

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves (TD)

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) + \rho_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

Time domain - Differential form

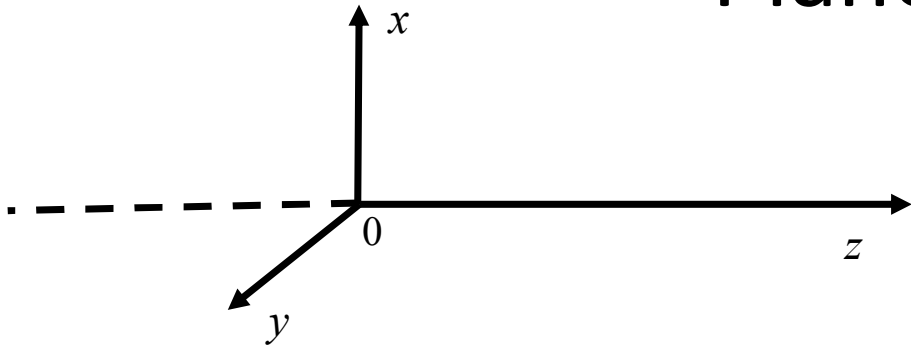
$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \\ \sigma = 0 \end{cases}$$

Plane Waves (TD)

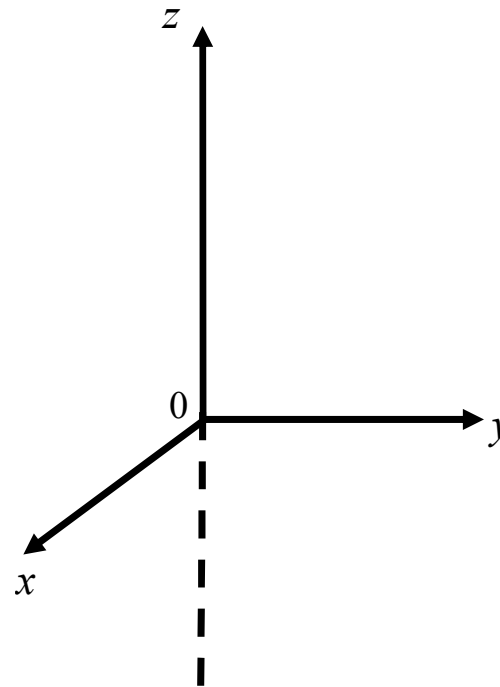


Medium

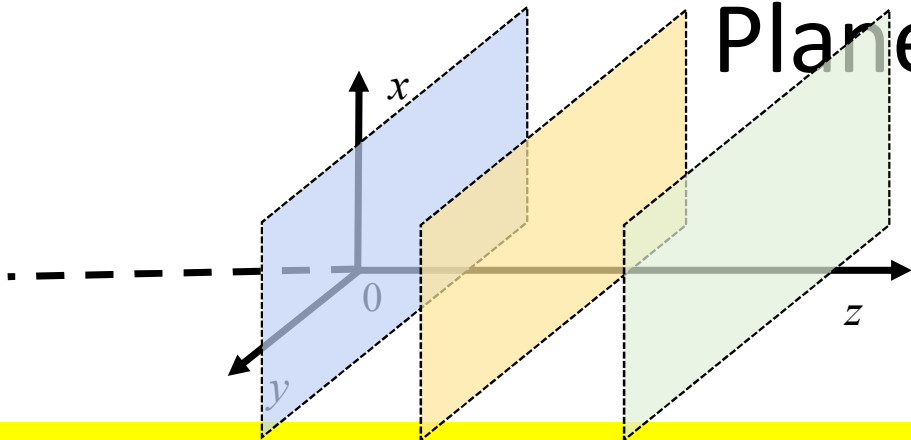
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Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$



Plane Waves (TD)



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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(\cancel{x}, \cancel{y}, z, t)\hat{i}_x + e_y(\cancel{x}, \cancel{y}, z, t)\hat{i}_y + e_z(\cancel{x}, \cancel{y}, z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(\cancel{x}, \cancel{y}, z, t)\hat{i}_x + h_y(\cancel{x}, \cancel{y}, z, t)\hat{i}_y + h_z(\cancel{x}, \cancel{y}, z, t)\hat{i}_z$$

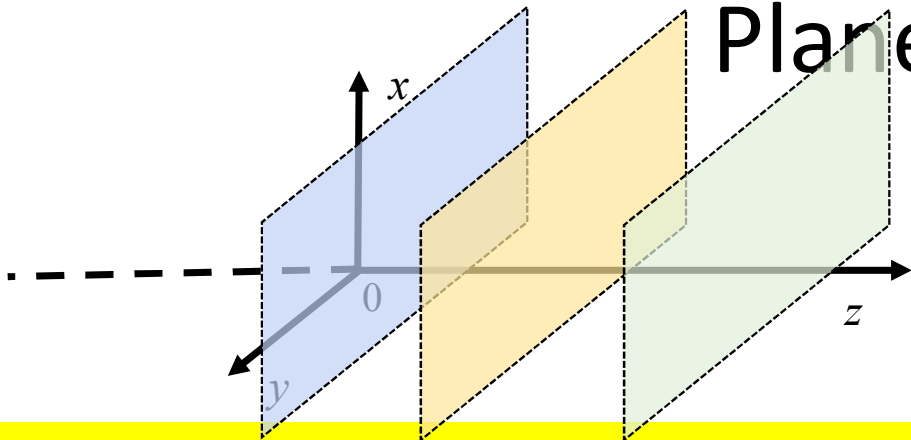
Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

$$\nabla \times \vec{\mathbf{e}} = \left(\frac{\partial e_z}{\partial y} - \frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial e_x}{\partial z} - \frac{\partial e_z}{\partial x} \right) \hat{i}_y + \left(\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} \right) \hat{i}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(\frac{\partial h_z}{\partial y} - \frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial h_x}{\partial z} - \frac{\partial h_z}{\partial x} \right) \hat{i}_y + \left(\frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y} \right) \hat{i}_z$$

Plane Waves (TD)



Source-free

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y + e_z(z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y + h_z(z, t)\hat{i}_z$$

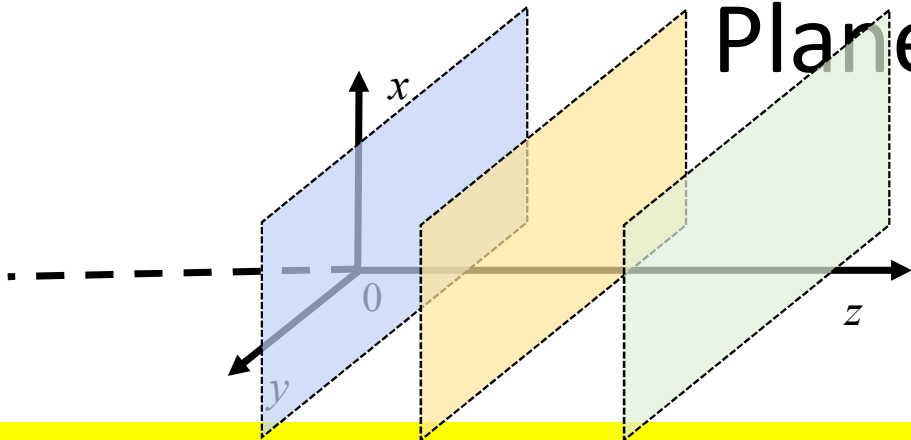
Time domain - Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial e_x}{\partial z} \right) \hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial h_x}{\partial z} \right) \hat{i}_y$$

Plane Waves (TD)



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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y + e_z(z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y + h_z(z, t)\hat{i}_z$$

Time domain - Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} \end{array} \right.$$

$$\varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = 0$$

$$\mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial e_x}{\partial z} \right) \hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial h_x}{\partial z} \right) \hat{i}_y$$

Plane Waves (TD)

$$\nabla \times \vec{e}(\vec{r}, t) = -\mu \frac{\partial \vec{h}(\vec{r}, t)}{\partial t}$$

$$\nabla \times \vec{h}(\vec{r}, t) = \varepsilon \frac{\partial \vec{e}(\vec{r}, t)}{\partial t}$$

$$\vec{e}(\vec{r}, t) = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y + \cancel{e_z(z, t)\hat{i}_z}$$

$$\vec{h}(\vec{r}, t) = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y + \cancel{h_z(z, t)\hat{i}_z}$$

$$\nabla \times \vec{e} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$-\mu \frac{\partial \vec{h}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{i}_x - \mu \frac{\partial h_y}{\partial t}\hat{i}_y - \mu \frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla \times \vec{h} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon \frac{\partial \vec{e}(\vec{r}, t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{i}_z$$

$$\frac{\partial e_z}{\partial t} = 0$$

$$\frac{\partial h_z}{\partial t} = 0$$



$$e_z(z, t) = \text{const}$$

$$h_z(z, t) = \text{const}$$



$$e_z(z, t) = 0$$

$$h_z(z, t) = 0$$



TEM fields

Source-free

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

Plane Waves (TD)

$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t}$$

$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(z, t) \hat{\mathbf{i}}_x + e_y(z, t) \hat{\mathbf{i}}_y$$

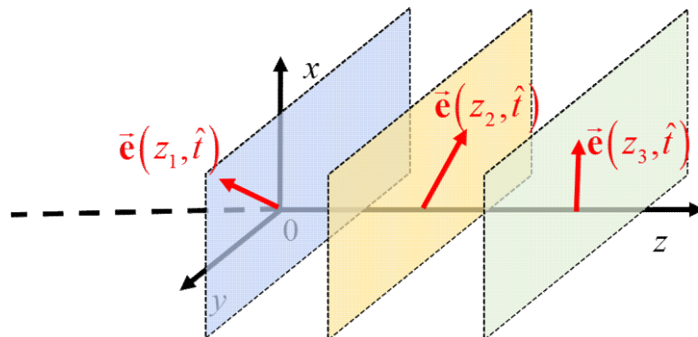
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(z, t) \hat{\mathbf{i}}_x + h_y(z, t) \hat{\mathbf{i}}_y$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z} \right) \hat{\mathbf{i}}_x + \left(\frac{\partial e_x}{\partial z} \right) \hat{\mathbf{i}}_y$$

$$-\mu \frac{\partial \vec{\mathbf{h}}}{\partial t} = -\mu \frac{\partial h_x}{\partial t} \hat{\mathbf{i}}_x - \mu \frac{\partial h_y}{\partial t} \hat{\mathbf{i}}_y - \mu \frac{\partial h_z}{\partial t} \hat{\mathbf{i}}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z} \right) \hat{\mathbf{i}}_x + \left(\frac{\partial h_x}{\partial z} \right) \hat{\mathbf{i}}_y$$

$$\varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t} \hat{\mathbf{i}}_x + \varepsilon \frac{\partial e_y}{\partial t} \hat{\mathbf{i}}_y + \varepsilon \frac{\partial e_z}{\partial t} \hat{\mathbf{i}}_z$$



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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

Plane Waves (TD)

$$\nabla \times \vec{e}(\vec{r}, t) = -\mu \frac{\partial \vec{h}(\vec{r}, t)}{\partial t}$$

$$\nabla \times \vec{h}(\vec{r}, t) = \varepsilon \frac{\partial \vec{e}(\vec{r}, t)}{\partial t}$$

$$\vec{e}(\vec{r}, t) = e_x(z, t) \hat{i}_x + e_y(z, t) \hat{i}_y$$

$$\vec{h}(\vec{r}, t) = h_x(z, t) \hat{i}_x + h_y(z, t) \hat{i}_y$$

$$\nabla \times \vec{e} = \left(-\frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial e_x}{\partial z} \right) \hat{i}_y$$

$$-\mu \frac{\partial \vec{h}}{\partial t} = -\mu \frac{\partial h_x}{\partial t} \hat{i}_x - \mu \frac{\partial h_y}{\partial t} \hat{i}_y - \mu \frac{\partial h_z}{\partial t} \hat{i}_z$$

$$\nabla \times \vec{h} = \left(-\frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left(\frac{\partial h_x}{\partial z} \right) \hat{i}_y$$

$$\varepsilon \frac{\partial \vec{e}(\vec{r}, t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t} \hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t} \hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t} \hat{i}_z$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t}$$

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$\{e_y, h_x\}$
 $\{e_x, h_y\}$ Independent each other

Plane Waves (TD)

$$\nabla \times \vec{e}(\vec{r}, t) = -\mu \frac{\partial \vec{h}(\vec{r}, t)}{\partial t}$$

$$\nabla \times \vec{h}(\vec{r}, t) = \varepsilon \frac{\partial \vec{e}(\vec{r}, t)}{\partial t}$$

$$\vec{e}(\vec{r}, t) = e_x(z, t) \hat{i}_x + e_y(z, t) \hat{i}_y$$

$$\vec{h}(\vec{r}, t) = h_x(z, t) \hat{i}_x + h_y(z, t) \hat{i}_y$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t}$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t}$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t}$$

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$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$\{e_y, h_x\}$
 $\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

$$\vec{e}(\vec{r}, t) = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y$$

$$\vec{h}(\vec{r}, t) = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y$$

$$\left\{ \begin{array}{l} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{array} \right.$$

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

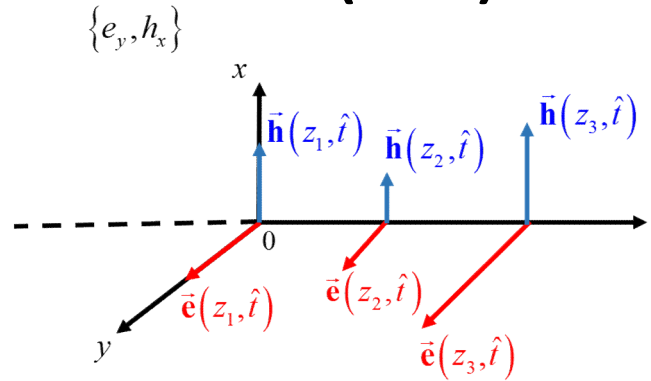
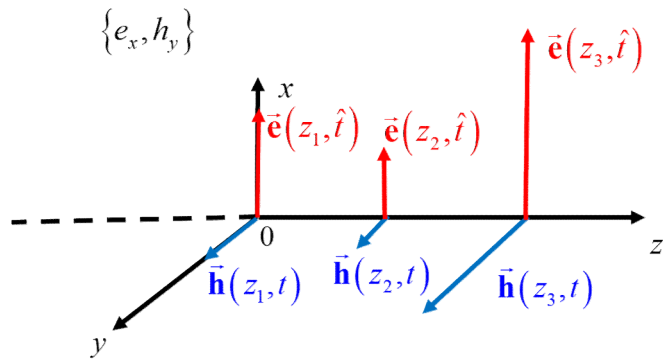
$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

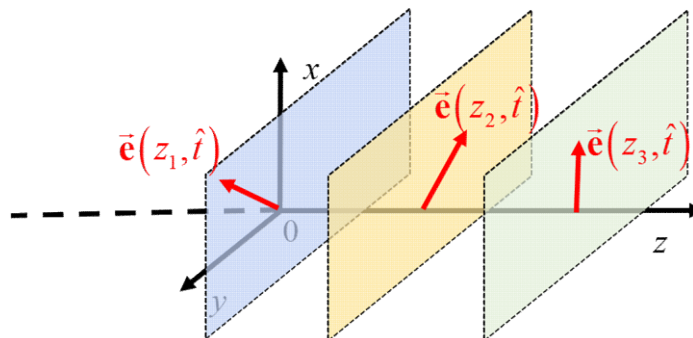
$\{e_y, h_x\}$
 $\{e_x, h_y\}$ Independent each other

Plane Waves (TD)



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial e_y}{\partial t} \end{cases}$$



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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

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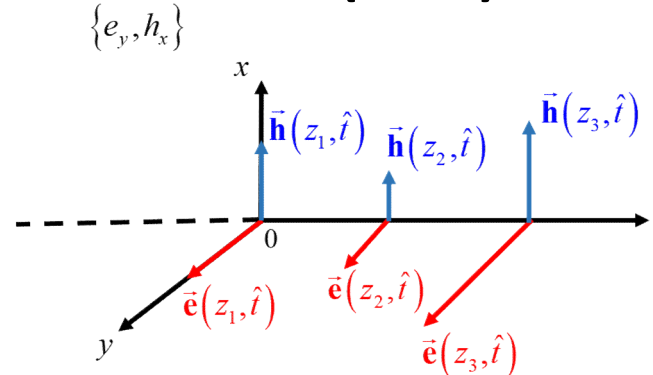
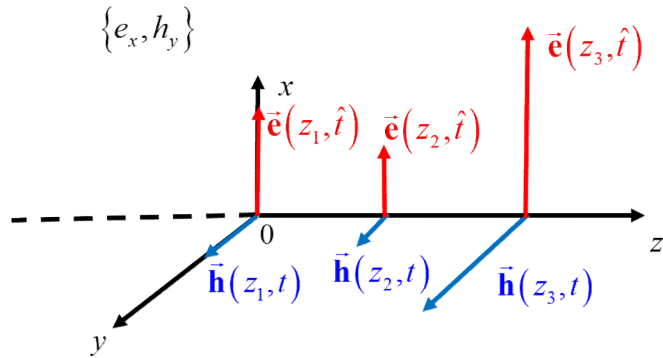


$$e_z(z, t) = h_z(z, t) = 0$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent
each other

Plane Waves (TD)



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases} \Rightarrow \begin{cases} \frac{\partial^2 e_x}{\partial z^2} = -\mu \frac{\partial}{\partial z} \frac{\partial h_y}{\partial t} = -\mu \frac{\partial}{\partial t} \frac{\partial h_y}{\partial z} = \mu \epsilon \frac{\partial^2 e_x}{\partial t^2} \\ -\frac{\partial}{\partial t} \frac{\partial h_y}{\partial z} = \epsilon \frac{\partial^2 e_x}{\partial t^2} \end{cases} \Rightarrow \frac{\partial^2 e_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 e_x}{\partial t^2}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial e_y}{\partial t} \end{cases} \Rightarrow \begin{cases} \frac{\partial^2 e_y}{\partial z^2} = \mu \frac{\partial}{\partial z} \frac{\partial h_x}{\partial t} = \mu \frac{\partial}{\partial t} \frac{\partial h_x}{\partial z} = \mu \epsilon \frac{\partial^2 e_y}{\partial t^2} \\ \frac{\partial}{\partial t} \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial^2 e_y}{\partial t^2} \end{cases} \Rightarrow \frac{\partial^2 e_y}{\partial z^2} = \mu \epsilon \frac{\partial^2 e_y}{\partial t^2}$$

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

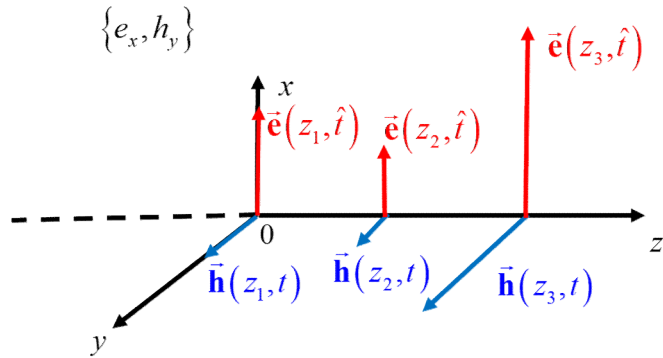
$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$e_z(z, t) = h_z(z, t) = 0$$

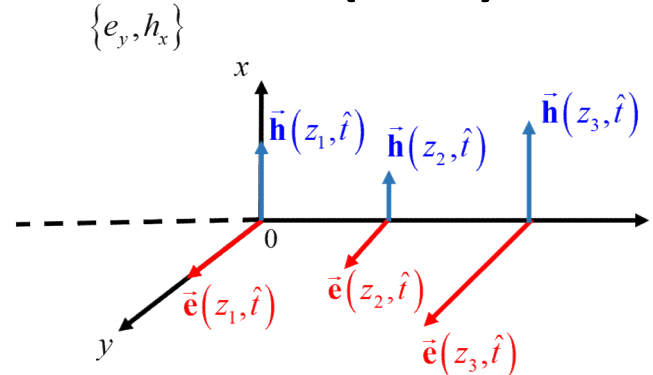
$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent each other

Plane Waves (TD)



$$c = \frac{1}{\sqrt{\mu\epsilon}}$$



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 e_x}{\partial t^2} \quad \frac{\partial^2 e_x(z,t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x(z,t)}{\partial t^2} = 0$$

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Source-free

Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent
each other

Plane Waves (TD)

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$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct)$$

$$\alpha = z - ct$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^+}{\partial z} = \frac{\partial e^+}{\partial \alpha} \frac{\partial \alpha}{\partial z} = \frac{\partial e^+}{\partial \alpha}$$



$$\frac{\partial e^+}{\partial \alpha} = \frac{\partial e^+}{\partial z}$$



$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^+}{\partial t} = \frac{\partial e^+}{\partial \alpha} \frac{\partial \alpha}{\partial t} = -c \frac{\partial e^+}{\partial \alpha}$$



$$\frac{\partial e^+}{\partial \alpha} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

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$$\Rightarrow \frac{\partial^2 e^+}{\partial z^2} = \frac{\partial}{\partial z} \left[-\frac{1}{c} \frac{\partial e^+}{\partial t} \right] = -\frac{1}{c} \frac{\partial}{\partial z} \frac{\partial e^+}{\partial t} = -\frac{1}{c} \frac{\partial}{\partial t} \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial}{\partial t} \left[-\frac{1}{c} \frac{\partial e^+}{\partial t} \right] = \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2}$$

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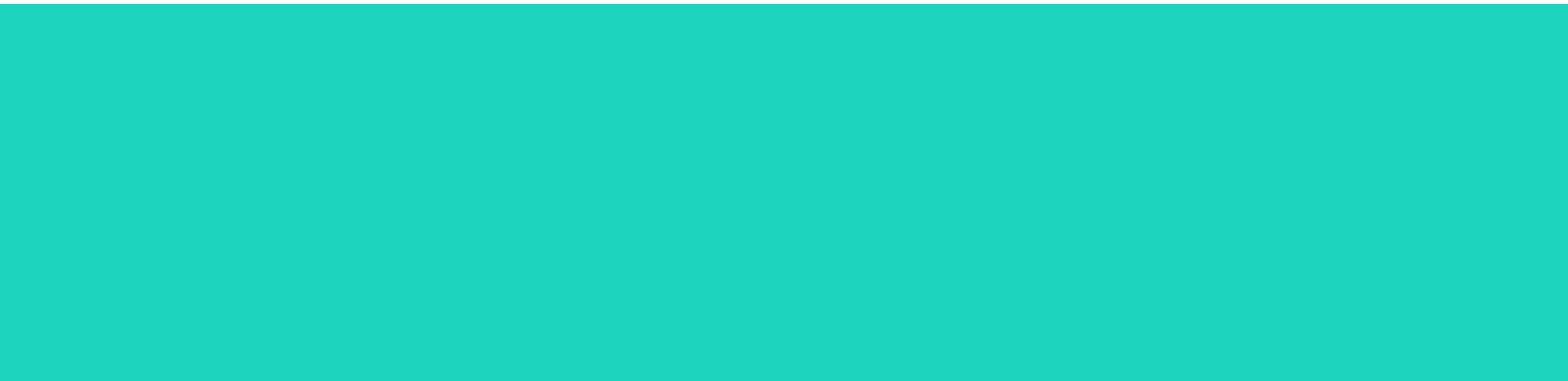
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$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0 \quad \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$



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Plane Waves (TD)

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$$\left\{ \begin{array}{l} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{array} \right. \quad \left\{ e_y, h_x \right\} \quad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0 \quad \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \quad \beta = z+ct \quad \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$\frac{\partial e^-}{\partial z} = \frac{\partial e^-}{\partial \beta} \frac{\partial \beta}{\partial z} = \frac{\partial e^-}{\partial \beta} \quad \rightarrow \quad \frac{\partial e^-}{\partial \beta} = \frac{\partial e^-}{\partial z}$$

$$\frac{\partial e^-}{\partial t} = \frac{\partial e^-}{\partial \beta} \frac{\partial \beta}{\partial t} = c \frac{\partial e^-}{\partial \beta} \quad \rightarrow \quad \frac{\partial e^-}{\partial \beta} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

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$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0 \quad \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \quad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0 \quad \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$\begin{aligned} \Rightarrow \frac{\partial^2 e^-}{\partial z^2} &= \frac{\partial}{\partial z} \left[\frac{1}{c} \frac{\partial e^-}{\partial t} \right] = \frac{1}{c} \frac{\partial}{\partial t} \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial}{\partial t} \left[\frac{1}{c} \frac{\partial e^-}{\partial t} \right] = \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} \\ \Rightarrow \frac{\partial^2 e^-}{\partial z^2} &= \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} \Rightarrow \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0 \end{aligned}$$

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$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \quad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0 \quad \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \quad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0 \quad \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

Source-free

Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent each other

Plane Waves (TD)

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \{e_x, h_y\}$$

$$\frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$$\frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

$$f(z,t) = e^+(z-ct)$$

$$f(z,t) = e^-(z+ct)$$

$$\frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

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$\{e_y, h_x\}$
 $\{e_x, h_y\}$ Independent each other

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$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

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$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct)$$

$$\frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct)$$

$$\frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0$$

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$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

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$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \quad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

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$\{e_y, h_x\}$

$\{e_x, h_y\}$

Independent each other

Plane Waves (TD)

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

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$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$$e_y(z, t) = e_y^+(z - ct) + e_y^-(z + ct)$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

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$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

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$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \left\{ e_x, h_y \right\}$$

$$\frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \quad \left\{ e_y, h_x \right\}$$

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$$e_y(z, t) = e_y^+(z - ct) + e_y^-(z + ct)$$

$$\mu c = \frac{\mu}{\sqrt{\varepsilon\mu}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$-\mu \frac{\partial h_y}{\partial t} = \frac{\partial e_x}{\partial z} = \frac{\partial e_x^+}{\partial z} + \frac{\partial e_x^-}{\partial z} = -\frac{1}{c} \frac{\partial e_x^+}{\partial t} + \frac{1}{c} \frac{\partial e_x^-}{\partial t}$$

$$\Rightarrow \mu c \frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t} (e_x^+ - e_x^-)$$

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$\left\{ e_y, h_x \right\}$
 $\left\{ e_x, h_y \right\}$ Independent each other

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$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x, h_y\}$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$$\zeta h_y(z, t) = e_x^+(z - ct) - e_x^-(z + ct) = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \quad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

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$$-\mu \frac{\partial h_y}{\partial t} = \frac{\partial e_x}{\partial z} = \frac{\partial e_x^+}{\partial z} + \frac{\partial e_x^-}{\partial z} = -\frac{1}{c} \frac{\partial e_x^+}{\partial t} + \frac{1}{c} \frac{\partial e_x^-}{\partial t}$$

$$\mu c \frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t} (e_x^+ - e_x^-) \implies \frac{\partial}{\partial t} [\zeta h_y - (e_x^+ - e_x^-)] = 0$$

$$\zeta h_y = (e_x^+ - e_x^-)$$

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$\{e_y, h_x\}$

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$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

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$$\mu \frac{\partial h_x}{\partial t} = \frac{\partial e_y}{\partial z} = \frac{\partial e_y^+}{\partial z} + \frac{\partial e_y^-}{\partial z} = -\frac{1}{c} \frac{\partial e_y^+}{\partial t} + \frac{1}{c} \frac{\partial e_y^-}{\partial t}$$

$$\Rightarrow \mu c \frac{\partial h_x}{\partial t} = -\frac{\partial}{\partial t} (e_y^+ - e_y^-) \Rightarrow \frac{\partial}{\partial t} [\zeta h_x + (e_y^+ - e_y^-)] = 0$$

$$-\zeta h_x = (e_y^+ - e_y^-)$$

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Independent each other

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$$[\vec{e}]: \frac{\text{Volt}}{m}$$

$$[\zeta] \frac{\text{Ampere}}{m} = \frac{\text{Volt}}{m} \quad \longrightarrow \quad [\zeta] = \frac{\text{Volt}}{\text{Ampere}} = \Omega$$

$$[\vec{h}]: \frac{\text{Ampere}}{m}$$

ζ : intrinsic resistance of the medium

in freespace

$$\zeta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent each other

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$$

Plane Waves (TD)

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \{e_x, h_y\}$$

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 $\{e_x, h_y\}$ Independent each other

$$[\vec{e}]: \frac{\text{Volt}}{m} \quad \frac{\text{Volt}}{m} \frac{1}{m^2} = \frac{1}{[c]^2} \frac{\text{Volt}}{m} \frac{1}{s^2} \quad \Rightarrow \quad [c]^2 = \left(\frac{m}{s}\right)^2 \quad \Rightarrow \quad [c] = \frac{m}{s}$$

c is a speed

in free space

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$$

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$$e_z(z, t) = h_z(z, t) = 0$$

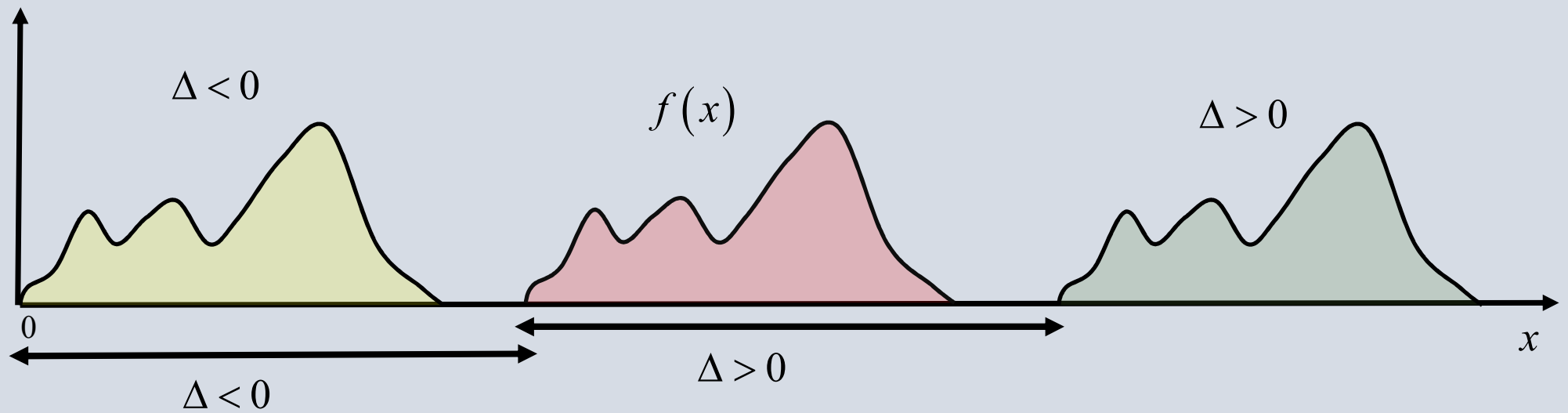
$\{e_y, h_x\}$

$\{e_x, h_y\}$

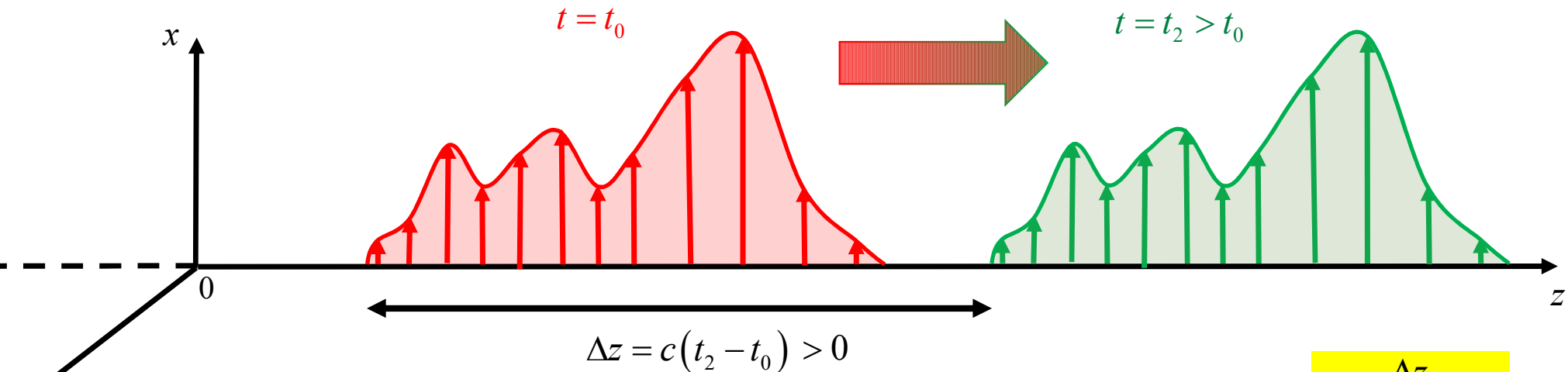
Independent
each other

MEMO

$$f(x - \Delta)$$



Plane Waves (TD)



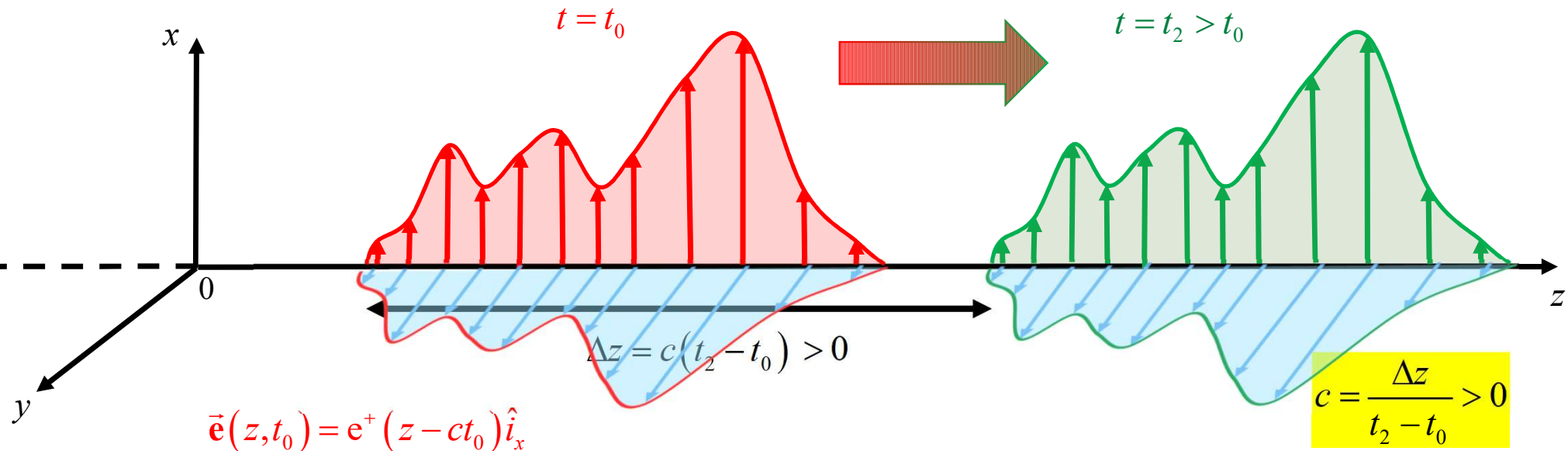
$$\vec{e}(z, t_0) = e^+(z - ct_0) \hat{i}_x$$

$$\vec{e}(z, t_2) = e^+(z - ct_2) \hat{i}_x = e^+(z - ct_0 + ct_0 - ct_2) \hat{i}_x = e^+(z - ct_0 - \underbrace{c[t_2 - t_0]}_{\Delta z}) \hat{i}_x$$

$$c = \frac{\Delta z}{t_2 - t_0} > 0$$

The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the positive sense of the z-axis

Plane Waves (TD)



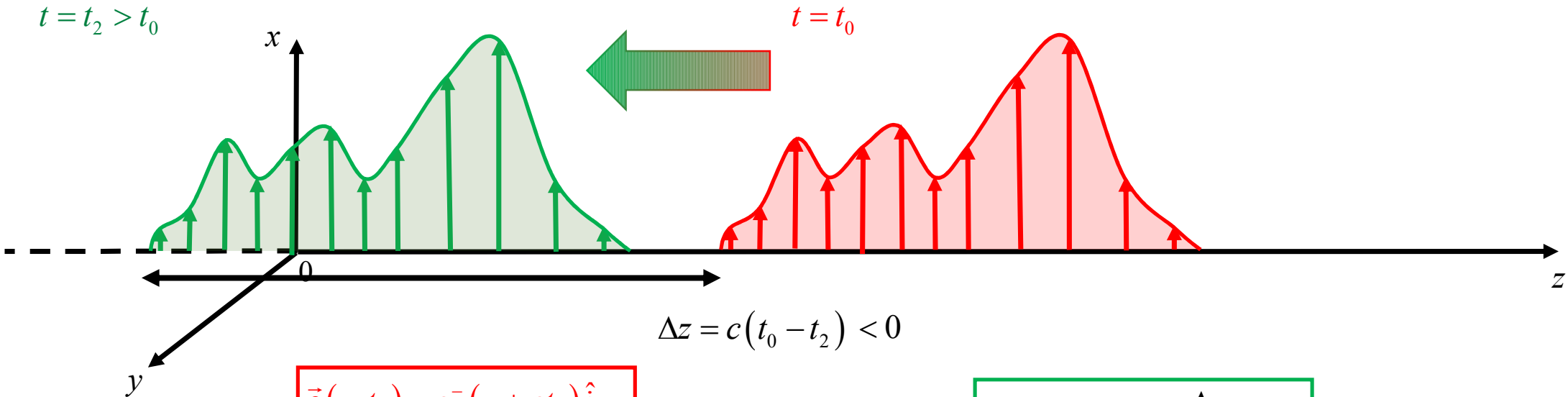
$$\vec{e}(z, t_0) = e^+(z - ct_0) \hat{i}_x$$

$$\vec{e}(z, t_2) = e^+(z - ct_2) \hat{i}_x = e^+(z - ct_0 + ct_0 - ct_2) \hat{i}_x = e^+(z - ct_0 - c[t_2 - t_0]) \hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed c along the positive sense of the z -axis

$\begin{cases} e^+(z - ct) \\ h^+(z - ct) \end{cases}$ is referred to as electromagnetic **progressive plane wave**

Plane Waves (TD)

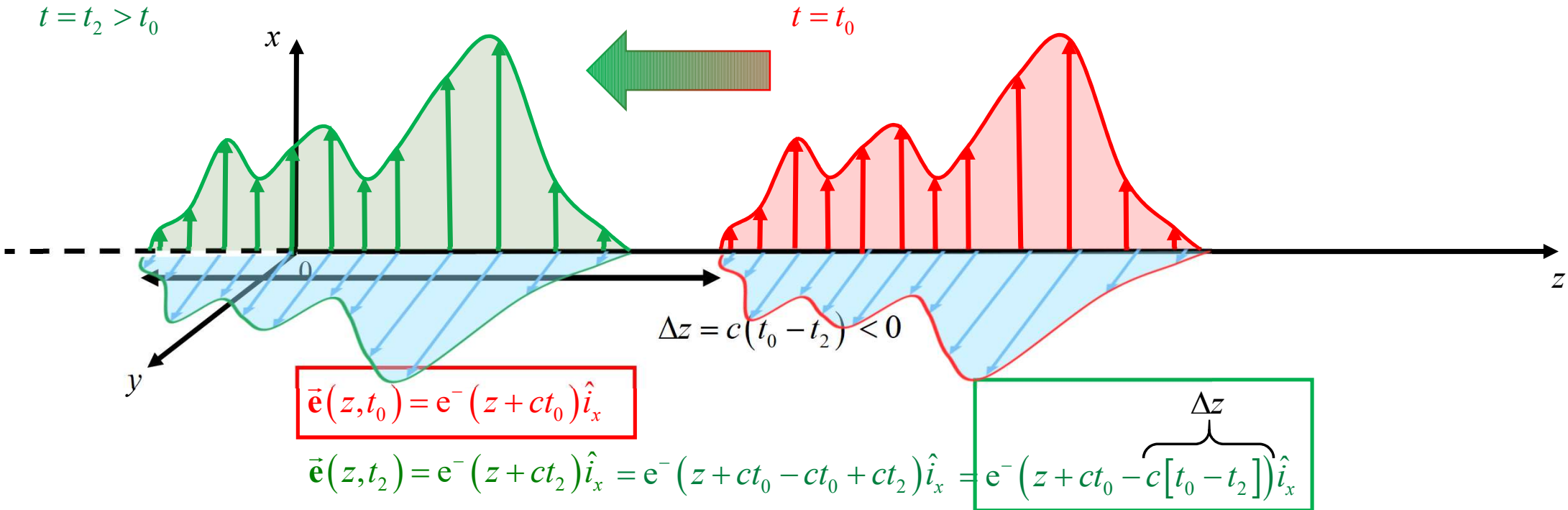


$$\vec{e}(z, t_0) = e^{- (z + ct_0)} \hat{i}_x$$

$$\vec{e}(z, t_2) = e^{- (z + ct_2)} \hat{i}_x = e^{- (z + ct_0 - ct_0 + ct_2)} \hat{i}_x = e^{- (z + ct_0 - \overbrace{c[t_0 - t_2]}^{\Delta z})} \hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the negative sense of the z-axis

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the negative sense of the z-axis

$\begin{cases} e^{- (z + ct)} \\ h^{- (z + ct)} \end{cases}$ is referred to as electromagnetic **regressive plane wave**