Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Maxwell Equations (Spectral Domains)

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega \vec{\mathbf{B}} \\ \nabla \times \vec{\mathbf{H}} = j\omega \vec{\mathbf{D}} + \vec{\mathbf{J}} \\ \nabla \cdot \vec{\mathbf{D}} = \rho \\ \nabla \cdot \vec{\mathbf{B}} = 0 \end{cases}$$

Maxwell Equations (Spectral Domains)

Magnetic Sources

James Clerk Maxwell 1831-1879

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\vec{\mathbf{B}} - \vec{\mathbf{J}}_{\mathrm{m}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\vec{\mathbf{D}} + \vec{\mathbf{J}} \\ \nabla \cdot \vec{\mathbf{D}} = \rho \\ \nabla \cdot \vec{\mathbf{B}} = \rho_{\mathrm{m}} \end{cases}$$

$$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right] : \frac{Volt}{m} \qquad \left[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right] : \frac{Weber}{m^2}$$

$$\left[\vec{\mathbf{j}}_m(\vec{\mathbf{r}},t)\right] : \frac{Volt}{m^2} \qquad \left[\rho_m(\vec{\mathbf{r}},t)\right] : \frac{Weber}{m^3}$$

Equivalence theorem

$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$

Consider a source distribution $\vec{\mathbf{J}}_0$ with its associated electromagnetic field $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$

Equivalence theorem

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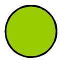
$\vec{\mathbf{J}}_0$ 

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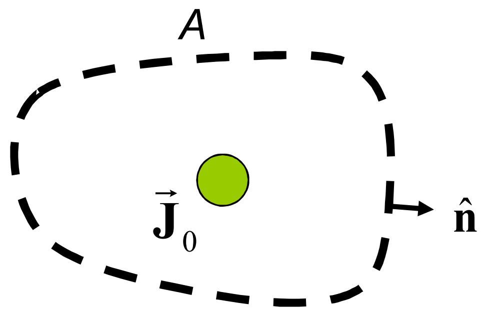
$$\vec{\mathbf{J}}_0 \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem



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Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

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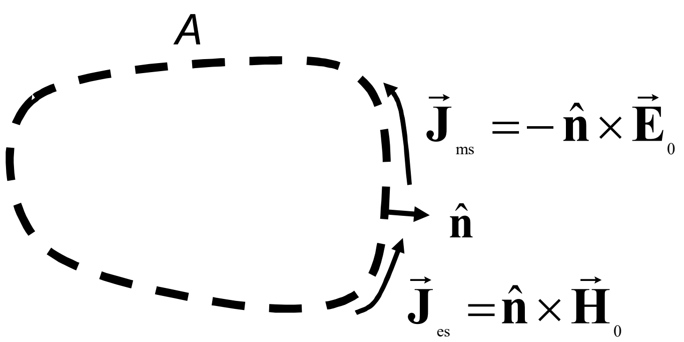

$\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m}$ $\left[\vec{\mathbf{j}}_m(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m^2}$

$\left[\vec{\mathbf{j}}_{ms}(\vec{\mathbf{r}},t)\right]: \frac{Volt}{m}$

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The original sources $\vec{\mathbf{J}}_0$ enclosed in *A* can be removed and substituted by equivalent sources, i.e., electric $\vec{\mathbf{J}}_{es} = \hat{\mathbf{n}} \times \vec{\mathbf{H}}_0$ and magnetic $\vec{\mathbf{J}}_{ms} = -\hat{\mathbf{n}} \times \vec{\mathbf{E}}_0$ current densities distributed over the surface *A*.

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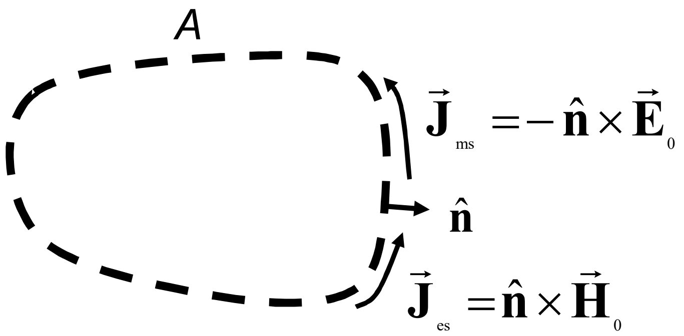

$\left[\vec{h}(\vec{\mathbf{r}},t)\right] : \dfrac{Ampere}{m}$	$\left[\vec{\mathbf{j}}_e(\vec{\mathbf{r}},t)\right] : \dfrac{Ampere}{m^2}$
$\left[\vec{\mathbf{j}}_{ms}(\vec{\mathbf{r}},t)\right] : \dfrac{Volt}{m}$	$\left[\vec{\mathbf{j}}_{es}(\vec{\mathbf{r}},t)\right] : \dfrac{Ampere}{m}$

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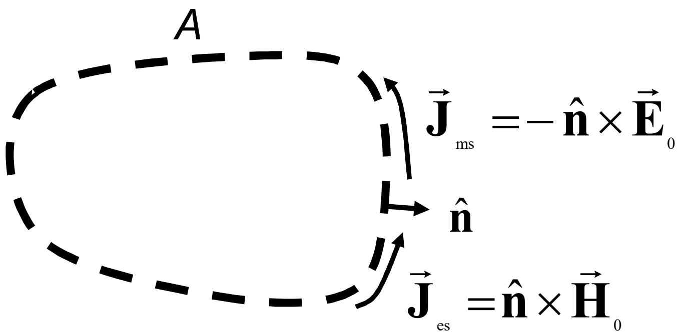

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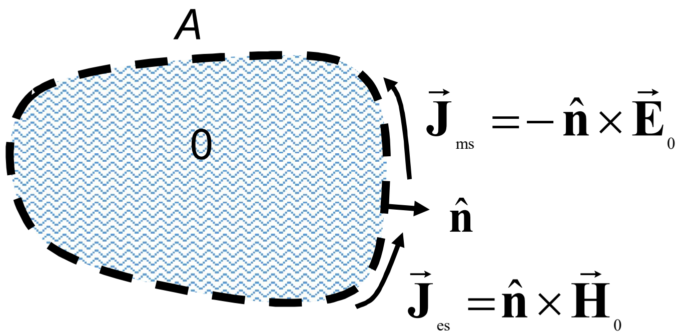

$$\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0$$

The Equivalence Theorem states that the equivalent sources $\vec{\mathbf{J}}_{es}$ and $\vec{\mathbf{J}}_{ms}$ generate a field $\left(\vec{\mathbf{E}}', \vec{\mathbf{H}}'\right)$ coincident with $\left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$ outside *A* and identically equal to zero inside

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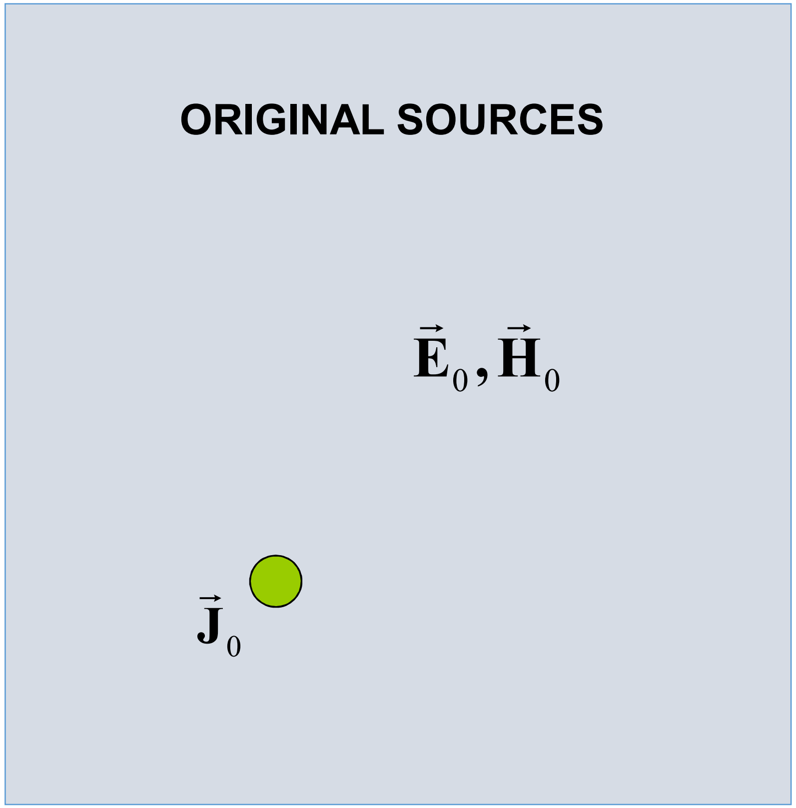
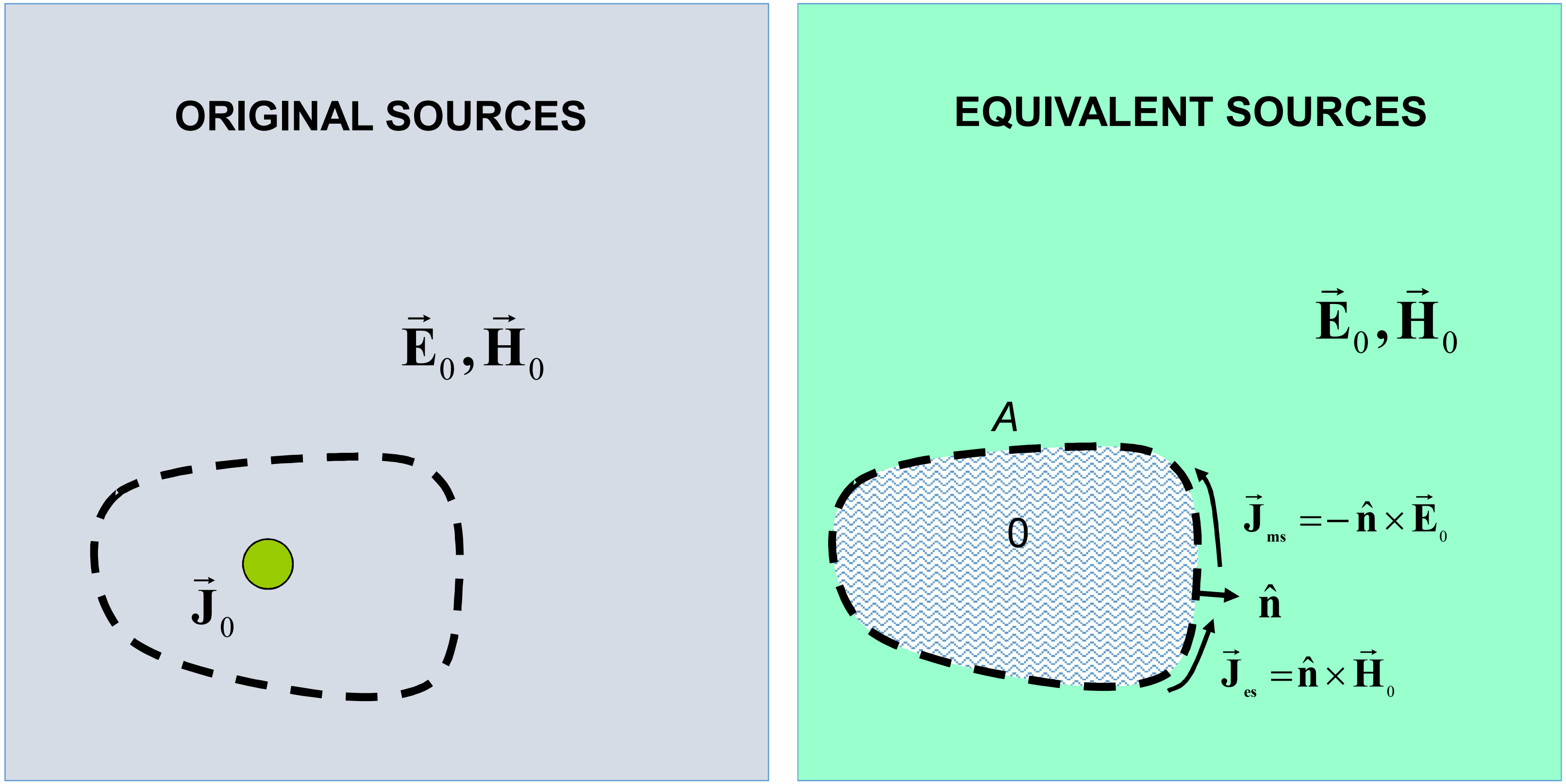
Equivalence theorem



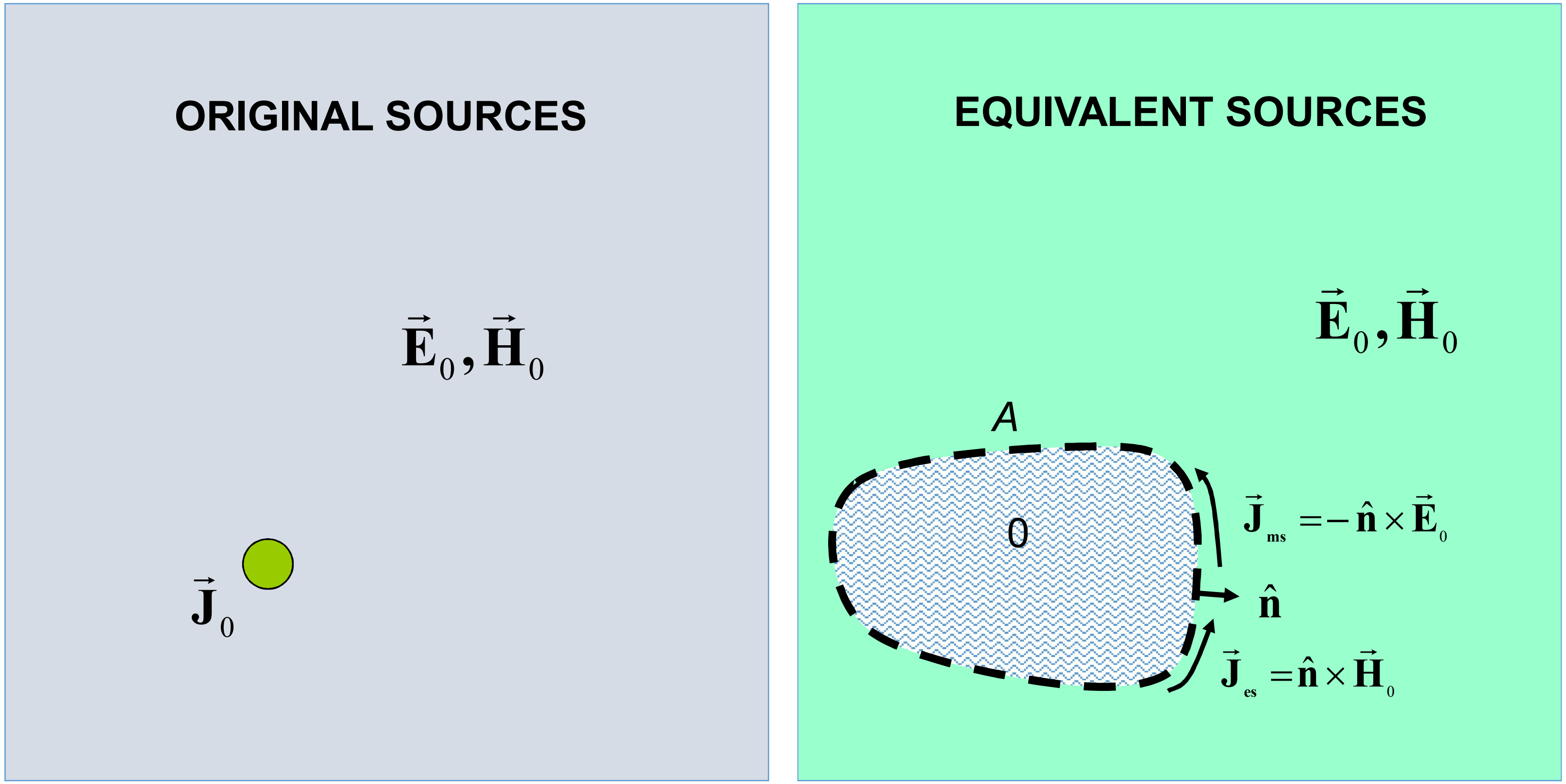
Equivalence theorem



Equivalence theorem

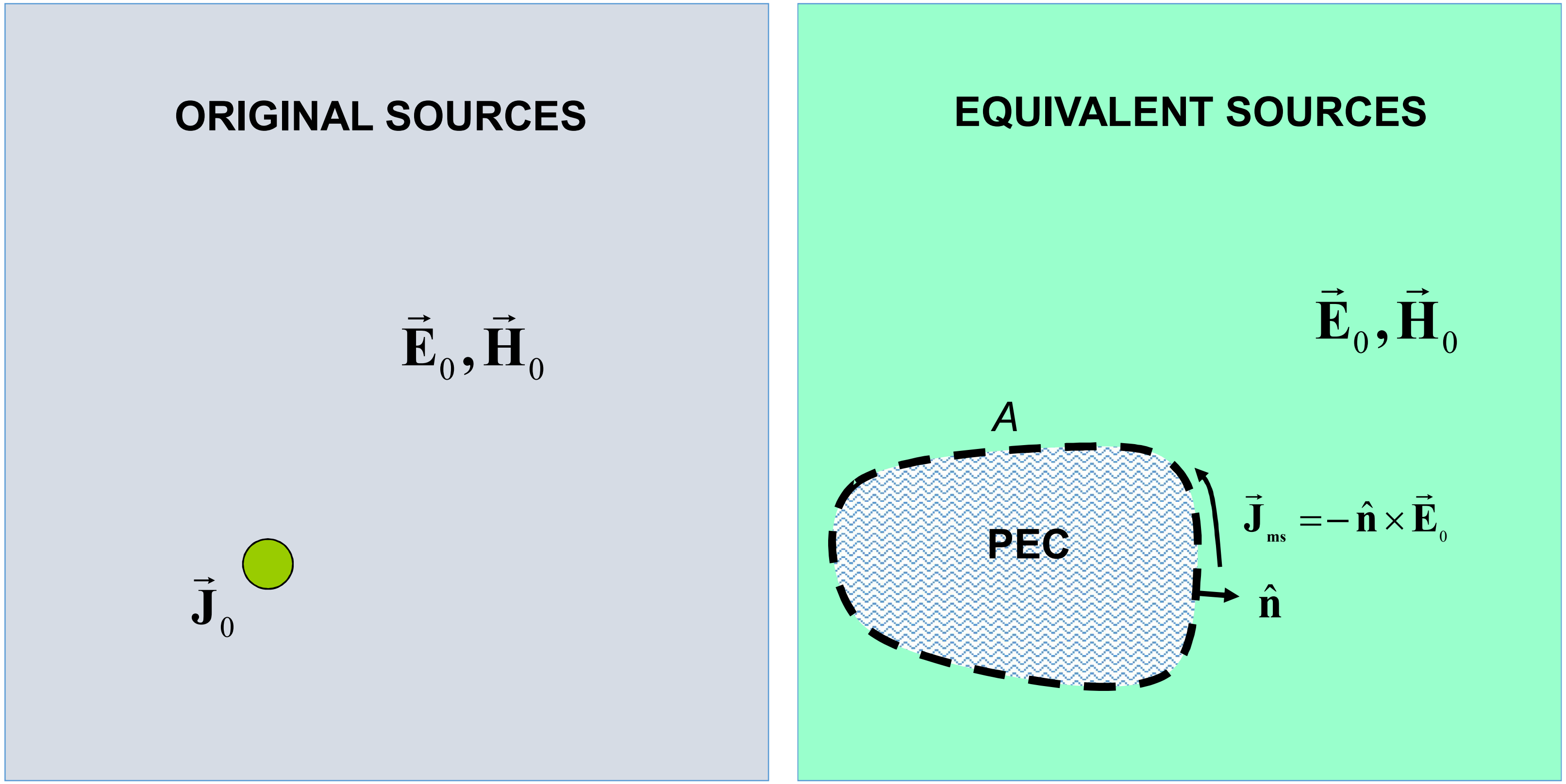
It's a powerful theorem that allows calculating the e.m. field in **all** the space, starting from the knowledge of its value **just on a surface**.

Equivalence theorem



Equivalence theorem

Alternative formulation



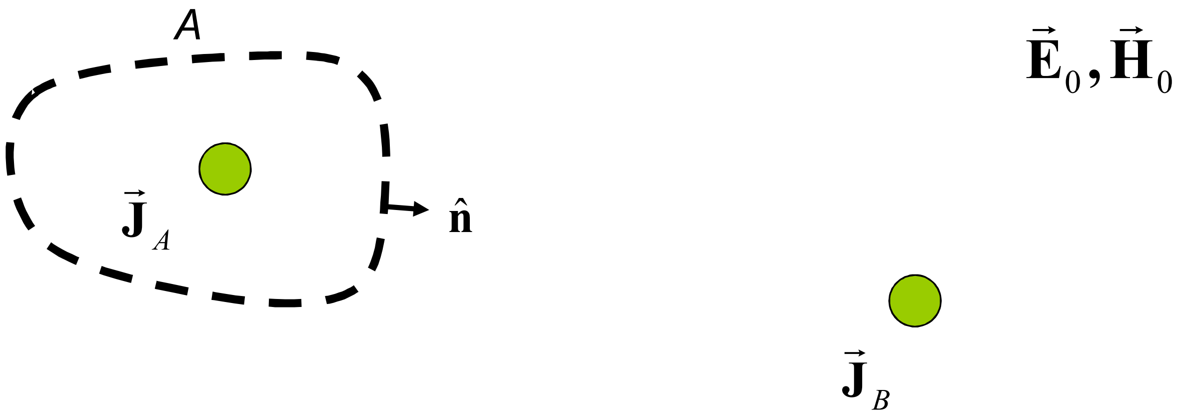
Equivalence theorem

More general formulation

$$\vec{\mathbf{J}}_A + \vec{\mathbf{J}}_B \rightarrow \left(\vec{\mathbf{E}}_0, \vec{\mathbf{H}}_0\right)$$

Equivalence theorem

More general formulation



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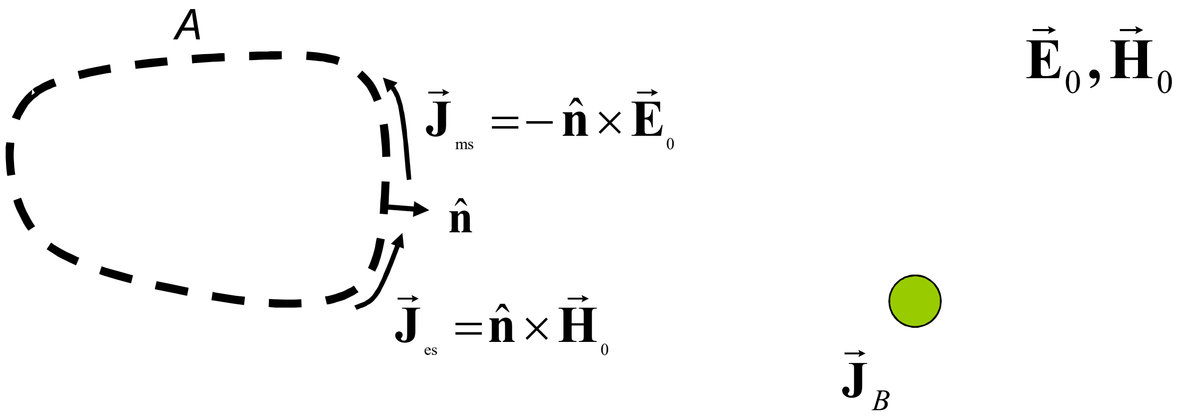
Equivalence theorem

More general formulation



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Equivalence theorem

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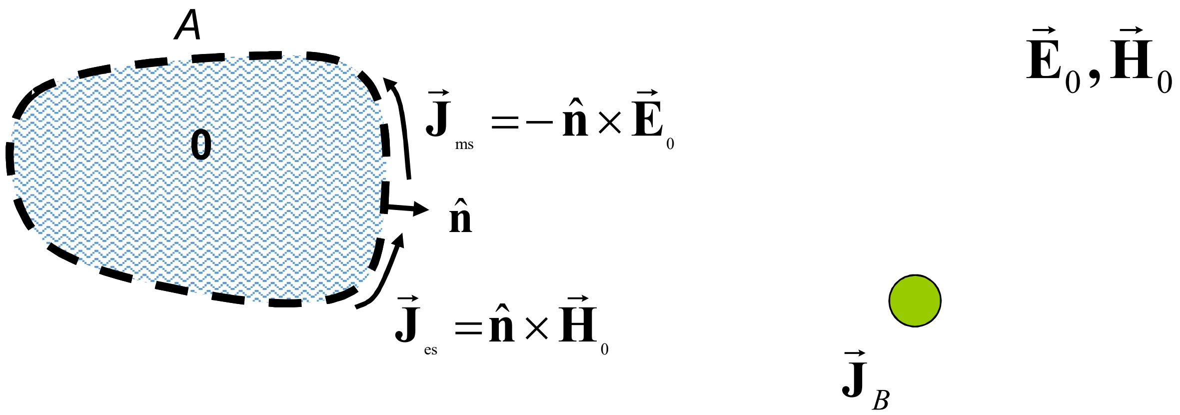

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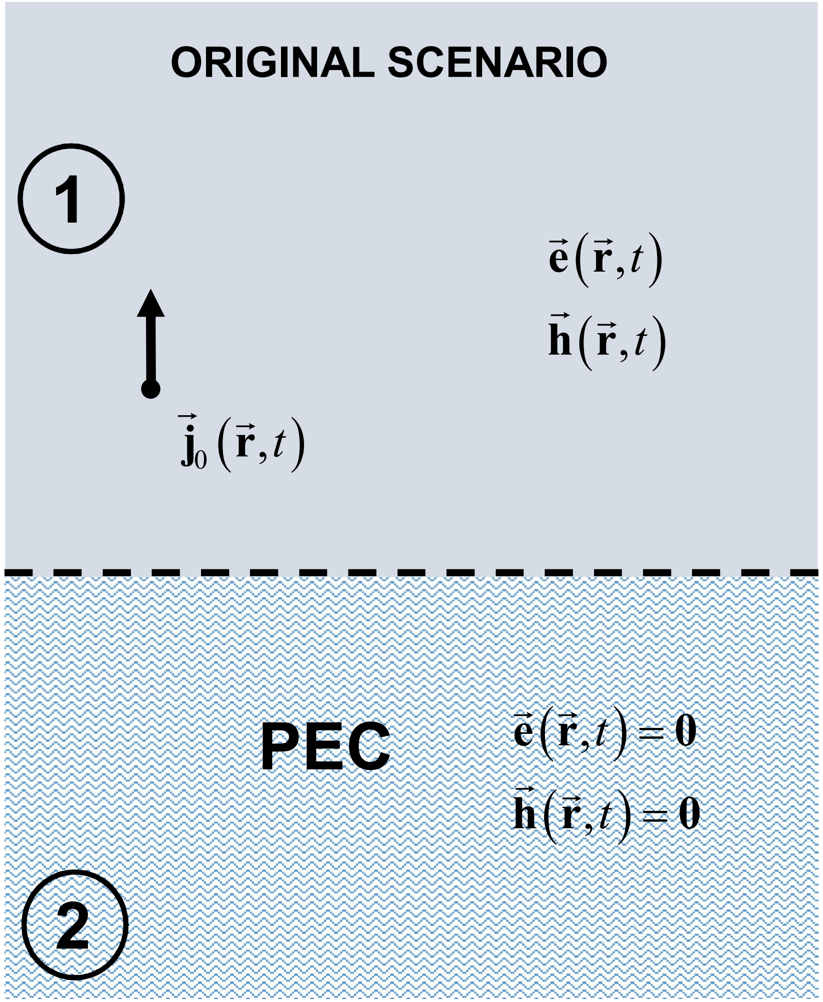


Image theory

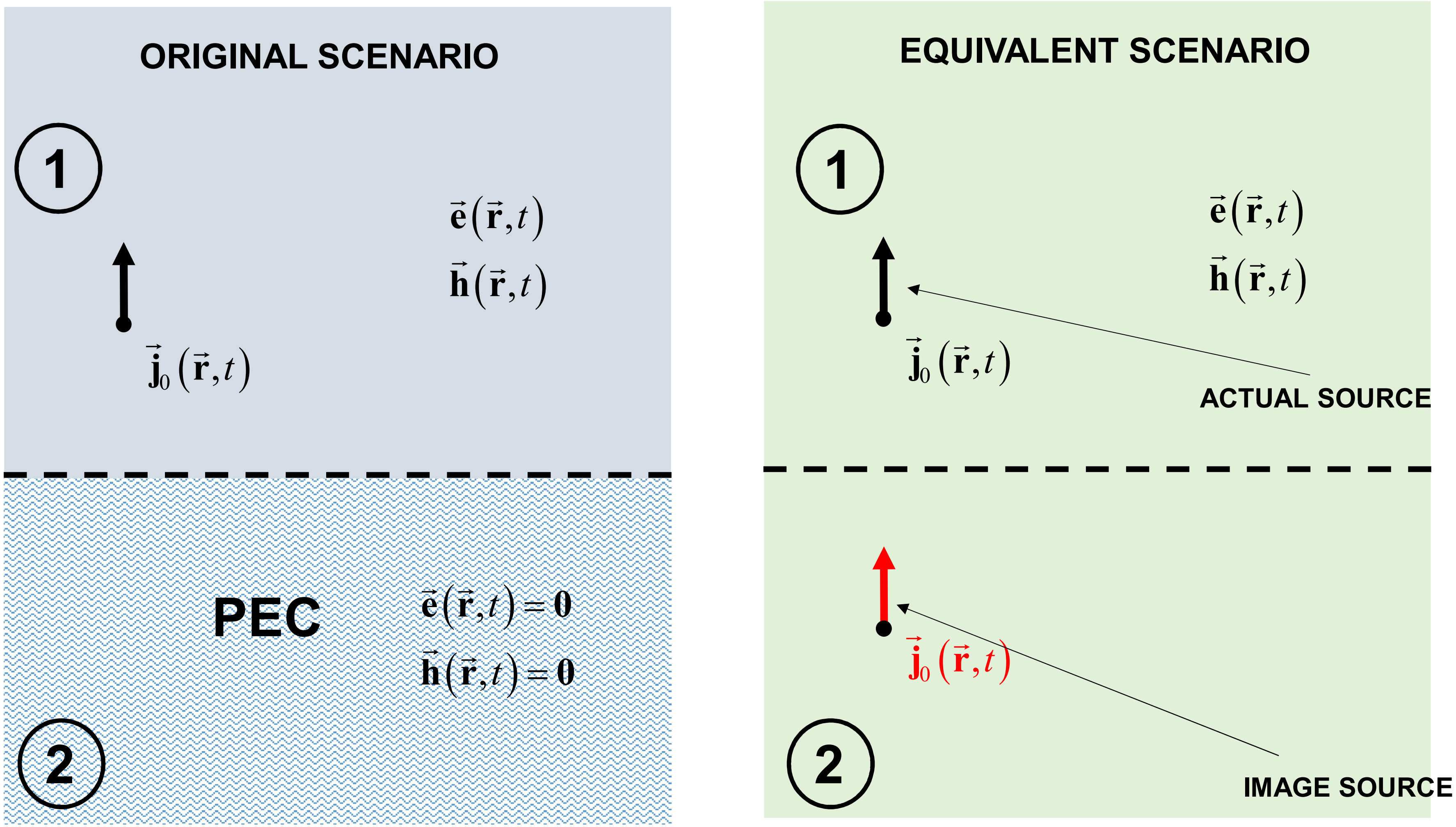
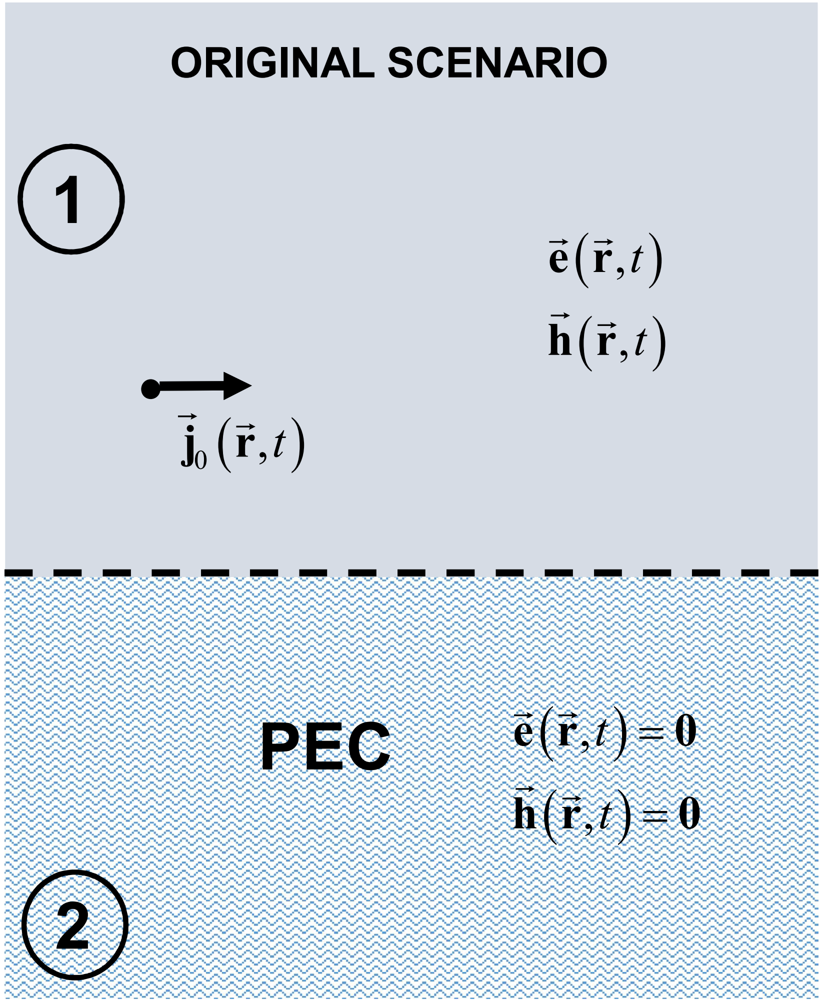


Image theory



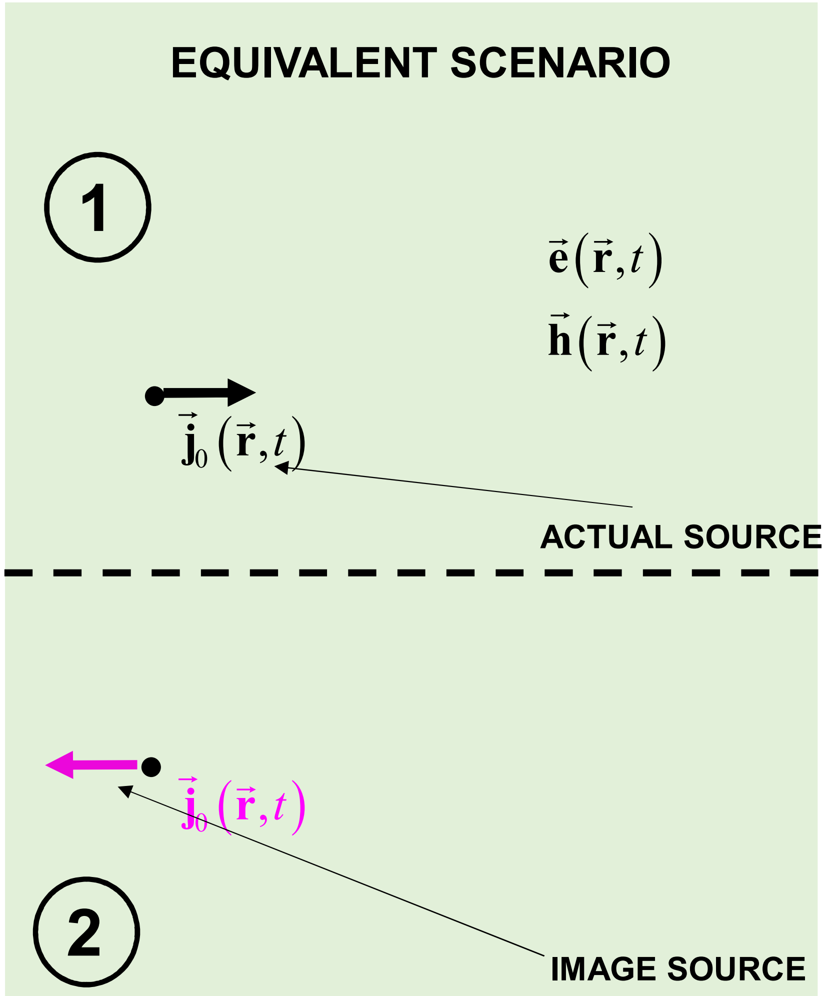
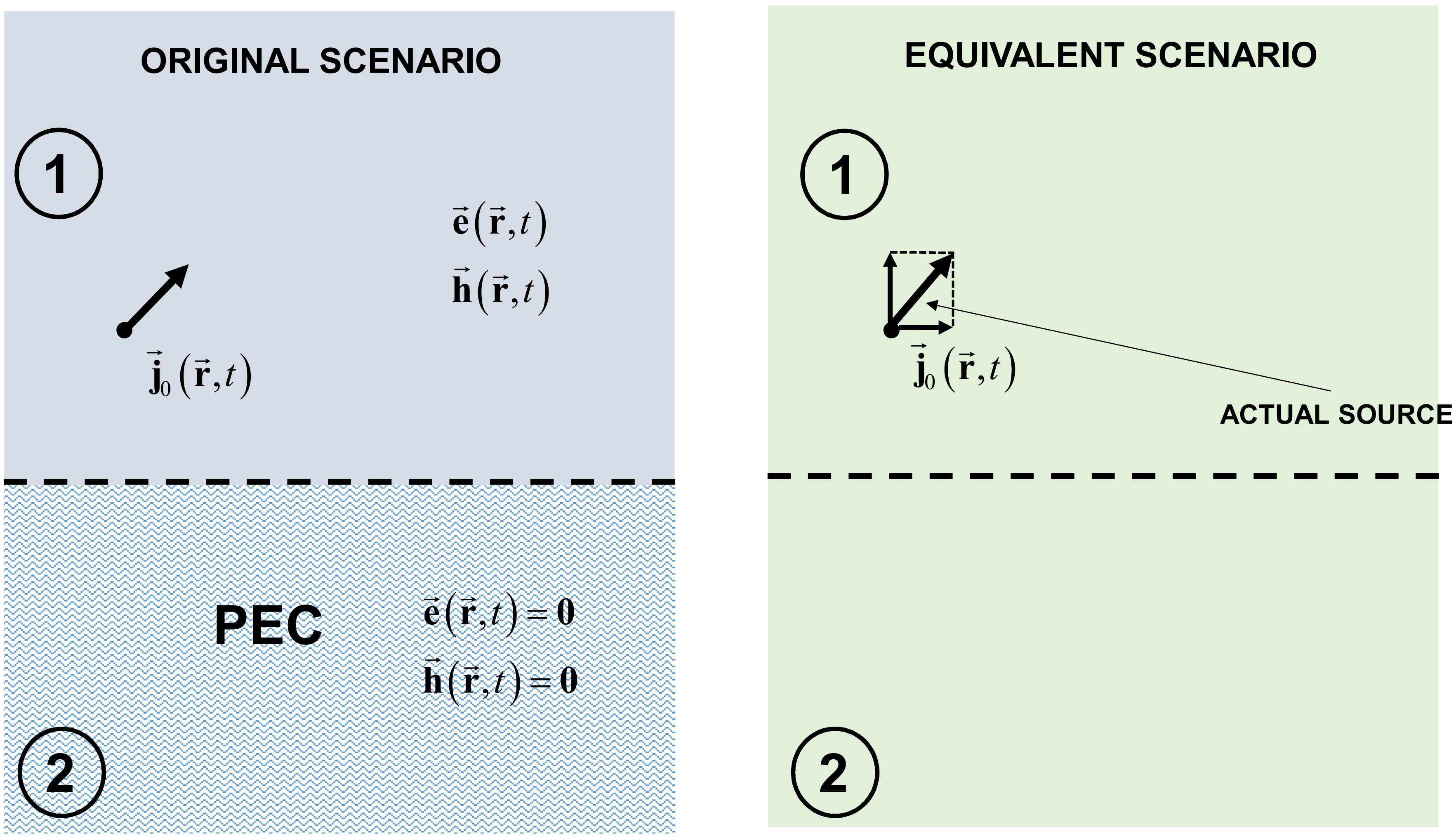


Image theory



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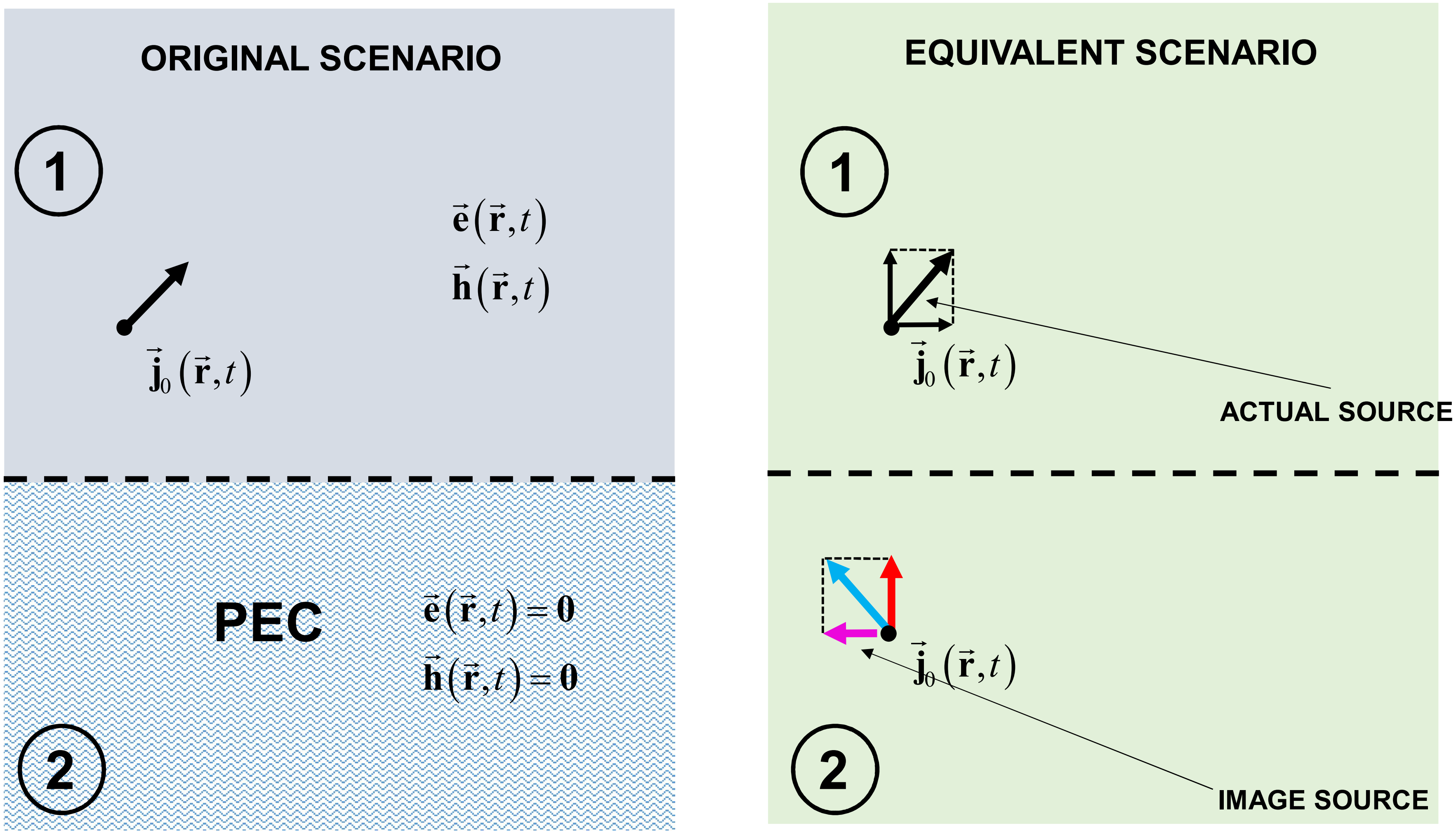

Image theory

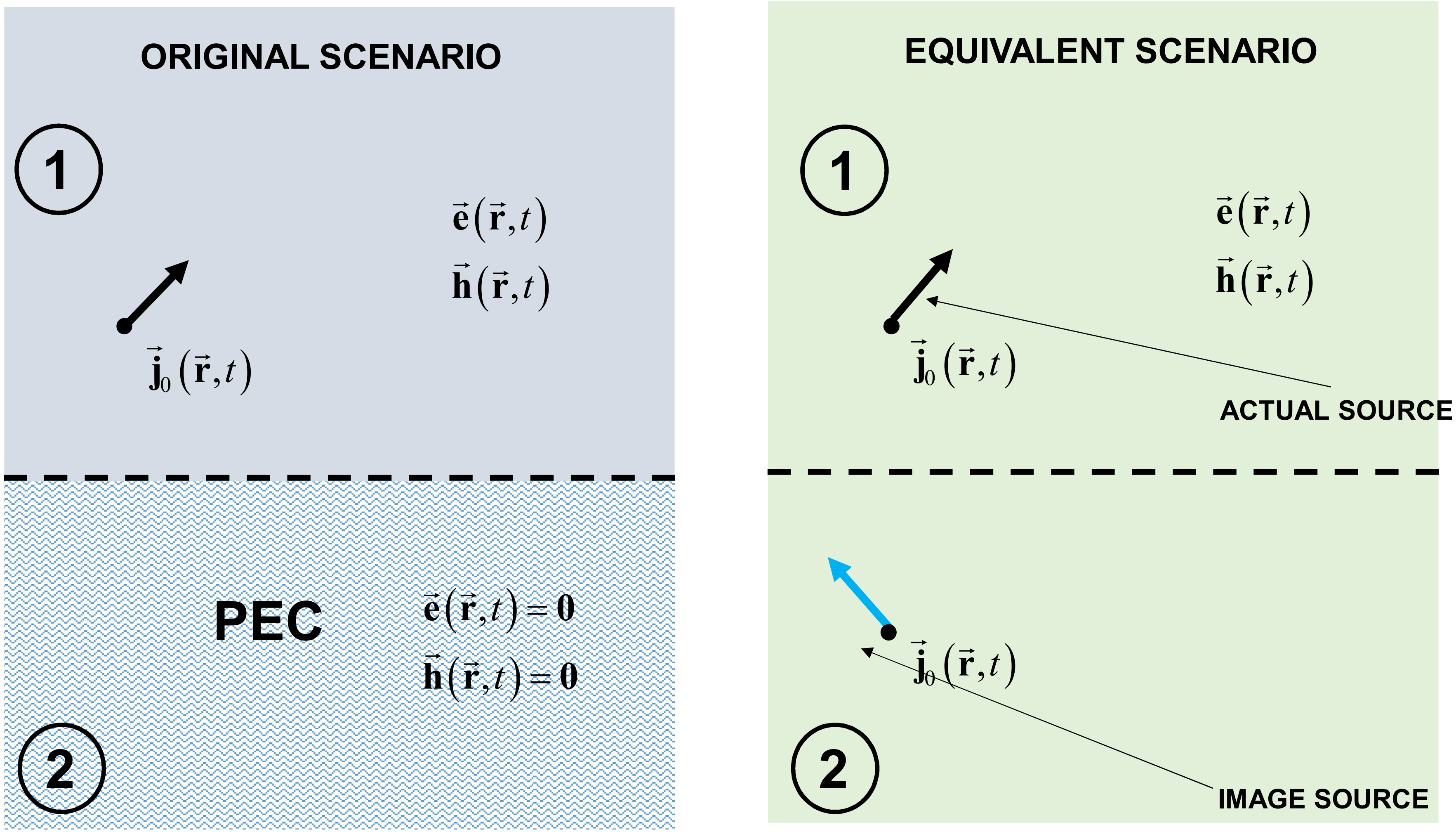

Image theory

Image theory (magnetic sources)