Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

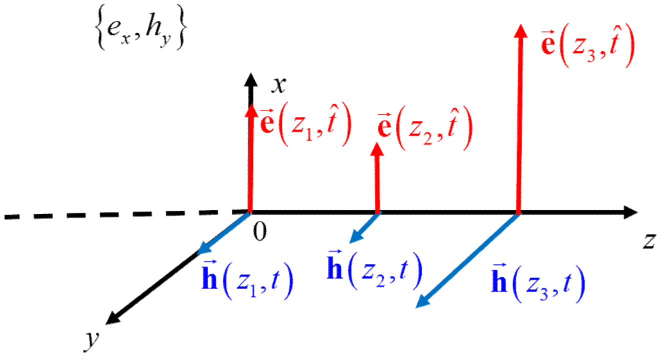
Very important for the discussion

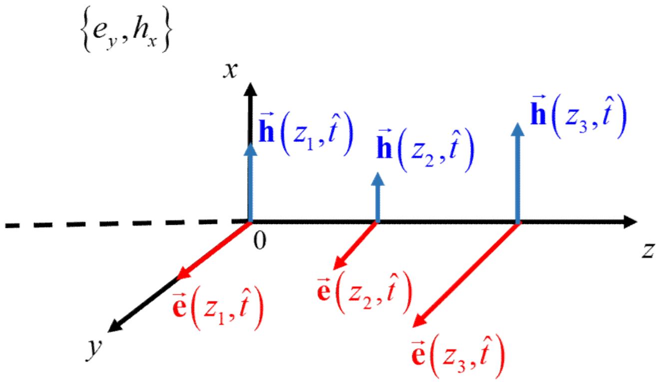
Memo

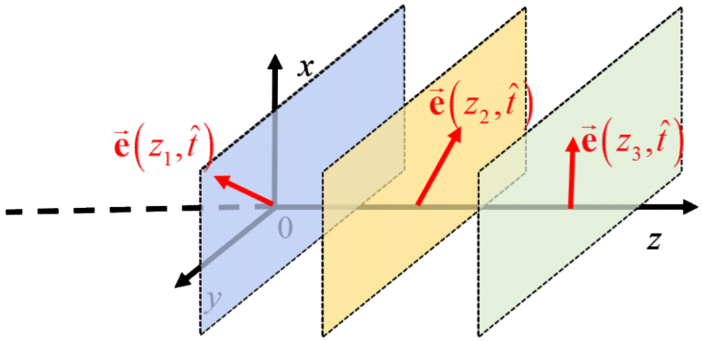
Mathematical tools to be exploited

Mathematics

Plane Waves (TD)







Source-free

Medium

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

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$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

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⬇

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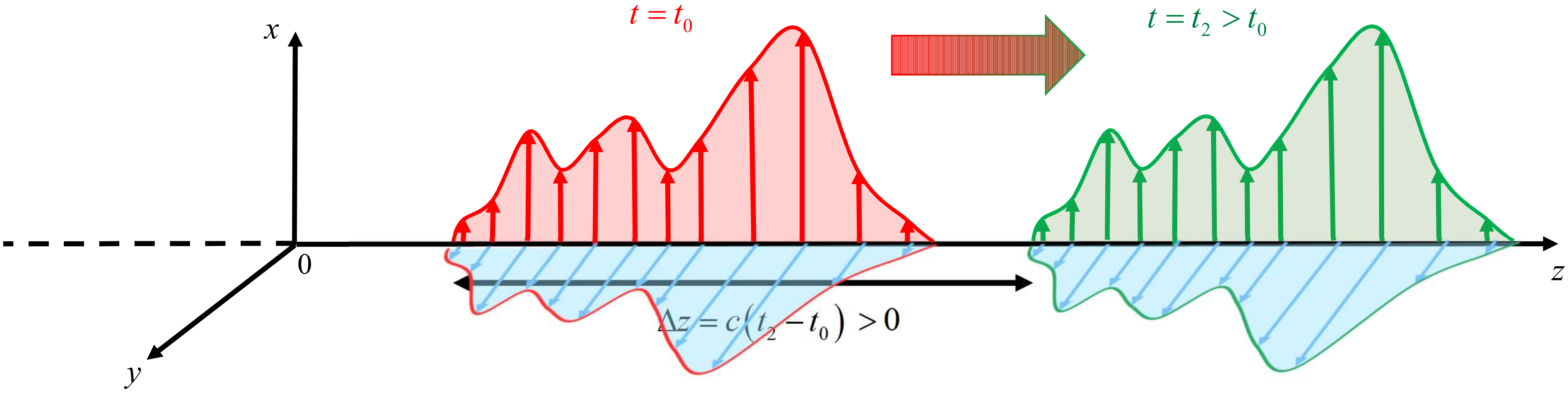
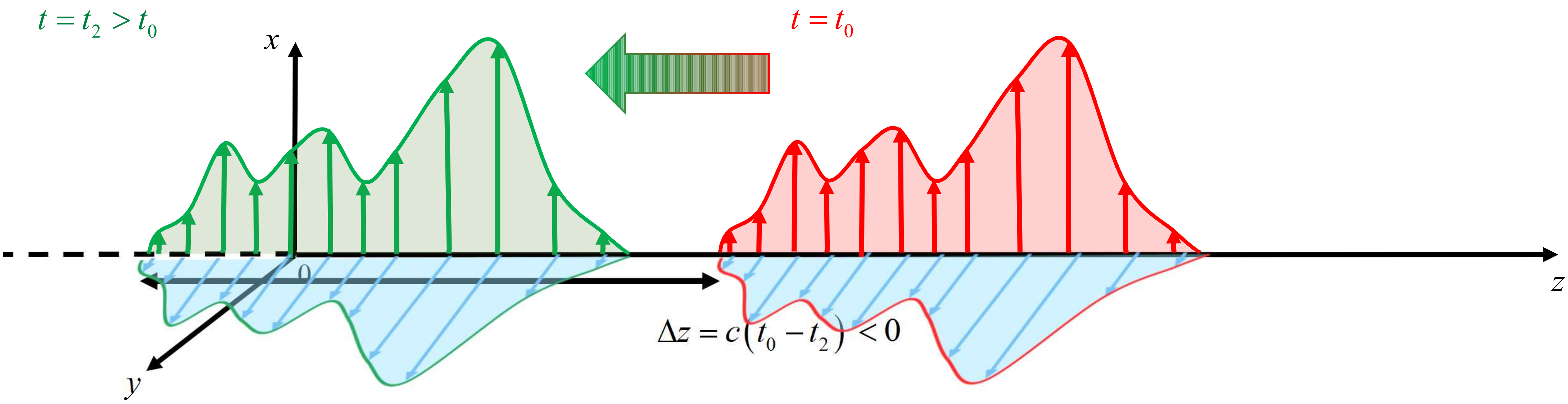


The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the *z*-axis

$$\begin{cases} e^{+}\left(z-ct\right) \\ h^{+}\left(z-ct\right) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^-(z+ct) \\ h^-(z+ct) \end{cases}$$ is referred to as electromagnetic **regressive plane wave**

Plane Waves (TD)

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$\{e_x, h_y\}$

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$$\Downarrow$$

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$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z$$

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Independent each other

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{s}}^+ = \vec{\mathbf{e}}^+ \times \vec{\mathbf{h}}^+ = e_x^+\hat{i}_x \times h_y^+\hat{i}_y = e_x^+\hat{i}_x \times \frac{e_x^+}{\zeta}\hat{i}_y = \frac{\left|e_x^+\right|^2}{\zeta}\left(\hat{i}_x \times \hat{i}_y\right) = \frac{\left|e_x^+\right|^2}{\zeta}\hat{i}_z$$

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$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{s}}^- = \vec{\mathbf{e}}^- \times \vec{\mathbf{h}}^- = e_x^- \hat{i}_x \times h_y^- \hat{i}_y = e_x^- \hat{i}_x \times \left(-\frac{e_x^-}{\zeta}\right)\hat{i}_y = -\frac{\left|e_x^-\right|^2}{\zeta}\left(\hat{i}_x \times \hat{i}_y\right) = -\frac{\left|e_x^-\right|^2}{\zeta}\hat{i}_z$$

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Plane Waves (TD)

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$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

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$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \qquad \hat{i}_y = \hat{i}_z \times \hat{i}_x \qquad \hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

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$\{e_x, h_y\}$

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$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In al these 4 cases the Poynting vector is directed along the direction of propagation

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

$$e_z(z,t)=h_z(z,t)=0$$

$\{e_y,h_x\}$
$\{e_x,h_y\}$ **Independent each other**

$\{e_x,h_y\}$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\[2mm]\dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

$$\begin{cases}e_x^+(z,t)=e_x^+(z-ct)\\ \zeta h_y^+(z,t)=e_x^+(z-ct)\end{cases}\qquad \{e_x^+,h_y^+\}$$

$$\begin{cases}e_x^-(z,t)=e_x^-(z+ct)\\ \zeta h_y^-(z,t)=-e_x^-(z+ct)\end{cases}\qquad \{e_x^-,h_y^-\}$$

$$\vec{\mathbf{s}}^+=\frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z\qquad \vec{\mathbf{s}}^-=-\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y,h_x\}$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\[2mm]\dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases}\qquad \frac{\partial^2 e_y}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2}=0$$

$$\begin{cases}e_y^+(z,t)=e_y^+(z-ct)\\ \zeta h_x^+(z,t)=-e_y^+(z-ct)\end{cases}\qquad \{e_y^+,h_x^+\}$$

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$$\zeta\vec{\mathbf{h}}=\hat{i}_p\times\vec{\mathbf{e}}$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

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$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$\{e_y, h_x\}$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

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$$\vec{\mathbf{s}}^+ = \frac{|e_y^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_y^-(z+ct)|^2}{\zeta}\hat{i}_z$$

Source-free

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$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$\{e_x^+, h_y^+\}$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^+ \hat{i}_y = e_x^+ \hat{i}_y$$

$$\hat{i}_p = \hat{i}_z \,;\, \vec{\mathbf{e}} = e_x^+ \hat{i}_x \;\Rightarrow\; \hat{i}_p \times \vec{\mathbf{e}} = \hat{i}_z \times e_x^+ \hat{i}_x = e_x^+ (\hat{i}_z \times \hat{i}_x) = e_x^+ \hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y \qquad \hat{i}_y = \hat{i}_z \times \hat{i}_x \qquad \hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

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$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

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$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

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$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

$\{e_x^-, h_y^-\}$

$$\zeta\vec{\mathbf{h}} = \zeta h_y^- \hat{i}_y = -e_x^- \hat{i}_y$$

$$\hat{i}_p = -\hat{i}_z \,;\, \vec{\mathbf{e}} = e_x^- \hat{i}_x \Rightarrow \hat{i}_p \times \vec{\mathbf{e}} = -\hat{i}_z \times e_x^- \hat{i}_x = -e_x^-(\hat{i}_z \times \hat{i}_x) = -e_x^- \hat{i}_y$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

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$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

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$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

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$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In al these 4 cases the electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ points to the propagation direction

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_x^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_x^-(z+ct)|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta |h_y^+(z-ct)|^2 \hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases} \qquad \{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases} \qquad \{e_y^-, h_x^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{|e_y^+(z-ct)|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{|e_y^-(z+ct)|^2}{\zeta}\hat{i}_z$$

$$|e_x^+(z-ct)| = \zeta |h_y^+(z-ct)|$$

$$\vec{\mathbf{s}}^+ = \frac{|e_x^+(z-ct)|^2}{\zeta}\hat{i}_z = \zeta |h_y^+(z-ct)|^2 \hat{i}_z$$

Source-free

Medium

- **Linear**
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_x^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_x^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left|h_y^+(z-ct)\right|^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left|h_y^-(z+ct)\right|^2\hat{i}_z$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

$$\vec{\mathbf{s}}^+ = \frac{\left|e_y^+(z-ct)\right|^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left|e_y^-(z+ct)\right|^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left|h_x^+(z-ct)\right|^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left|h_x^-(z+ct)\right|^2\hat{i}_z$$

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$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (TD)

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta \vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
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- **Homogeneous (TI – SI)**
- **Lossless**

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$
Independent each other

Plane Waves (Spectral Domains)

Source-free

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- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

Plane Waves (Spectral Domains)

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$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

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Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

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Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

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PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

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Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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Plane Waves (Spectral Domains)

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- **Space non-dispersive**
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Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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Plane Waves (Spectral Domains)

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Medium
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Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) + \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0 \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) + \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = 0 \\ \nabla\cdot\mu\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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Fourier domain (FD)

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$$\varepsilon_{eq} = \varepsilon\left[1-\frac{j\sigma}{\omega\varepsilon}\right]$$

$$\varepsilon_{eq} = \varepsilon\left[1-\frac{j\sigma}{\omega_0\varepsilon}\right]$$

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- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega \mu \vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega \varepsilon \vec{\mathbf{E}}(z,\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \end{cases}$$

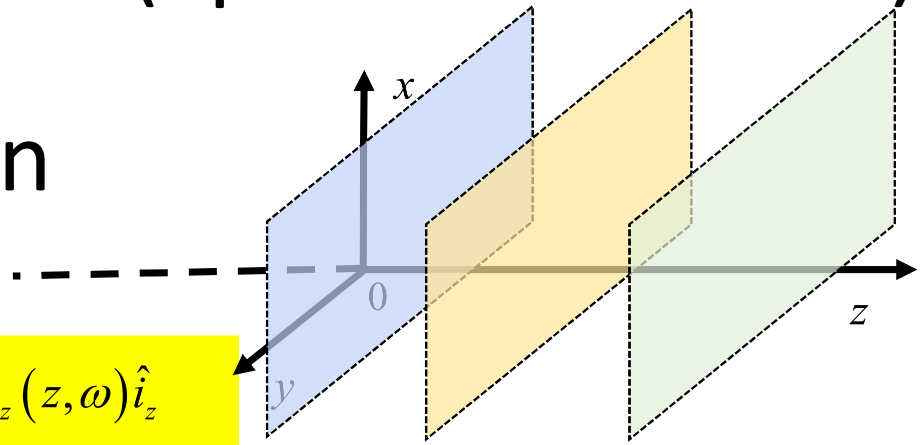
FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

Plane Waves (Spectral Domains)

Fourier Domain



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

Fourier domain (FD)

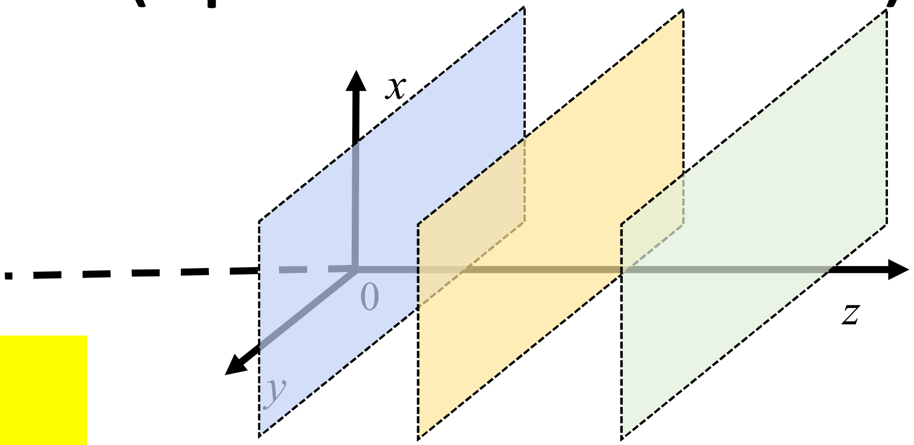
$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega\mu\vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega\varepsilon\vec{\mathbf{E}}(z,\omega) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

Phasor Domain



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y + \cancel{E_z\hat{i}_z}$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y + \cancel{H_z\hat{i}_z}$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$E_z = 0 \qquad H_z = 0$$

$$\Rightarrow \quad e_z(z,t) = 0 \qquad h_z(z,t) = 0$$

$\Rightarrow$ **TEM fields**

Source-free

Medium
- **Linear**
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- **Space non-dispersive**
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- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

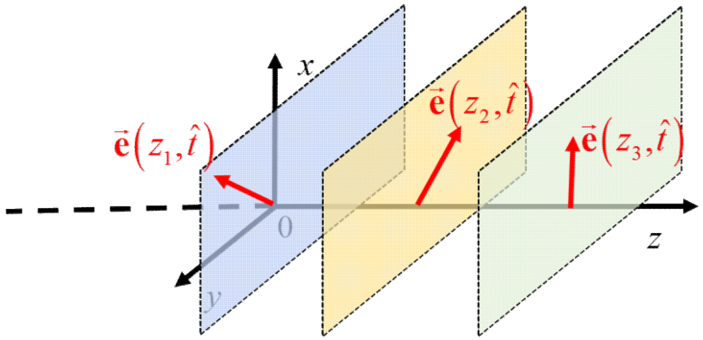
$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$



Source-free

Medium

- **Linear**
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- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x \hat{i}_x + E_y \hat{i}_y$$

$$\vec{\mathbf{H}} = H_x \hat{i}_x + H_y \hat{i}_y$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$\{E_y, H_x\}$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \Rightarrow \frac{d^2E_x}{dz^2} = -j\omega\mu\frac{dH_y}{dz} = -\omega^2\mu\varepsilon E_x \Rightarrow \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \Rightarrow \frac{d^2E_y}{dz^2} = j\omega\mu\frac{dH_x}{dz} = -\omega^2\mu\varepsilon E_y \Rightarrow \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

k : (complex) propagation constant

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

Source-free

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

Source-free

Medium

- Linear
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$: **Independent each other**

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

Source-free

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↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other