) \qquad \boxed{\omega_o = 2\pi f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c} \qquad \boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad \boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

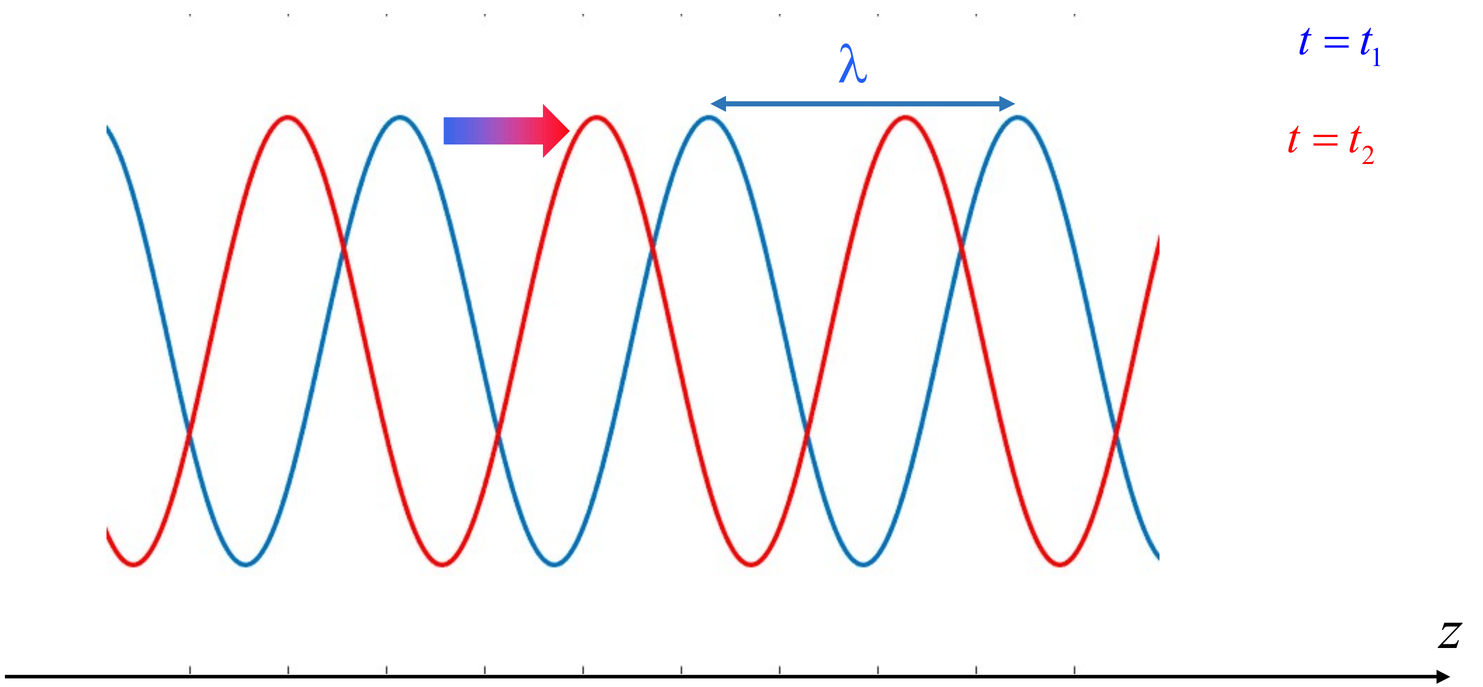
<u>Progressive</u> plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis

Plane Waves (Phasor Domain)

$$e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c}$$

$$\boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

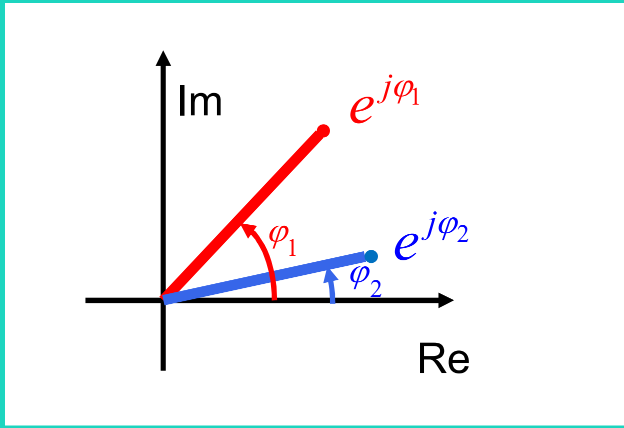
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$e^{-j\beta z} = e^{-j\frac{2\pi}{\lambda}z}$$

$$z << \lambda \Rightarrow \frac{2\pi}{\lambda}z << 2\pi \Rightarrow e^{-j\frac{2\pi}{\lambda}z} \approx 1$$



Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

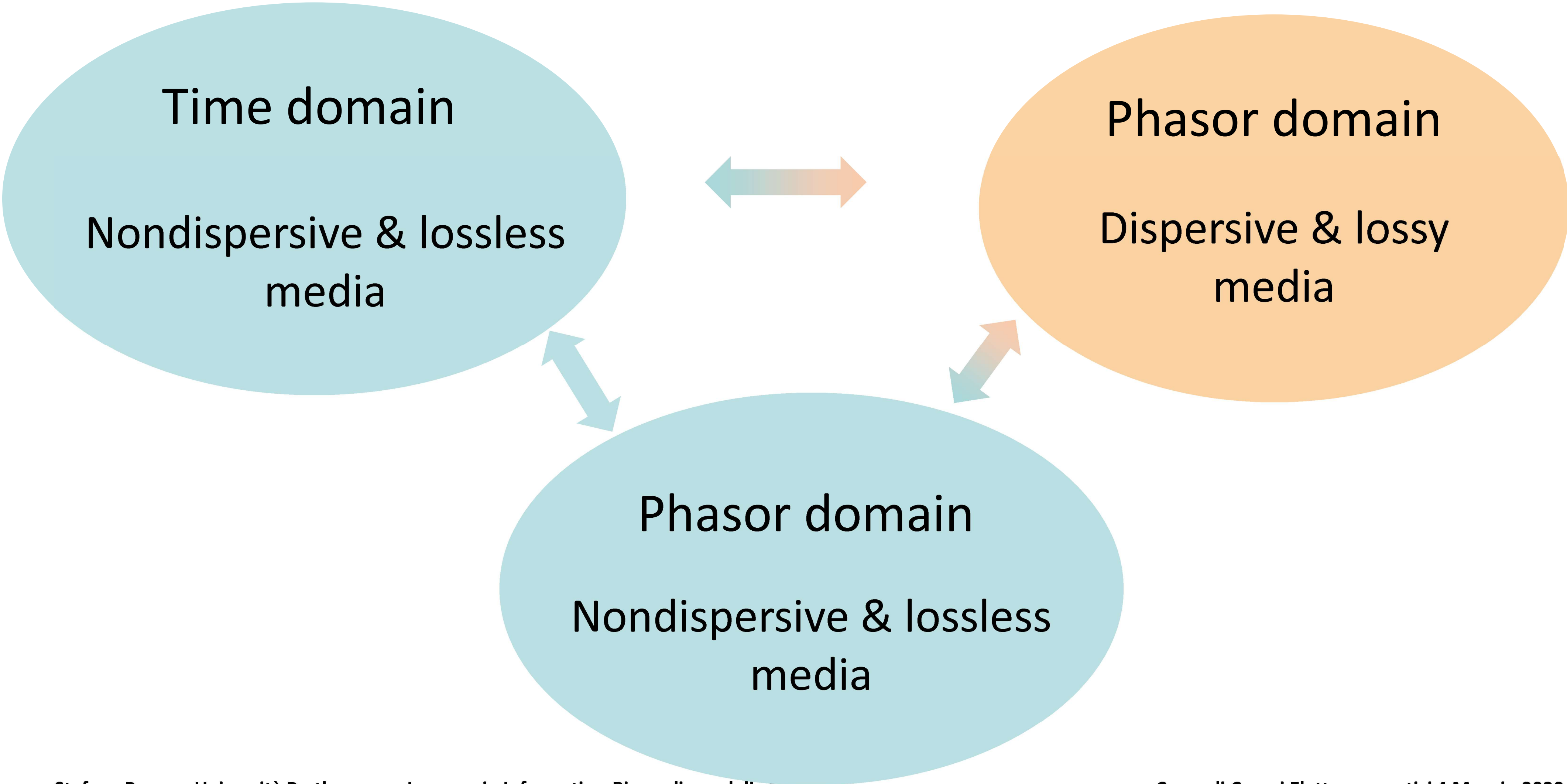
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis
- When z is small with respect to λ, the propagation effects become negligible

Rationale



Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$\boxed{v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c}$$

$$\boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

<u>Progressive</u> plane wave

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_o = 2\pi f_0$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

<u>Progressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

<u>Progressive</u> plane wave

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

$$\{E_x, H_y\}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$\cancel{k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)}$

$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$

$E_x^+(z) = E^+ e^{-j\beta z}$

$\zeta H_y^+(z) = E^+ e^{-j\beta z}$

$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$

$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$

$\omega_o = 2\pi f_0$

$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$

<u>Progressive</u> plane wave

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$

$E_z = H_z = 0$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma = 0 \end{cases}$$

$c = \frac{1}{\sqrt{\mu\varepsilon}}$

Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \cancel{= \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

$$\boxed{\cancel{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}}$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

$$\boxed{\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} = \cancel{\frac{c}{f_0}}}$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$\lambda = \frac{2\pi}{\beta} = \cancel{\frac{2\pi}{\omega_0\sqrt{\mu\varepsilon}}} = \cancel{\frac{c}{f_0}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0} v_p = \frac{v_p}{f_0}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

$$\boxed{\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}}$$

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}}$$

$$\boxed{\omega_o = 2\pi f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+| \cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$\lambda = \frac{2\pi}{\beta} = \cancel{\frac{2\pi}{\omega_0\sqrt{\mu\varepsilon}}} \cancel{= \frac{c}{f_0}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0} v_p = \frac{v_p}{f_0}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$\boxed{c = \frac{1}{\sqrt{\mu\varepsilon}}}$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ \cancel{e^{-j\beta z}}$$

$$\zeta H_y^+(z) = E^+ \cancel{e^{-j\beta z}}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o = 2\pi f_0$$

$$E^+ \cancel{e^{-j\beta z}} \rightarrow \cancel{e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)}$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$\omega_o = 2\pi f_0$$

$E^+ e^{-jkz} \quad \rightarrow$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

$$E^+ e^{-jkz} = E^+ e^{-j(\beta - j\alpha)z} = E^+ e^{(-j\beta z - \alpha z)} = E^+ e^{-j\beta z} e^{-\alpha z}$$

$$E^+ = \left|E^+\right| e^{j\varphi^+} \qquad \Rightarrow E^+ e^{-jkz} = \left|E^+\right| e^{j\varphi^+} e^{-j\beta z} e^{-\alpha z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{\left|E^+\right| e^{j\varphi^+} e^{-j\beta z} \underbrace{e^{-\alpha z}} e^{j\omega_0 t}\right\} = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

$$= e^{-\alpha z}\left|E^+\right|\cos\left(-\beta\left[z - \frac{\omega_0}{\beta}t\right] + \varphi^+\right) = e^{-\alpha z}\left|E^+\right|\cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z} \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o = 2\pi f_0$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}|E^+|\cos(\omega_o t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : real \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_o = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$\{E_y, H_x\}$$
$$\{E_x, H_y\}$$

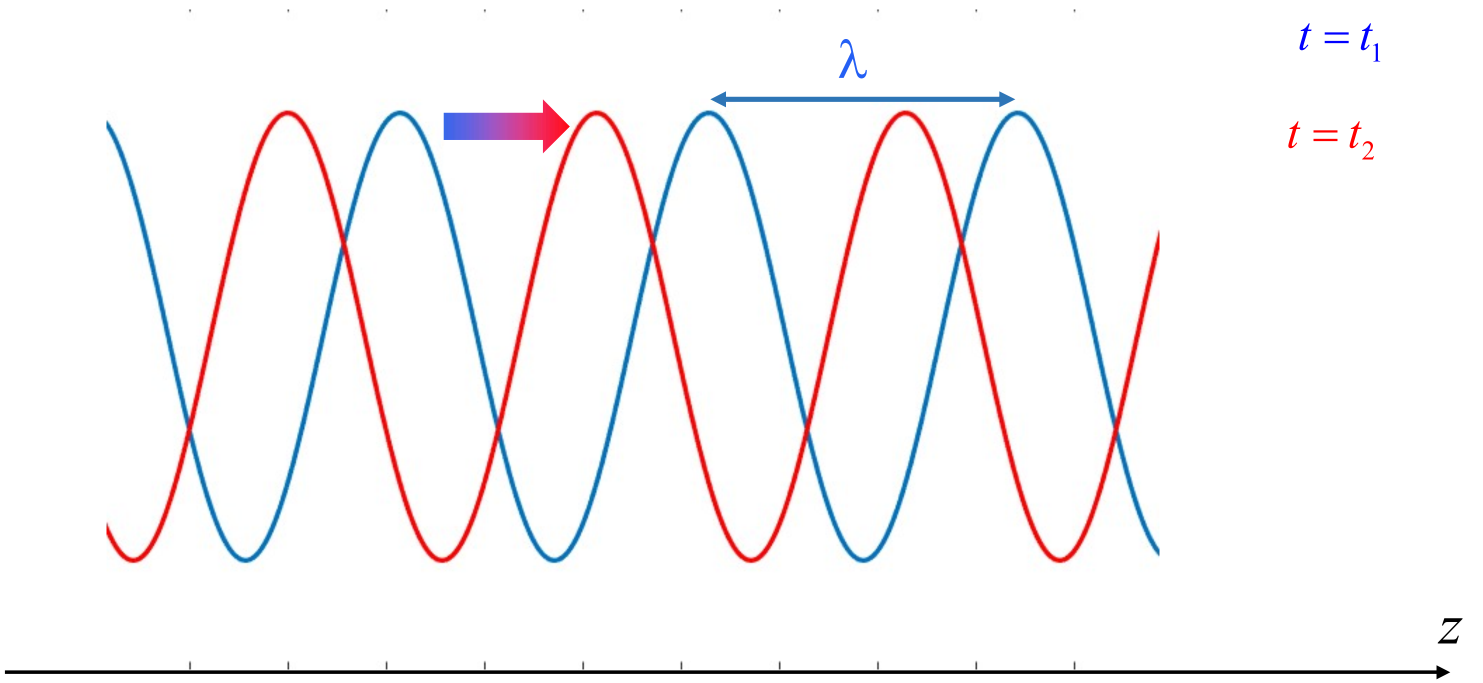
Independent each other

Plane Waves (Phasor Domain)

Time nondispersive & lossless medium

$$e_x^+(z,t) = \left|E^+\right| \cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$



Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

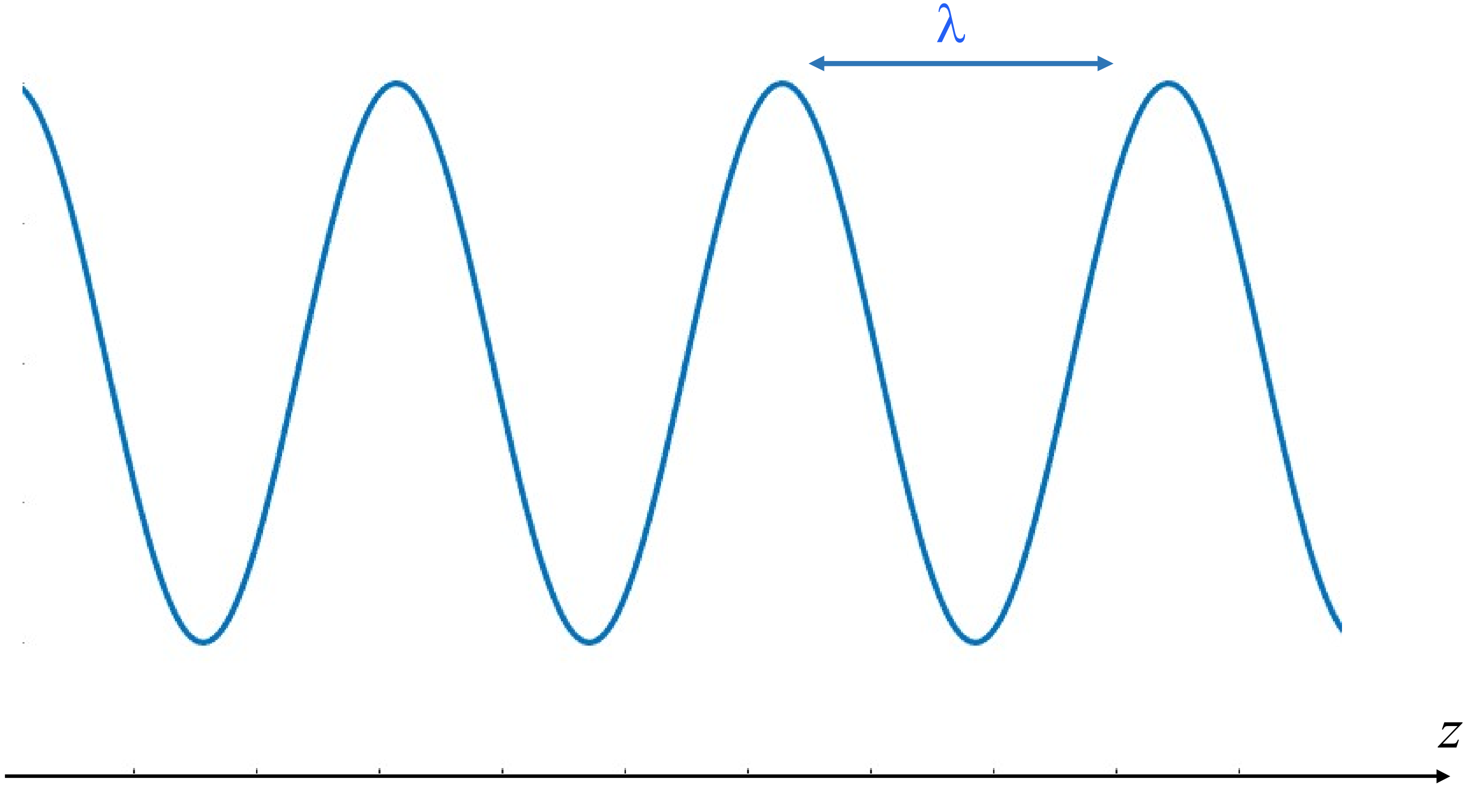
z

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_o t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$



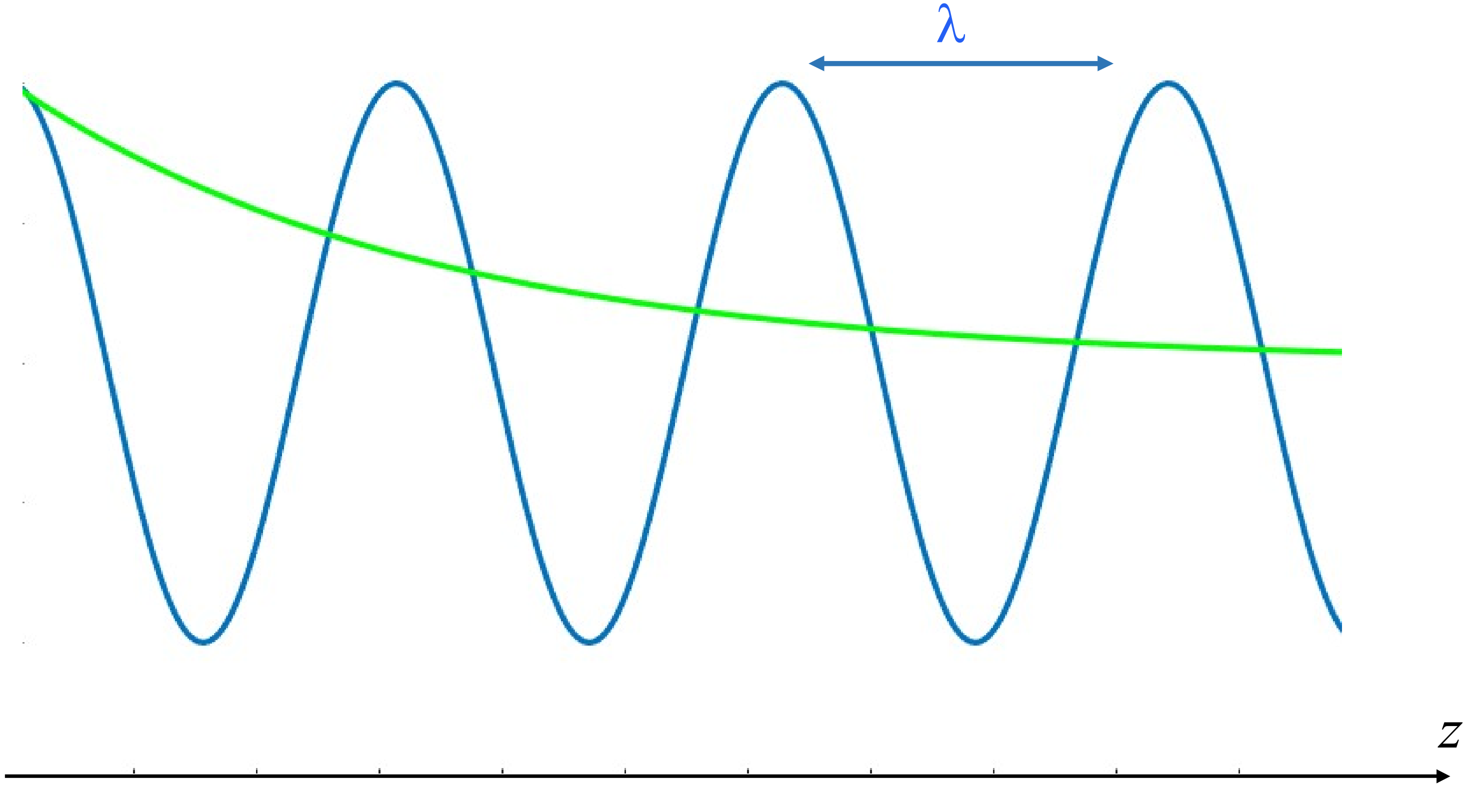
$$\cos(\omega_o t - \beta z + \varphi^+)$$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$



$$\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

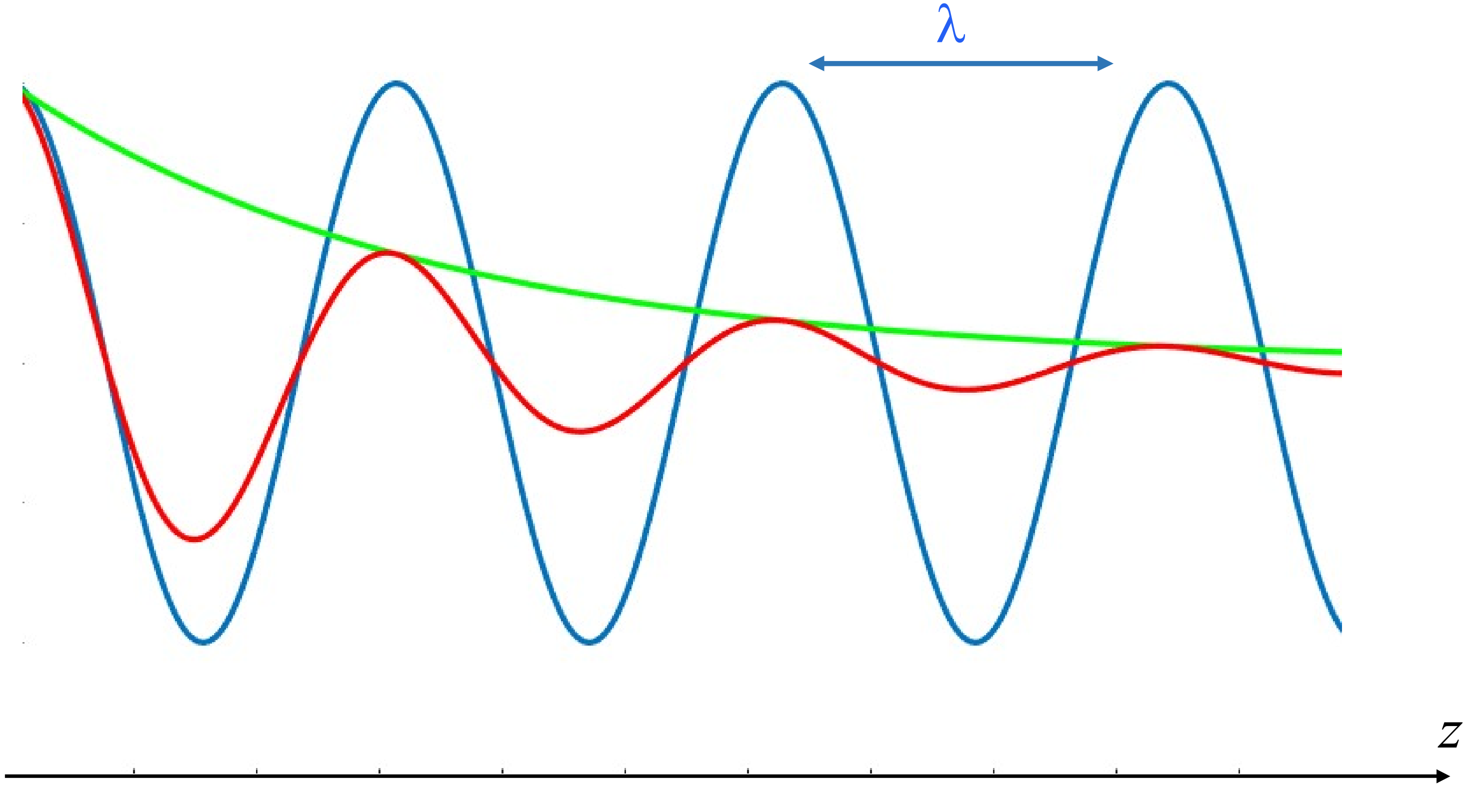
$$e^{-\alpha z}$$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$



$\cos\left(\omega_o t - \beta z + \varphi^+\right)$

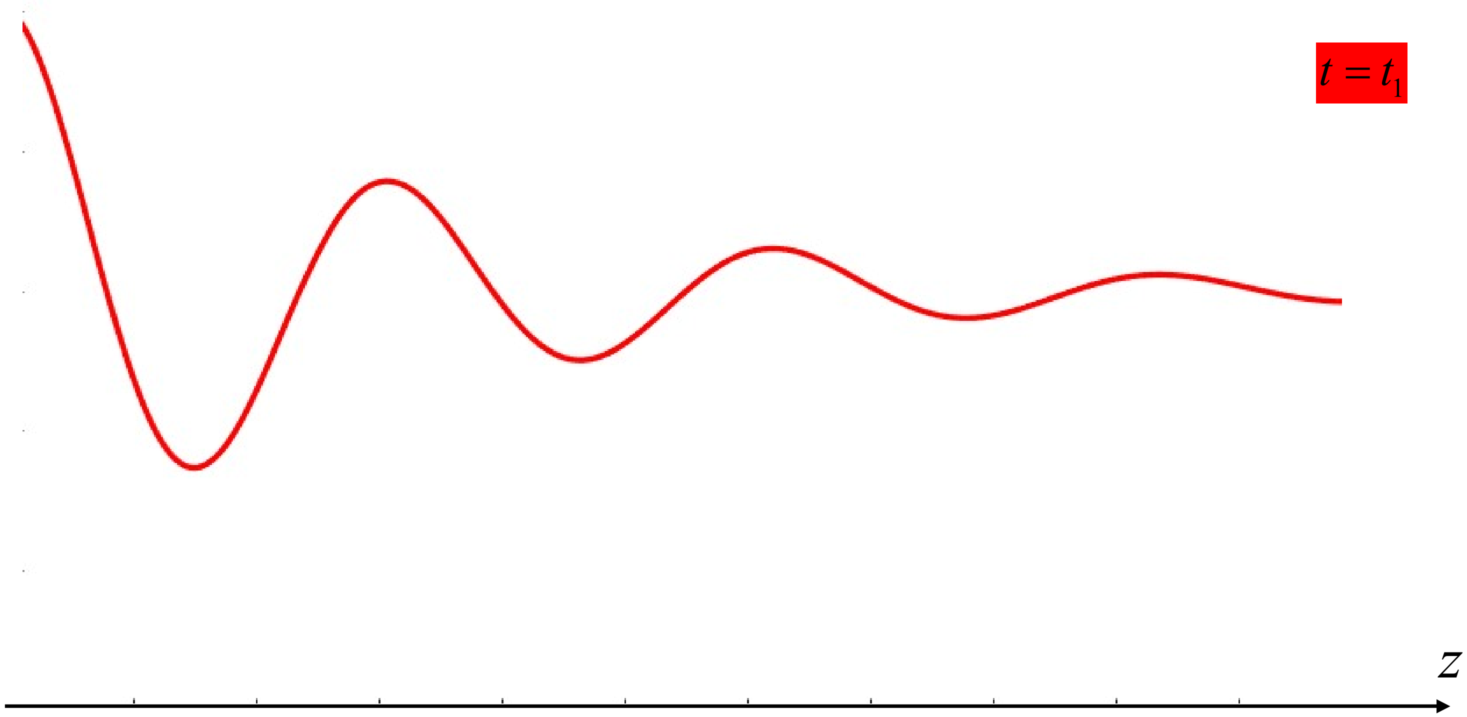
$e^{-\alpha z}$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$

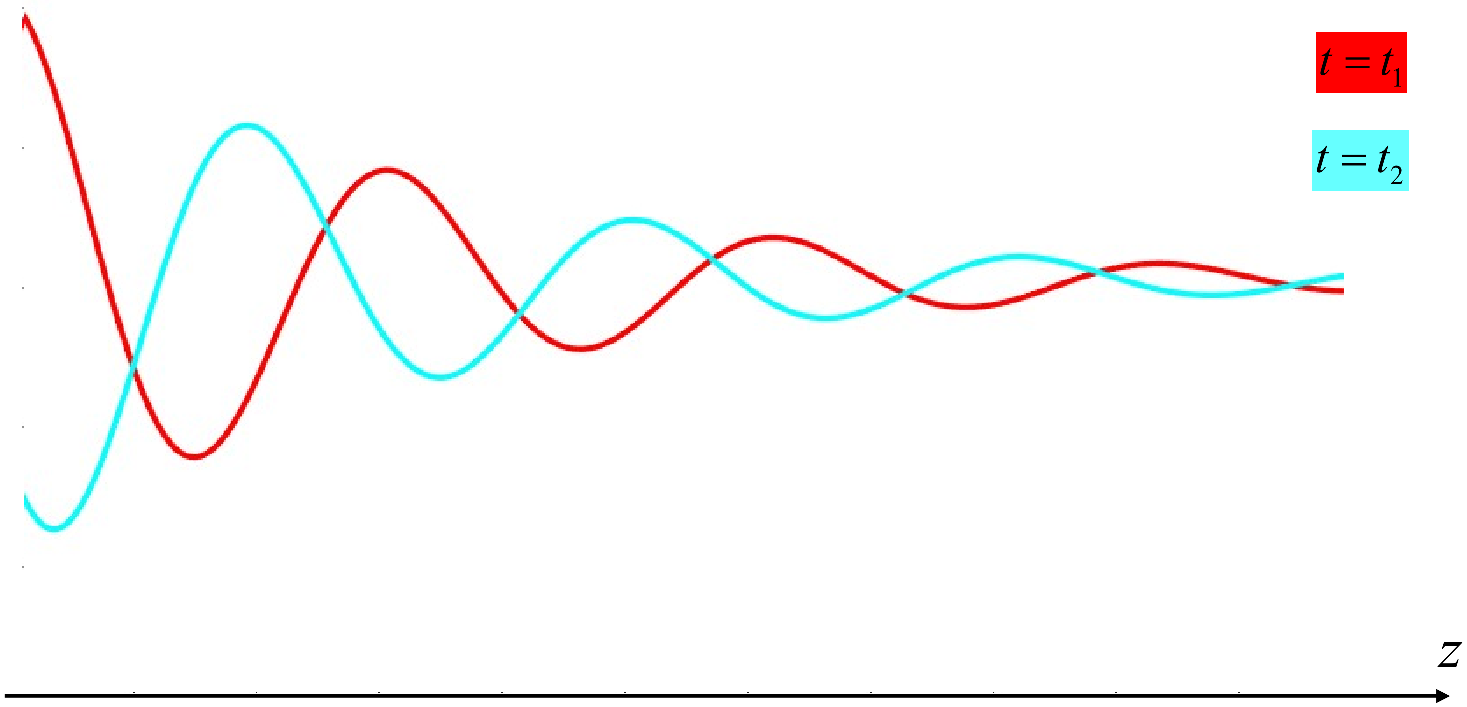


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = \left|E^+\right| e^{-\alpha z} \cos\left(\omega_o t - \beta z + \varphi^+\right) = e^{-\alpha z} e_x^+\left(z - v_p t\right)$$

$$\boxed{v_p = \frac{\omega_0}{\beta}}$$



Plane Waves (Phasor Domain)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$E_z=H_z=0$$

$$\{E_y,H_x\}$$
$$\{E_x,H_y\}$$

Independent each other

$$\{E_x,H_y\}$$

$$E_x^+(z)=E^+e^{-jkz}$$
$$\zeta H_y^+(z)=E^+e^{-jkz}$$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$\lambda=\frac{2\pi}{\beta}=\frac{v_p}{f_0}$$

$$v_p=\frac{\omega_0}{\beta}$$

$$\zeta(\omega_0)=\sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_o=2\pi f_0$$

$$E^+e^{-jkz}\quad\rightarrow\quad e^+(z,t)=e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$E_x^+(z)=E^+e^{-j\beta z}$$
$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

$$\omega_o=2\pi f_0$$

$$v_p=\frac{\omega_0}{\beta}$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda=\frac{2\pi}{\beta}=\frac{v_p}{f_0}$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon:real\\ \mu:real\\ \sigma=0\end{cases}$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+e^{-j\beta z}\ \rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+\left(z-v_p t\right)$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$
$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$E^+ e^{-jkz} \quad \rightarrow \quad e^+(z,t) = e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \ real \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+\left(z - v_p t\right)$$

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0)=\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}=\beta(\omega_0)-j\alpha(\omega_0)$$

$$E_x^+(z)=E^+e^{-jkz}$$
$$\zeta H_y^+(z)=E^+e^{-jkz}$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\mathrm{Re}\left\{\omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=\frac{1}{\mathrm{Re}\left\{\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}\right\}}=v_p(\omega_0)$$

$$E^+e^{-jkz}\quad\rightarrow\quad e^+(z,t)=e^{-\alpha z}\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)$$

Time dispersive (lossy)

$$\begin{cases}\varepsilon(\omega_0)=\varepsilon_1(\omega_0)-j\varepsilon_2(\omega_0)\\ \mu=\mu_1(\omega_0)-j\mu_2(\omega_0)\\ \sigma:\ real\end{cases}$$

$$E_x^+(z)=E^+e^{-j\beta z}$$
$$\zeta H_y^+(z)=E^+e^{-j\beta z}$$

$$k(\omega_0)=\omega_0\sqrt{\mu\varepsilon}=\beta(\omega_0)$$

$$v_p=\frac{\omega_0}{\beta}=\frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}}=\frac{1}{\sqrt{\mu\varepsilon}}=c$$

Time nondispersive & lossless

$$\begin{cases}\varepsilon:real\\ \mu:real\\ \sigma=0\end{cases}$$

$$\boxed{c=\frac{1}{\sqrt{\mu\varepsilon}}}$$

$$E^+e^{-j\beta z}\rightarrow e_x^+(z,t)=\left|E^+\right|\cos\left(\omega_o t-\beta z+\varphi^+\right)=e_x^+\left(z-v_p t\right)$$

Source-free

Medium
- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- **Lossy**

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

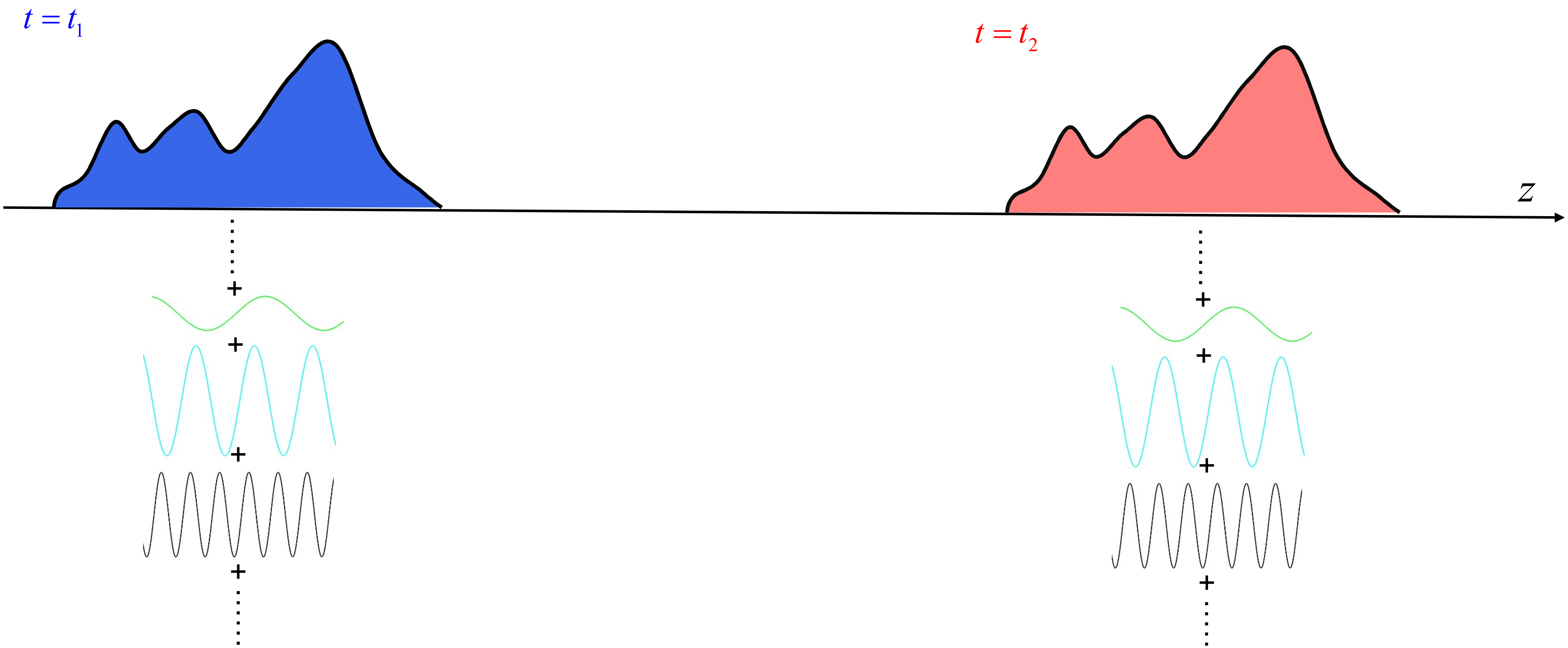
$$E_z=H_z=0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves

Time nondispersive & lossless medium

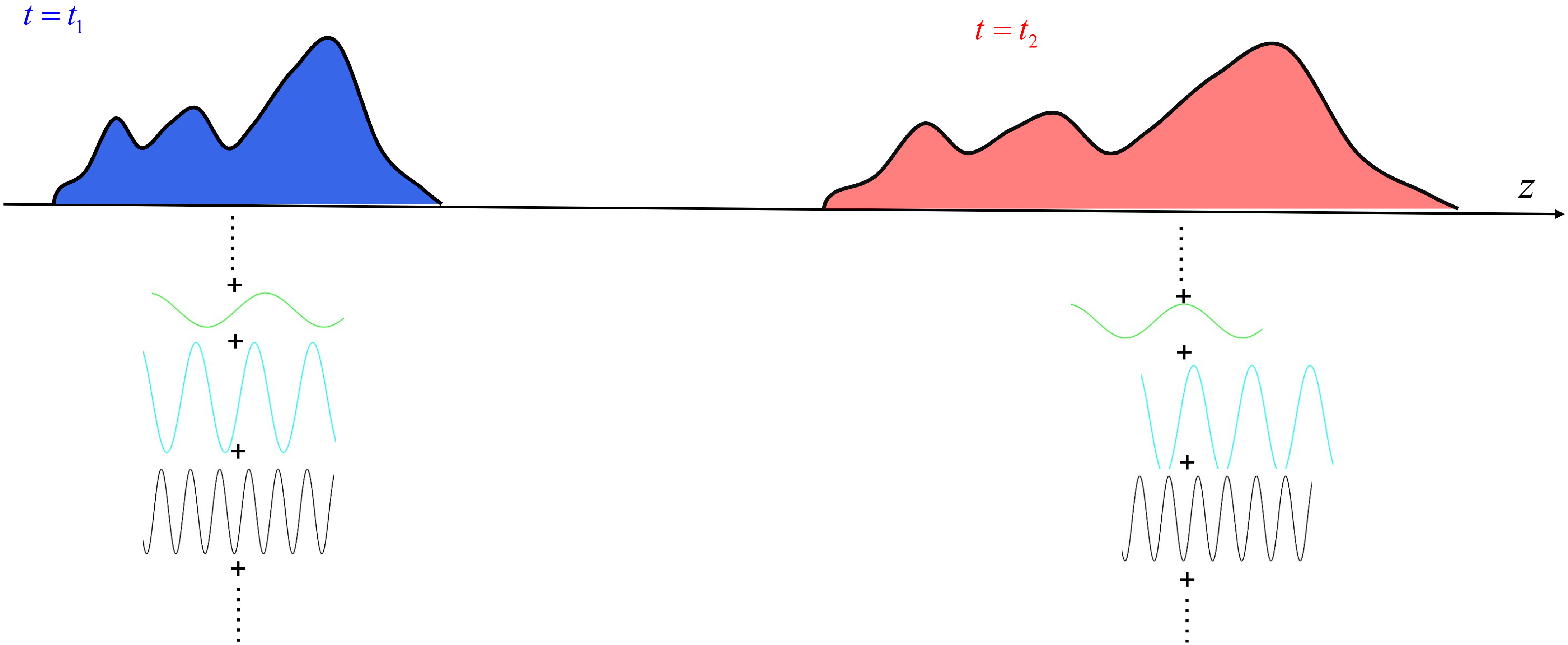
$\boxed{v_p = c}$



Plane Waves

Time dispersive medium

$$\boxed{v_p = v_p(\omega_0)}$$



Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence