Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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THEOREMS

Poynting

Time domain – Phasor domain

Uniqueness (Interior problem – Exterior problem)

Time domain – Phasor domain

Equivalence

Phasor domain

Image Theory

Reciprocity

Phasor domain

Summary of the past lectures: Poynting theorem

TD

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{\partial}{\partial t}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

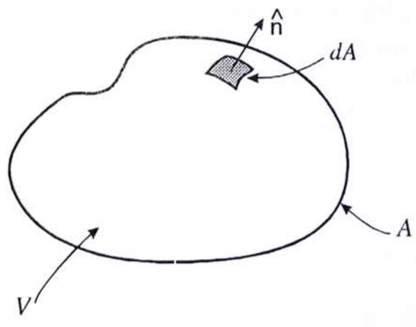
$$P_S(t)+\frac{\partial}{\partial t}W(t)+P_j(t)=P_0(t)$$

$$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2$$

$$p_j(\vec{\mathbf{r}},t)=\sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2$$

$$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

Linear **Isotropic** **Space- Nondispersive** **Time-Nondispersive** **Time-invariant**



PD

$$\mathbf{S}(\vec{\mathbf{r}})=\frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]=\mathbf{S}_1(\vec{\mathbf{r}})+j\mathbf{S}_2(\vec{\mathbf{r}})$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

Linear **Isotropic** **Space- Nondispersive** **Time-Dispersive** **Time-invariant**

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today

$$\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{B}}(\vec{\mathbf{r}})\times\vec{\mathbf{C}}(\vec{\mathbf{r}})\right]=\vec{\mathbf{C}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{A}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]=\vec{\mathbf{B}}(\vec{\mathbf{r}})\cdot\left[\vec{\mathbf{C}}(\vec{\mathbf{r}})\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right]$$

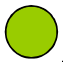
Uniqueness (TD)

$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

- Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

Uniqueness (TD)



$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

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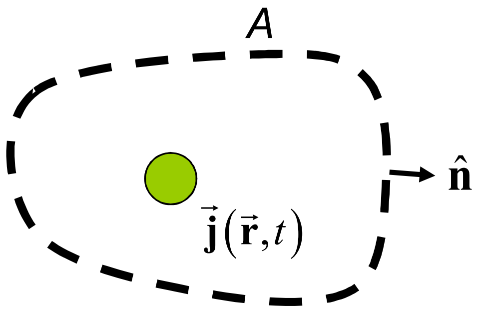
- Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

Uniqueness (TD)

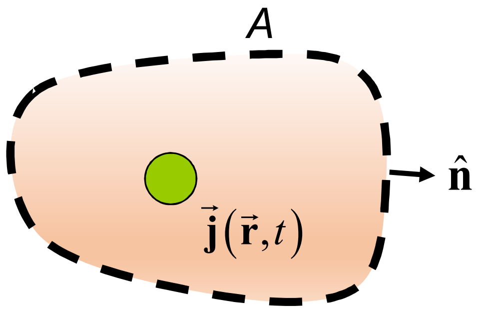


$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

Uniqueness (TD)



$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

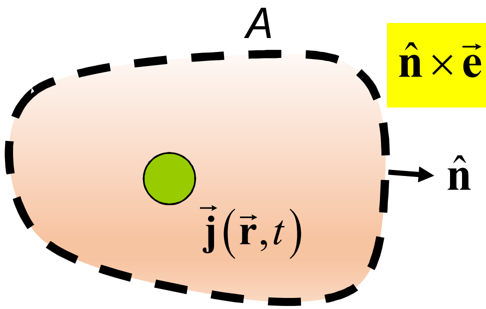
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Uniqueness (TD)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

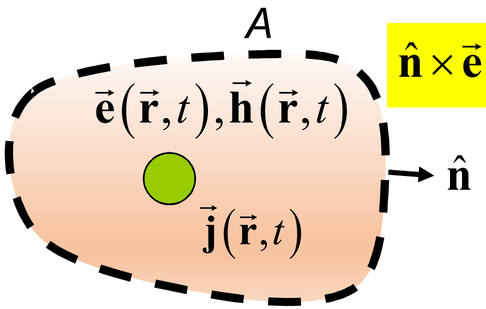
I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

Uniqueness (TD)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Interior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

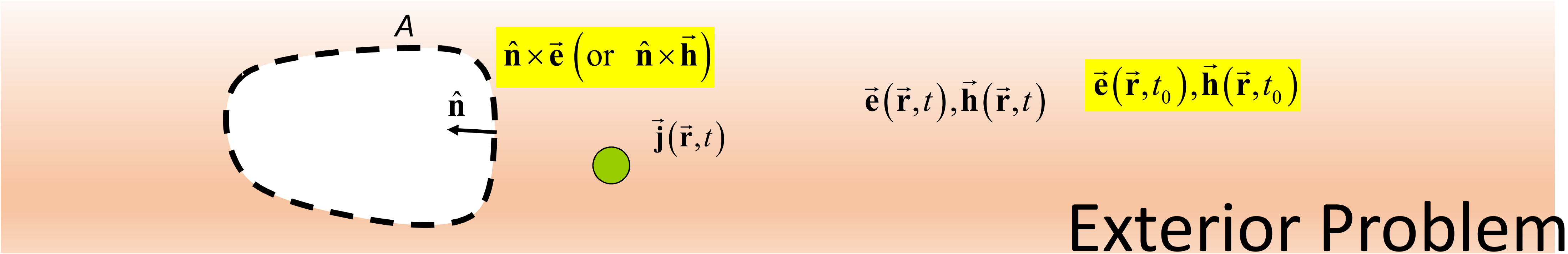
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD)



Exterior Problem

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}}, t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}}, \vec{\mathbf{h}}\right)$

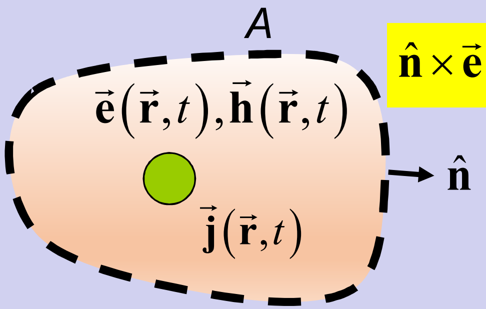
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}}, t_0), \vec{\mathbf{h}}(\vec{\mathbf{r}}, t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}} \times \vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}} \times \vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\;\left(\text{or}\;\;\hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

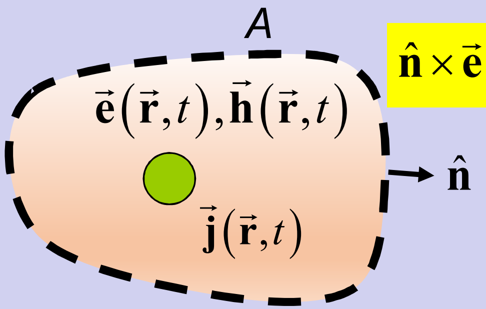
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Field difference: source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ = 0

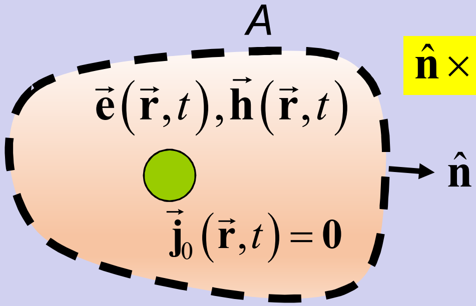
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\quad\text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

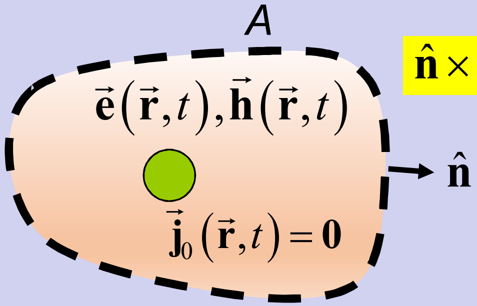
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad \text{on the boundary}}$

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\boxed{\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

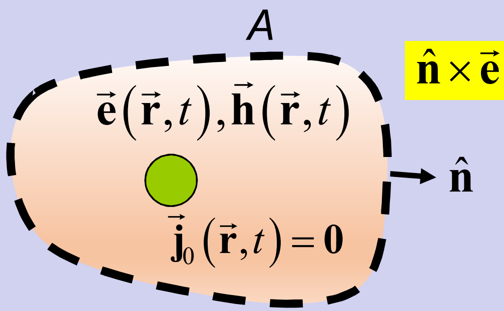
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=\cancel{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}$$

$$\iiint_V dV\,\boxed{\vec{\mathbf{j}}_0}\cdot\vec{\mathbf{e}}=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}} \times \vec{\mathbf{e}} = \mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0) = \mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0) = \mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}_1(\vec{\mathbf{r}},t) - \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) - \vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

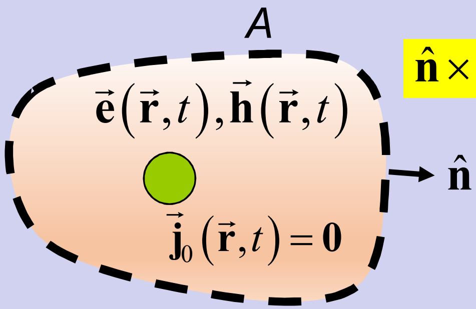
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0) = \mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0) = \mathbf{0}$

$\hat{\mathbf{n}} \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\!\oint_A dA\, \vec{\mathbf{s}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}} + \frac{\partial}{\partial t} \iiint_V dV \left[\frac{1}{2}\mu |\vec{\mathbf{h}}|^2 + \frac{1}{2}\varepsilon |\vec{\mathbf{e}}|^2 \right] + \iiint_V dV\, \sigma |\vec{\mathbf{e}}|^2 = \cancel{-\iiint_V dV\, \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

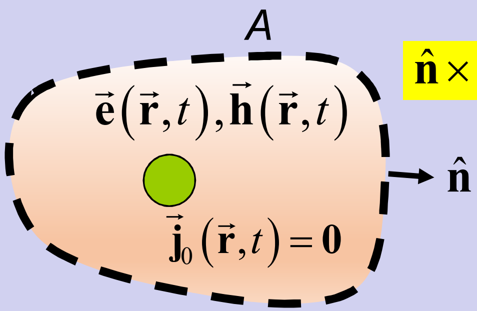
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=\mathbf{0}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

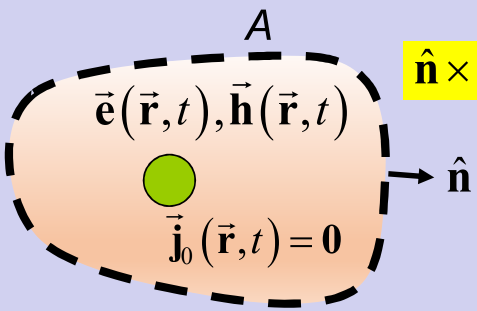
$$\boxed{\begin{aligned}\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)&=\mathbf{0}\\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)&=\mathbf{0}\end{aligned}}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=0$$

$$\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=P_j(t)\geq 0 \qquad \iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right]=W(t)\geq 0 \qquad \iiint_V dV\boxed{\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)\right|^2\right]}=W(t_0)=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

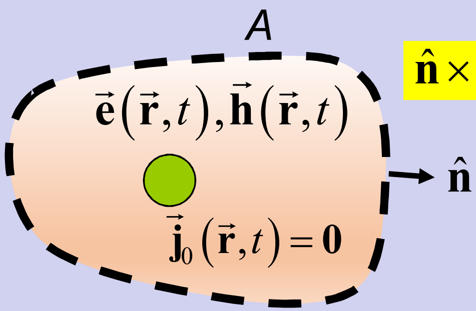
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=0 \qquad \frac{\partial}{\partial t}W(t)+P_j(t)=0$$

$$\iiint_V dV\sigma\left|\vec{\mathbf{e}}\right|^2=P_j(t)\geq 0 \qquad \iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right]=W(t)\geq 0 \qquad \iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)\right|^2\right]=W(t_0)=0$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\frac{\partial}{\partial t}W(t)+P_j(t)=0 \;\Rightarrow\; \frac{\partial}{\partial t}W(t)=-P_j(t) \;\Rightarrow\; \begin{array}{l} W(t_0)=0 \\ \frac{\partial}{\partial t}W(t)\le 0 \\ W(t)\ge 0 \end{array} \;\Rightarrow\; W(t)=0 \;\Rightarrow\; \begin{array}{l} \vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0} \\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0} \end{array}$$

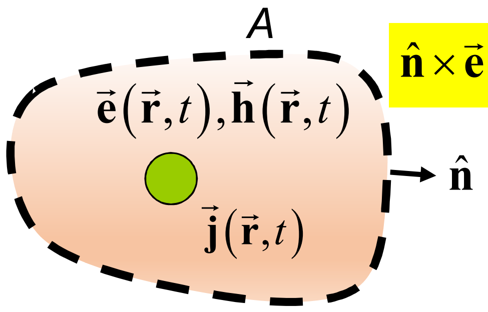
cvd

$$\iiint_V dV\,\sigma|\vec{\mathbf{e}}|^2=P_j(t)\ge 0$$

$$\boxed{\iiint_V dV\left[\frac{1}{2}\mu|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)|^2\right]}=\boxed{W(t)\ge 0}$$

$$\iiint_V dV\left[\frac{1}{2}\mu|\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)|^2\right]=\boxed{W(t_0)=0}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $\left(\vec{\mathbf{e}},\vec{\mathbf{h}}\right)$

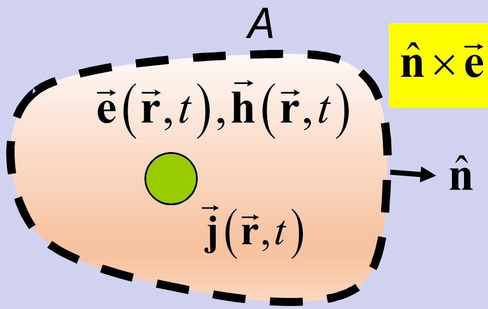
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution= 0

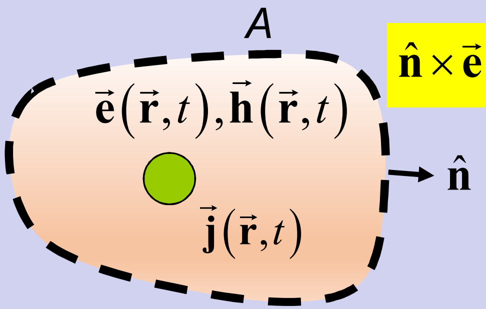
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}\ \left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{h}}\right)$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution= 0

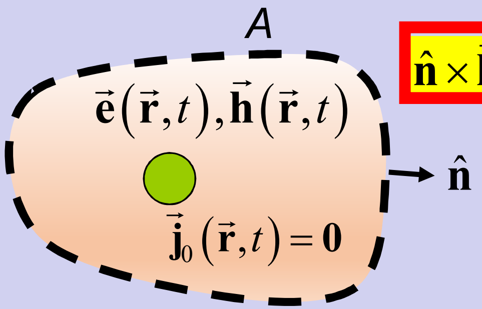
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)=0\ \text{on the boundary}$$

Uniqueness (TD-Interior Problem)



$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}=\mathbf{0}}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

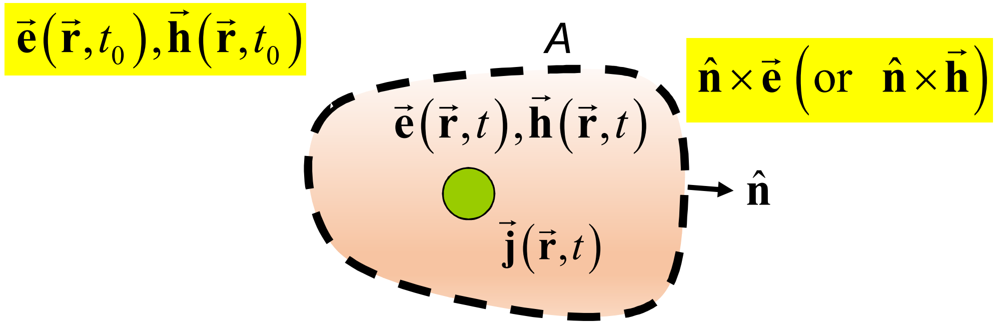
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\quad\text{on the boundary}}$

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\oiint_A dA\boxed{\left[\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\times\hat{\mathbf{n}}\right]}\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Interior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

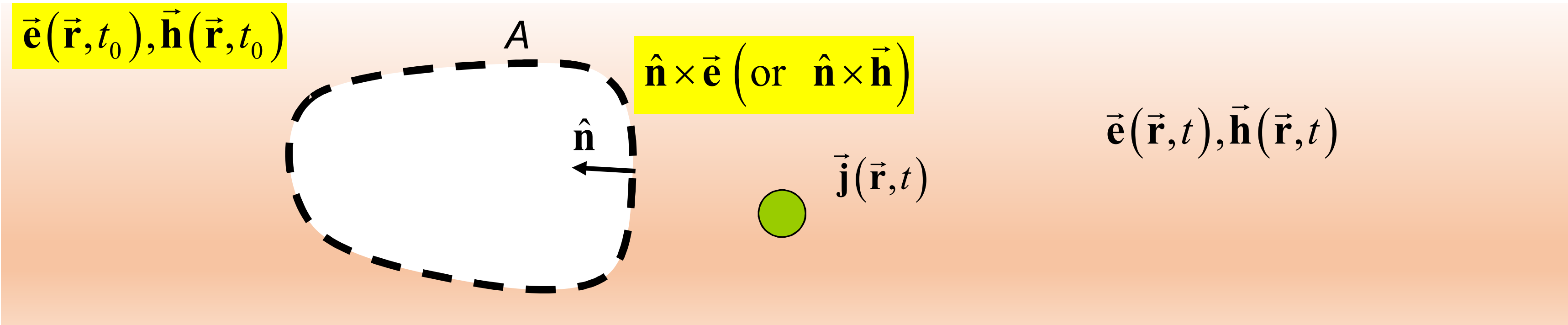
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

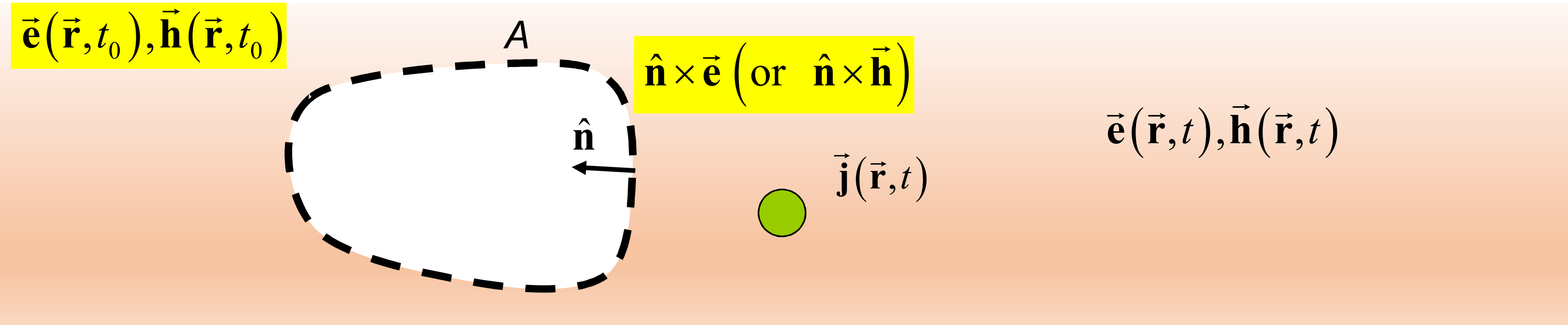
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Exterior Problem)



Source distribution: $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{e}}_2(\vec{\mathbf{r}},t),\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)$$

$$\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)\ \text{on the boundary}$$

Field difference: source distribution= 0

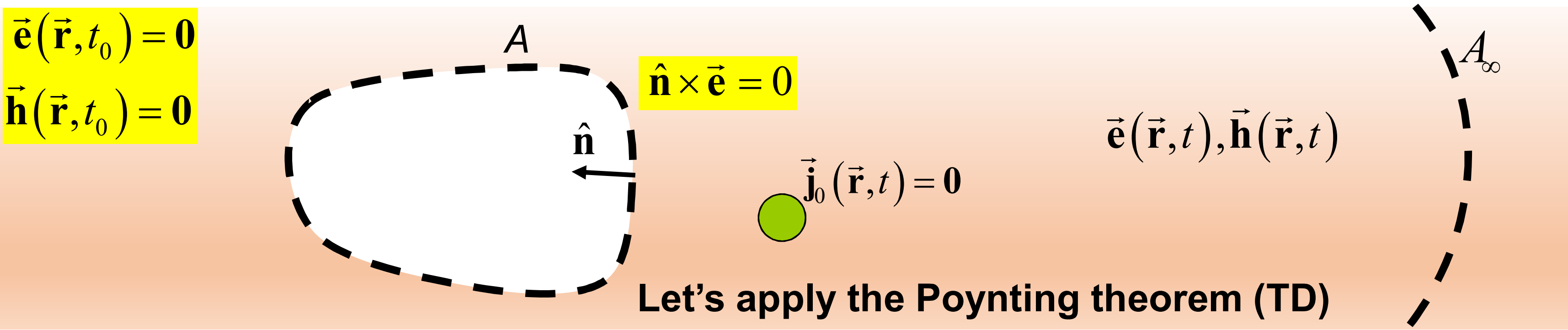
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t) \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t_0)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t_0)=0$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\hat{\mathbf{n}}\times\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)=0\ \ \text{on the boundary}$$

Uniqueness (TD-Exterior Problem)



Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

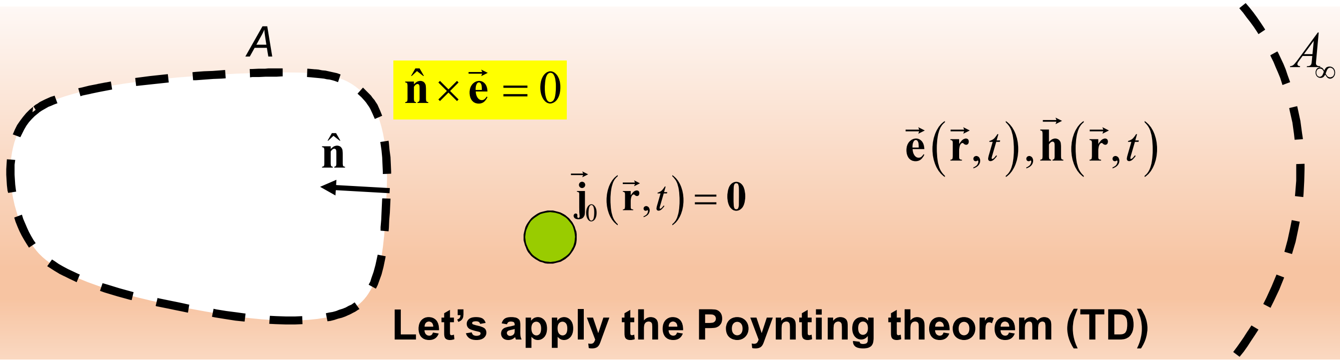
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

Uniqueness (TD-Exterior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$



$\hat{\mathbf{n}}\times\vec{\mathbf{e}}=0$

$\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)=\mathbf{0}$

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t),\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

Let's apply the Poynting theorem (TD)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

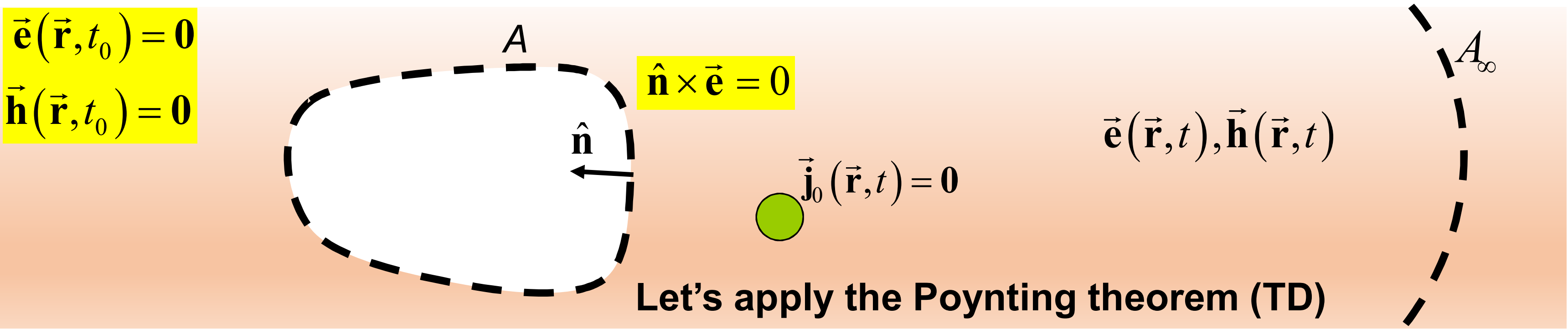
$\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\ \text{ on the boundary}}$

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\oiint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\boxed{\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]}\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (TD-Exterior Problem)



Medium

- Linear
- Isotropic
- Space-Nondispersive
- Time-Nondispersive
- Time-invariant

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ = 0

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0} \qquad \vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$$

$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\oiint_{A_\infty} dA_\infty\,\cancel{\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

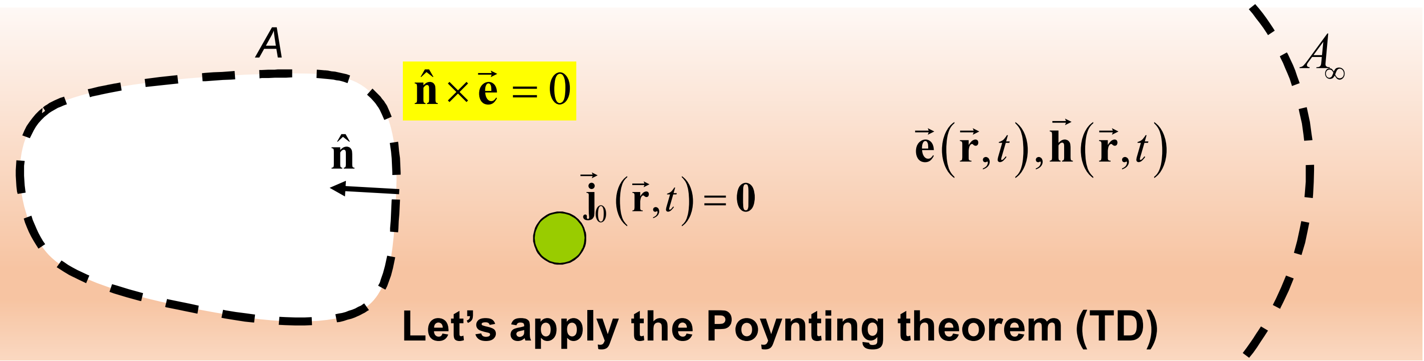
$$\oiint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=0$$

A_∞ is a large sphere whose radius $R>ct$, c being the speed of the light

Uniqueness (TD-Exterior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$



Let's apply the Poynting theorem (TD)

Medium
- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Nondispersive**
- **Time-invariant**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{e}}_2(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}_1(\vec{\mathbf{r}},t)-\vec{\mathbf{h}}_2(\vec{\mathbf{r}},t)$$

Source distribution $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)$ **= 0**

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

$\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)=\mathbf{0}$

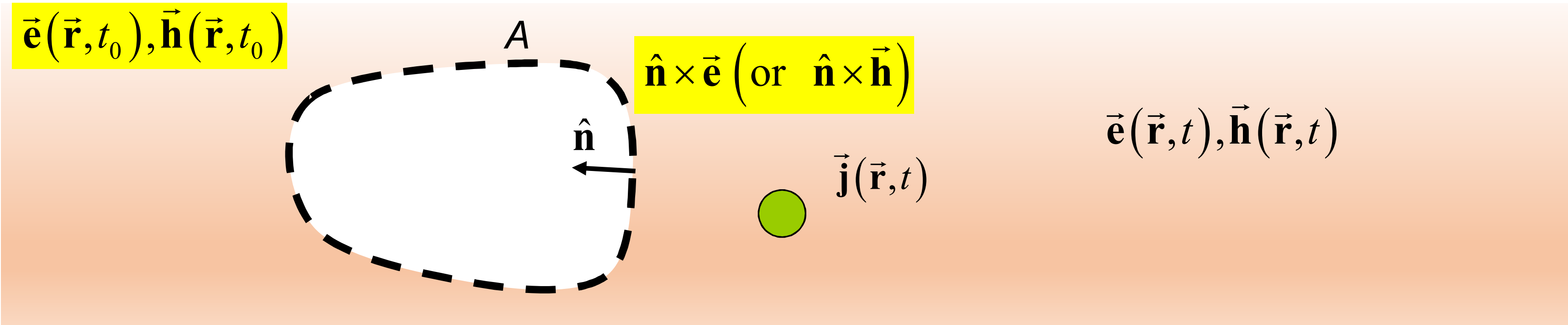
$\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}$ on the boundary

$$\cancel{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\cancel{\oint\!\!\!\oint_{A_\infty} dA_\infty\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right]+\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2=-\cancel{\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}$$

$$\Rightarrow\quad \begin{aligned}&W(t_0)=0\\ &\frac{\partial}{\partial t}W(t)\le 0\\ &W(t)\ge 0\end{aligned}\quad\Rightarrow\quad \begin{aligned}&\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\mathbf{0}\\ &\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\mathbf{0}\end{aligned}$$

cvd

Uniqueness (TD-Exterior Problem)



I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

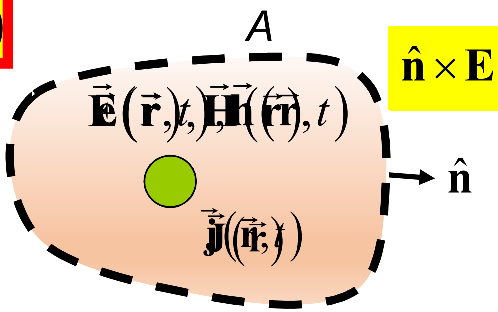
III Consider the values of the electromagnetic field everywhere in **the infinite volume outside** the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **infinite volume V outside** the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (TD-Interior Problem)

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$



$\hat{\mathbf{n}}\times\mathbf{E}\ (\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{H}})$

I Consider a source distribution $\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$ with its associated electromagnetic field $(\vec{\mathbf{e}},\vec{\mathbf{h}})$

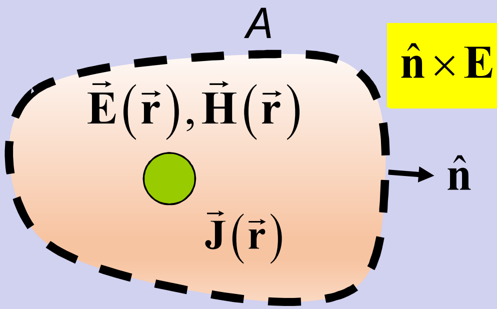
II Consider a (smooth) surface *A* with an everywhere defined unit normal $\hat{\mathbf{n}}$

III Consider the values of the electromagnetic field everywhere in **the finite volume V** bounded by the surface A **at the initial time**; that is, consider $\vec{\mathbf{e}}(\vec{\mathbf{r}},t_0),\vec{\mathbf{h}}(\vec{\mathbf{r}},t_0)$

IV Consider the values of the tangential component of the electric (**or** magnetic) field upon the surface A at any time after the initial one; that is, consider $\hat{\mathbf{n}}\times\vec{\mathbf{e}}$ (**or** $\hat{\mathbf{n}}\times\vec{\mathbf{h}}$) **on the boundary at any time**

The Uniqueness Theorem states that the electromagnetic field produced by the source in (I) within the **finite volume V bounded** by the surface A in (II), enforcing **the initial condition** in (III) and **the boundary condition** in (IV) is unique.

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}}\times\mathbf{E}\left(\text{or}\ \ \hat{\mathbf{n}}\times\vec{\mathbf{H}}\right)$

Source distribution: $\vec{\mathbf{J}}(\vec{\mathbf{r}})$

$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}),\vec{\mathbf{H}}_1(\vec{\mathbf{r}})$ $\quad$ $\vec{\mathbf{E}}_2(\vec{\mathbf{r}}),\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$

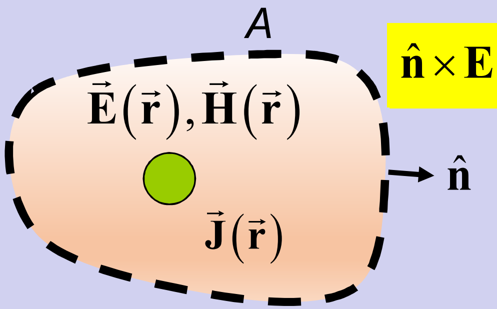
$$\hat{\mathbf{n}}\times\vec{\mathbf{E}}_1(\vec{\mathbf{r}})=\hat{\mathbf{n}}\times\vec{\mathbf{E}}_2(\vec{\mathbf{r}})\ \text{on the boundary}$$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}})=\vec{\mathbf{H}}_1(\vec{\mathbf{r}})-\vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\hat{\mathbf{n}}\times\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\hat{\mathbf{n}}\times\vec{\mathbf{E}}_2(\vec{\mathbf{r}})=0\quad \text{on the boundary}$$

Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}} \times \mathbf{E} \left(\text{or } \ \hat{\mathbf{n}} \times \vec{\mathbf{H}}\right)$

Field difference: source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) \qquad \vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \hat{\mathbf{n}} \times \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \hat{\mathbf{n}} \times \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = 0 \quad \text{on the boundary}$$

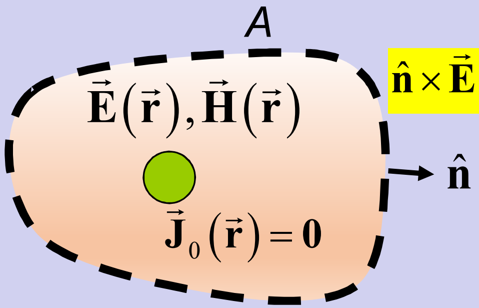
Uniqueness (PD-Interior Problem)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

Medium

- **Linear**
- **Isotropic**
- **Space-Nondispersive**
- **Time-Dispersive**
- **Time-invariant**

$$\vec{\mathbf{E}}(\vec{\mathbf{r}})=\vec{\mathbf{E}}_1(\vec{\mathbf{r}})-\vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

$\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=\mathbf{0}$ on the boundary

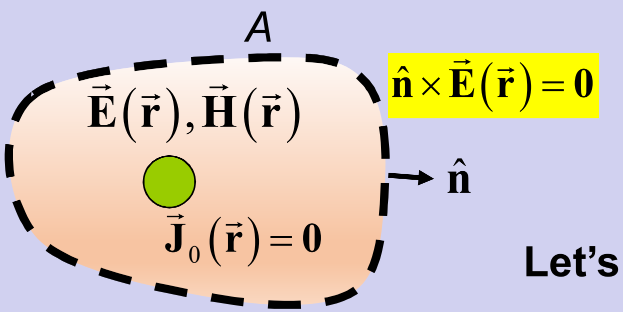
$$\cancel{\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}}+\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}+2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2-\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=\iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]\cdot\hat{\mathbf{n}}\right\}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\boxed{\hat{\mathbf{n}}\times\vec{\mathbf{E}}(\vec{\mathbf{r}})}\right]\cdot\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}=\mathrm{Re}\left\{\oiint_A dA\left[\frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\times\hat{\mathbf{n}}\right]\cdot\vec{\mathbf{E}}(\vec{\mathbf{r}})\right\}=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Uniqueness (PD-Interior Problem)



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Medium
- **Linear**
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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{E}}_2(\vec{\mathbf{r}})$$
$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}_1(\vec{\mathbf{r}}) - \vec{\mathbf{H}}_2(\vec{\mathbf{r}})$$

Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}$

$$\boxed{\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0} \quad \text{on the boundary}}$$

$$\cancel{\oiint_A dA\,\vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}} + \iiint_V dV \left[\frac{1}{2}\omega_0\mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0\varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right] = \iiint_V dV \left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\}\right]$$

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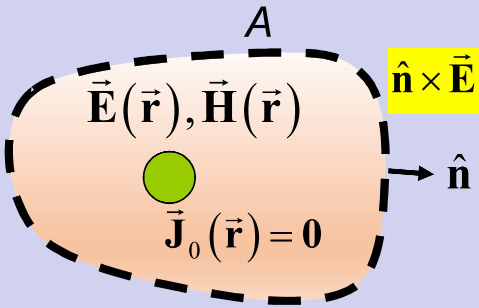
Uniqueness (PD-Interior Problem)



$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$

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$\boxed{\textbf{Source distribution } \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) = \mathbf{0}}$

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$ on the boundary

$$\cancel{\oiint_A dA\, \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}} + \iiint_V dV \left[\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = \cancel{\iiint_V dV \left[-\frac{1}{2} \mathrm{Re}\left\{ \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \boxed{\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})} \right\} \right]}$$

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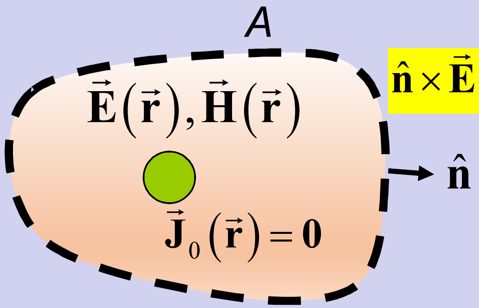

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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

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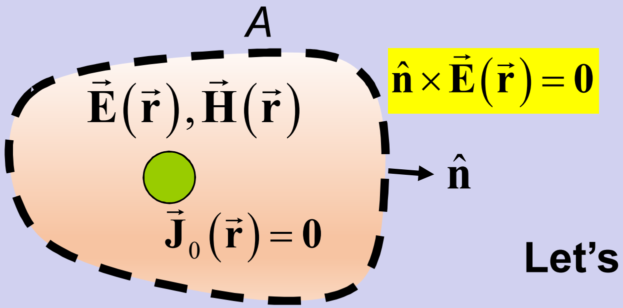
$$\iiint_V dV \left[\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 + \frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = 0$$

$$2\omega_0 \iiint_V dV \left[\frac{1}{4}\mu_1 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 - \frac{1}{4}\varepsilon_1 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 \right] = 0$$

Uniqueness (PD-Interior Problem)



Let's apply the Poynting theorem (PD)

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$$\iiint_V dV \frac{1}{4}\mu_1 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \iiint_V dV \frac{1}{4}\varepsilon_1 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2$$

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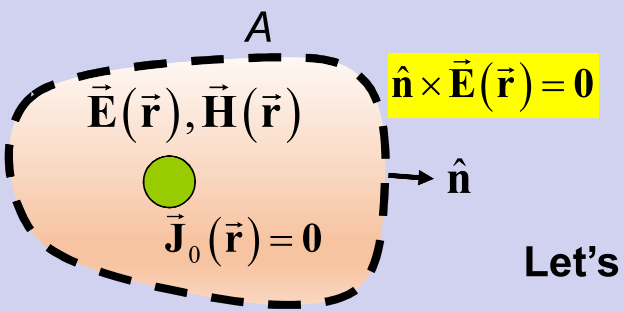

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$$\iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2+\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right]=0$$

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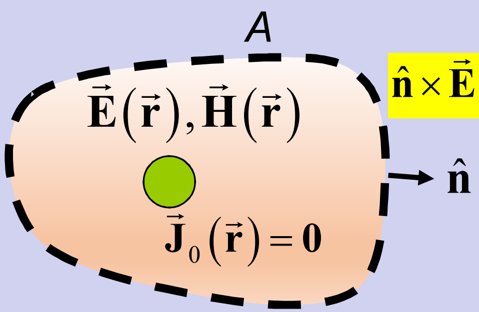

$\hat{\mathbf{n}} \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = \mathbf{0}$

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Source distribution $\vec{\mathbf{J}}_0(\vec{\mathbf{r}})$ **= 0**

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$$\iiint_V dV \left[\cancel{\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2} + \cancel{\frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2} + \cancel{\frac{1}{2}\sigma \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2} \right] = 0$$

$$\boxed{\iiint_V dV \frac{1}{4}\mu_1 \left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \iiint_V dV \frac{1}{4}\varepsilon_1 \left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2}$$

$$\boxed{\begin{cases} \varepsilon_2 = 0 \\ \mu_2 = 0 \end{cases}} + \boxed{\text{No Homic losses} \quad \sigma = 0}$$

Uniqueness is not ensured anymore!