Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2019-2020 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$ **Poynting vector** $$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

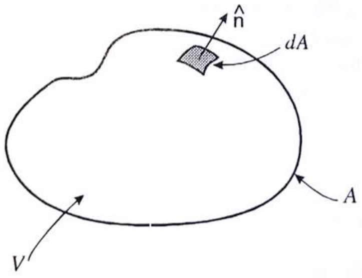
$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)+\frac{\partial}{\partial t}w(\vec{\mathbf{r}},t)+p_j(\vec{\mathbf{r}},t)=p_0(\vec{\mathbf{r}},t)$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}+\frac{\partial}{\partial t}\iiint_V dV\,w(\vec{\mathbf{r}},t)+\iiint_V dV p_j(\vec{\mathbf{r}},t)=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

Electromagnetic power flux

$$P_S(t)+\frac{\partial}{\partial t}W(t)+P_j(t)=P_0(t)$$

Hypotheses on the medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**



$$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2+\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2$$ **Energy density of the e.m. field** ➡ $$\iiint_V dV\,w(\vec{\mathbf{r}},t)=W(t)$$ **Energy of the e.m. field**

$$p_j(\vec{\mathbf{r}},t)=\sigma\left|\vec{\mathbf{e}}\right|^2$$ **Power density dissipated in the conducting medium** ➡ $$\iiint_V dV p_j(\vec{\mathbf{r}},t)=P_j(t)$$ **Power dissipated in the conducting medium**

$$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$ **Power density delivered by the sources to the field** ➡ $$\iiint_V dV\,p_0(\vec{\mathbf{r}},t)=P_0(t)$$ **Power delivered by the sources to the field**

Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$ **Poynting vector** $$[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$$

$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

Electromagnetic power flux

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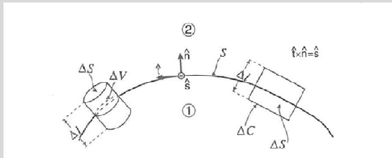
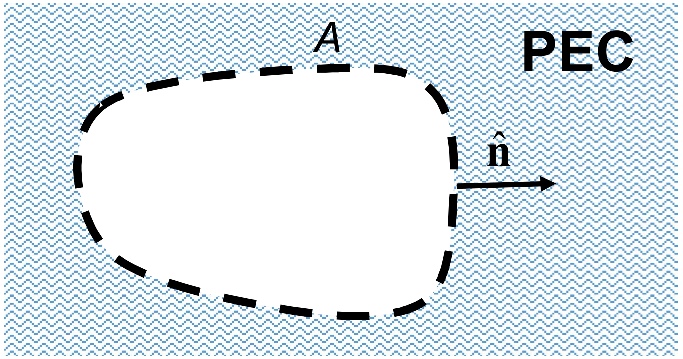
MEMO: Fields at boundaries

One example: the medium 1 is a PEC

$$\hat{\mathbf{n}}\times(\vec{\mathbf{e}}_2-\vec{\mathbf{e}}_1)=\mathbf{0}$$
$$\hat{\mathbf{n}}\times(\vec{\mathbf{h}}_2-\vec{\mathbf{h}}_1)=\vec{\mathbf{j}}_s$$
$$(\vec{\mathbf{d}}_2-\vec{\mathbf{d}}_1)\cdot\hat{\mathbf{n}}=\rho_s$$
$$(\vec{\mathbf{b}}_2-\vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}}=0$$
$$(\vec{\mathbf{j}}_2-\vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}}=-\frac{\partial\rho_s}{\partial t}$$



$\vec{\mathbf{e}}_1=\mathbf{0}$	$\hat{\mathbf{n}}\times\vec{\mathbf{e}}_2=\mathbf{0}$
$\vec{\mathbf{h}}_1=\mathbf{0}$	$\hat{\mathbf{n}}\times\vec{\mathbf{h}}_2=\vec{\mathbf{j}}_s$
$\vec{\mathbf{d}}_1=\mathbf{0}$	$\vec{\mathbf{d}}_2\cdot\hat{\mathbf{n}}=\rho_s$
$\vec{\mathbf{b}}_1=\mathbf{0}$	$\vec{\mathbf{b}}_2\cdot\hat{\mathbf{n}}=0$



$$\oiint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right]\cdot\hat{\mathbf{n}}=\oiint_A dA\left[\hat{\mathbf{n}}\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right]\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=0$$

$$\vec{\mathbf{A}}\cdot\left[\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right]=\vec{\mathbf{C}}\cdot\left[\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right]=\vec{\mathbf{B}}\cdot\left[\vec{\mathbf{C}}\times\vec{\mathbf{A}}\right]$$

Poynting theorem (PD)

$$\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]$$

Poynting vector

$$\left[\vec{\mathbf{S}}\right]: \frac{Watt}{m^2}$$

$$\nabla\cdot\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\nabla\cdot\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right] = \frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}})\right] - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right] =$$

$$= \frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}})\right] - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}})\right]^* =$$

$$= \frac{1}{2}\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\cdot\left[-j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}})\right] - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})\right]^*$$

$$= -\frac{1}{2}j\omega_0\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2}j\omega_0\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

$$\nabla\cdot\left[\vec{\mathbf{A}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right] = \vec{\mathbf{B}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right] - \vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]$$

Phasor domain

$$\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}})$$

$$\nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0$$

Poynting theorem (PD)

$$\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]$$

Poynting vector

$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^2}$$

$$\nabla \cdot \mathbf{S}(\vec{\mathbf{r}}) = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^*(\vec{\mathbf{r}}) \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) + \frac{1}{2} j\omega_0 \vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{D}}^*(\vec{\mathbf{r}}) - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}^*(\vec{\mathbf{r}}) - \frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})$$

Hypotheses on the medium (TD)

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Local (TND & SND)**

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t) \\ \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \end{cases}$$

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$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma : real \end{cases}$$

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- **Time-Dispersive**
- **Space-Nondispersive**

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$$\nabla\cdot\mathbf{S} = -\frac{1}{2}j\omega_0\vec{\mathbf{H}}^*\cdot\vec{\mathbf{B}} + \frac{1}{2}j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^* - \frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^* - \frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*$$

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$$\begin{cases}\varepsilon = \varepsilon_1 - j\varepsilon_2\\ \mu = \mu_1 - j\mu_2\\ \sigma : real\end{cases}$$

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$$= -\frac{1}{2} j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2$$

$$-j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} = -j\omega_0 \vec{\mathbf{H}}^* \cdot \left[\left(\mu_1 - j\mu_2\right)\vec{\mathbf{H}}\right] = -j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2$$

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$$= -\frac{1}{2} j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 \left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}\right|^2$$

$$j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* = j\omega_0 \vec{\mathbf{E}} \cdot \left[(\varepsilon_1 - j\varepsilon_2)\vec{\mathbf{E}}\right]^* = j\omega_0 \vec{\mathbf{E}} \cdot \left[(\varepsilon_1 + j\varepsilon_2)\vec{\mathbf{E}}^*\right] = j\omega_0 \varepsilon_1 \left|\vec{\mathbf{E}}\right|^2 - \omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}\right|^2$$

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$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : real \end{cases}$$

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$\mathbf{S}(\vec{\mathbf{r}})=\frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]$ **Poynting vector** $\left[\vec{\mathbf{S}}\right]:\frac{Watt}{m^{2}}$

$$\nabla\cdot\mathbf{S}=-\frac{1}{2}j\omega_0\vec{\mathbf{H}}^{*}\cdot\vec{\mathbf{B}}+\frac{1}{2}j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^{*}-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^{*}-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}$$

$$=-\frac{1}{2}j\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2-\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2+\frac{1}{2}j\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}$$

$$-\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^{*}=-\vec{\mathbf{E}}\cdot\sigma\vec{\mathbf{E}}^{*}=-\sigma\left|\vec{\mathbf{E}}\right|^2$$

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$$\mathbf{S}(\vec{\mathbf{r}}) = \mathbf{S}_1(\vec{\mathbf{r}}) + j\mathbf{S}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \mathbf{S} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2}\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2}\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$= -\frac{1}{2} j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2} j\omega_0 \varepsilon_1 \left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

$$\nabla \cdot \mathbf{S}(\vec{\mathbf{r}}) = \nabla \cdot \mathbf{S}_1(\vec{\mathbf{r}}) + j\nabla \cdot \mathbf{S}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \mathbf{S}_1 = -\frac{1}{2}\omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0 \varepsilon_2 \left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$

$$\nabla \cdot \mathbf{S}_2 = -\frac{1}{2}\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0 \varepsilon_1 \left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$

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$$\nabla \cdot \mathbf{S}_2 + \frac{1}{2}\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0 \varepsilon_1 \left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$

$$\nabla \cdot \mathbf{S}(\vec{\mathbf{r}}) = \nabla \cdot \mathbf{S}_1(\vec{\mathbf{r}}) + j\nabla \cdot \mathbf{S}_2(\vec{\mathbf{r}})$$

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$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : real \end{cases}$$

Poynting theorem (PD)

$$\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]$$

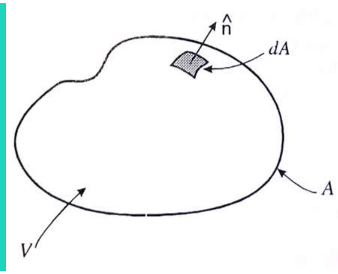
Poynting vector

$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^2}$$

$$\mathbf{S}(\vec{\mathbf{r}}) = \mathbf{S}_1(\vec{\mathbf{r}}) + j\mathbf{S}_2(\vec{\mathbf{r}})$$

$$\nabla\cdot\mathbf{S}_1 + \frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}$$

$$\nabla\cdot\mathbf{S}_2 + \frac{1}{2}\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}$$



$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{S}}_1 + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

$$\oint\!\!\oint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}} + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Time-Dispersive**
- **Space-Nondispersive**

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

$$\begin{cases}\varepsilon = \varepsilon_1 - j\varepsilon_2\\ \mu = \mu_1 - j\mu_2\\ \sigma : \ real\end{cases}$$

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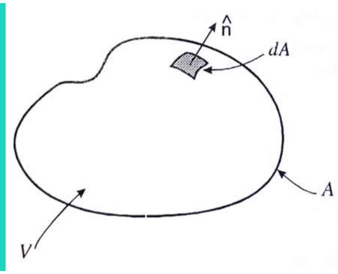
$$\nabla \cdot \mathbf{S}_2 + \frac{1}{2}\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$



$$\oiint_A dA\,\vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_2 \cdot \hat{\mathbf{n}} + 2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

Hypotheses on the medium (PD)

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$$\oiint_A dA\,\vec{\mathbf{S}}_2 \cdot \hat{\mathbf{n}} + 2\omega_0\iiint_V dV\left[\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

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TD

$$\oiint_A dA\, \vec{\mathbf{s}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{\partial}{\partial t} \iiint_V dV \left[\frac{1}{2}\mu \left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon \left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV\, \sigma \left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV\, \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
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$$\oint\!\!\oint_A dA\,\vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV\left[\xcancel{\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2} + \xcancel{\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2} + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}\right]$$

TD

$$\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV\,\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

- **Power flux associated to the e.m. field**
- **Time derivative of the energy of the e.m. field**
- **Power dissipated in the conducting medium**
- **Power delivered by the sources to the field**

Hypotheses on the medium (PD)

- **- Linear**
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Poynting theorem

…MEMO : phasors and time averages

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\cdot\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\times\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\mathbf{S}_1(\vec{\mathbf{r}}) = \mathrm{Re}\left\{\vec{\mathbf{S}}(\vec{\mathbf{r}})\right\} = \mathrm{Re}\left\{\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\} = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}$$

$$= \left\langle \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right\rangle = \left\langle \vec{\mathbf{s}}(\vec{\mathbf{r}},t)\right\rangle$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}} + \iiint_V dV\,\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

Time averaged power flux associated to the e.m. field

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

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TD

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

- **Power flux associated to the e.m. field**
- **Time derivative of the energy of the e.m. field**
- **Power dissipated in the conducting medium**
- **Power delivered by the sources to the field**

Poynting theorem

$$\mathrm{Re}\left\{-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\} = \left\langle -\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right\rangle$$

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Time averaged power flux associated to the e.m. field

Time averaged power delivered by the sources to the field

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

$$\begin{cases}\varepsilon = \varepsilon_1 - j\varepsilon_2\\ \mu = \mu_1 - j\mu_2\\ \sigma : real\end{cases} \qquad \begin{cases}\varepsilon_2 = 0\\ \mu_2 = 0\end{cases}$$

TD

$$\oint\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$$

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

Poynting theorem (PD)

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$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\times\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right\} \quad = \frac{1}{2}\mathrm{Re}\left\{\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{E}}^*(\vec{\mathbf{r}})\right\} \quad = \left\langle \sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right\rangle$$

$$= \left\langle \sigma\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right\rangle$$

$$\oint\!\!\oint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}} + \iiint_V dV\,\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

Time averaged power flux associated to the e.m. field

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

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Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

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Poynting theorem (PD)

$$\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]$$

$$\mathbf{S}(\vec{\mathbf{r}}) = \mathbf{S}_1(\vec{\mathbf{r}}) + j\mathbf{S}_2(\vec{\mathbf{r}})$$

Where is the energy?

Remember that the energy is a state function

$$\int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w(t) = w(t_2) - w(t_1)$$

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Time-Non Dispersive**
- **Space-Nondispersive**

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

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- **Time averaged power dissipated in the conducting medium**
- **Time averaged power delivered by the sources to the field**

TD

$$\oint\!\!\!\oint_A dA \vec{\mathbf{s}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{\partial}{\partial t}\iiint_V dV \left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV \sigma\left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV \vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

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- **Power delivered by the sources to the field**

Poynting theorem (PD)

$$\mathbf{S}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]$$

Poynting vector

$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^2}$$

$$\mathbf{S}(\vec{\mathbf{r}}) = \mathbf{S}_1(\vec{\mathbf{r}}) + j\mathbf{S}_2(\vec{\mathbf{r}})$$

$$\oiint_A dA\, \vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^{*}\right\}\right]$$

Time averaged power flux associated to the e.m. field

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

TD

$$\oiint_A dA\, \vec{\mathbf{s}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{\partial}{\partial t}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^2 + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^2\right] + \iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^2 = -\iiint_V dV\,\vec{\mathbf{j}}_0 \cdot \vec{\mathbf{e}}$$

Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

Hypotheses on the medium (PD)

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Time- Dispersive**
- **Space-Nondispersive**

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : real \end{cases}$$

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Time averaged power flux associated to the e.m. field

LOSSES
(ε_2) electric losses
(μ_2) magnetic losses

Time averaged power dissipated in the conducting medium

Time averaged power delivered by the sources to the field

$$\varepsilon_2 > 0;\ \mu_2 > 0;\ \sigma > 0$$

Dispersion and losses are related each other: a (time) dispersive medium presents losses

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…MEMO : phasors and time averages

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$
$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\cdot\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\times\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

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$$\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\vec{\mathbf{H}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{H}}(\vec{\mathbf{r}})^*\right\} = \left\langle\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2\right\rangle$$

$$\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{E}}^*(\vec{\mathbf{r}})\right\} = \left\langle\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right\rangle$$

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Time averaged magnetic energy density

Time averaged electric energy density

$$\frac{1}{4}\mu_1\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\left|\vec{\mathbf{H}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\mu\vec{\mathbf{H}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{H}}(\vec{\mathbf{r}})^*\right\} = \left\langle\frac{1}{2}\mu\left|\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right|^2\right\rangle$$

$$\frac{1}{4}\varepsilon_1\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right\} = \frac{1}{2}\mathrm{Re}\left\{\frac{1}{2}\varepsilon\vec{\mathbf{E}}(\vec{\mathbf{r}}) \cdot \vec{\mathbf{E}}^*(\vec{\mathbf{r}})\right\} = \left\langle\frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right|^2\right\rangle$$

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